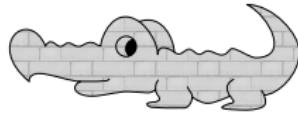


# Density functional theory and quantum Monte Carlo calculations of solid molecular hydrogen

**Jonathan Lloyd-Williams**<sup>1</sup>, Bartomeu Monserrat<sup>1</sup>,  
Pablo López Ríos<sup>1</sup>, Neil Drummond<sup>2</sup>, and Richard Needs<sup>1</sup>



UNIVERSITY OF  
CAMBRIDGE

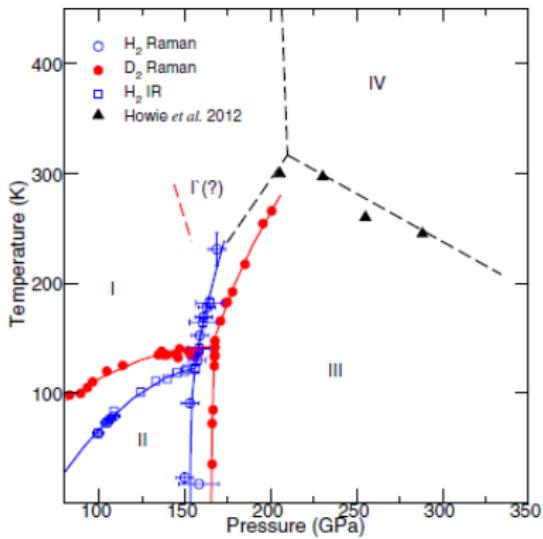
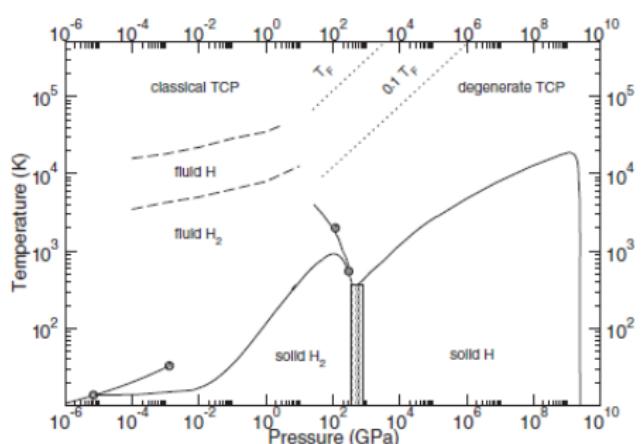


<sup>1</sup>Theory of Condensed Matter Group, Cavendish Laboratory,  
University of Cambridge, UK

<sup>2</sup>Condensed Matter Theory Group, Department of Physics,  
Lancaster University, UK

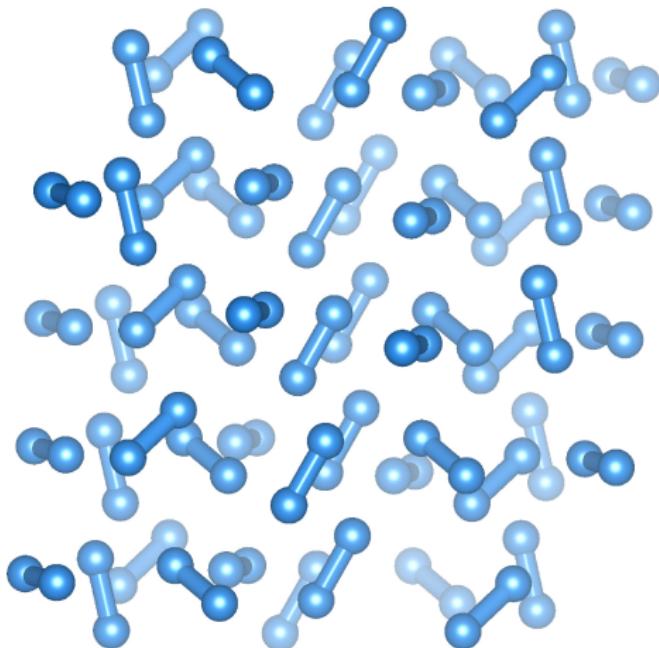
QMC in the Apuan Alps IX – Tuesday 29th July, 2014

# Hydrogen phase diagram

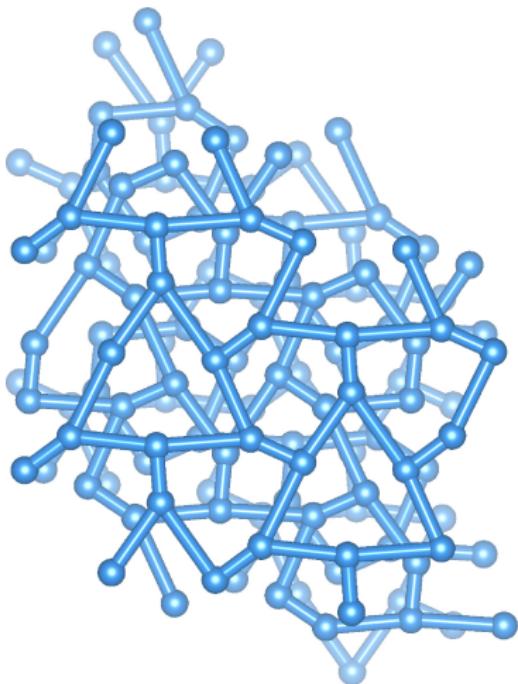
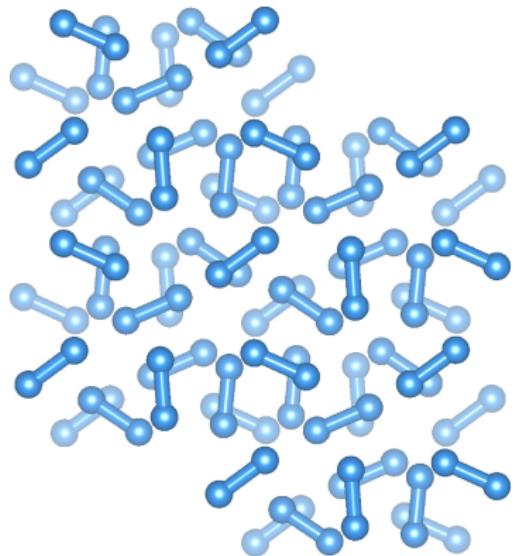


Figures from: J. M. McMahon, M. A. Morales, C. Pierleoni, and D. Ceperley, Rev. Mod. Phys. **84**, 1607 (2012).

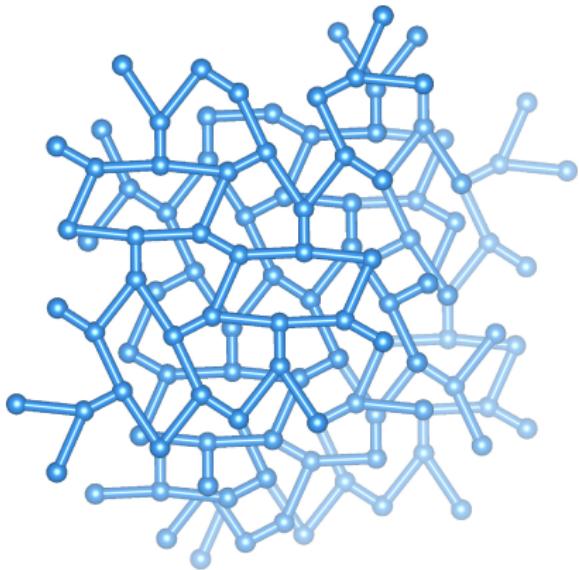
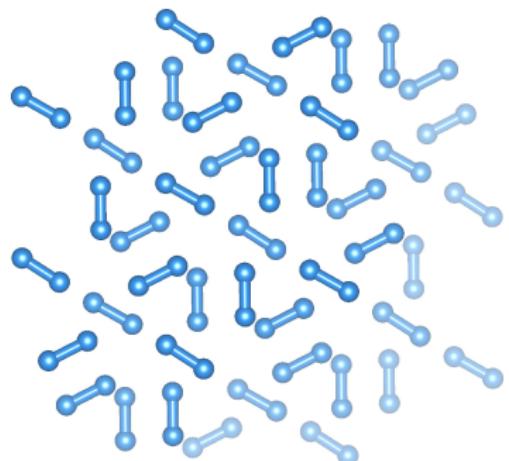
## Candidate structures - P<sub>2</sub><sub>1</sub>/c-24



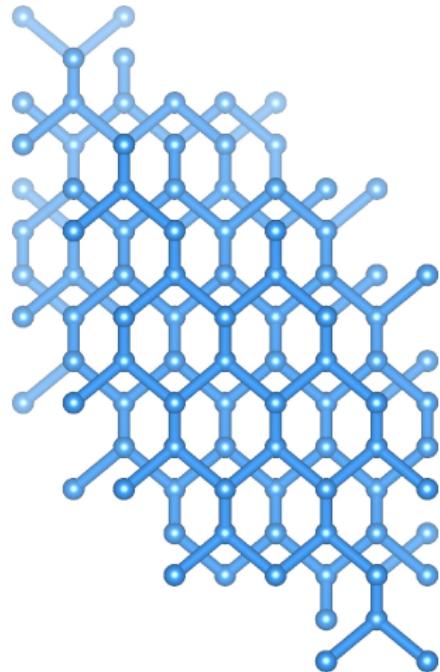
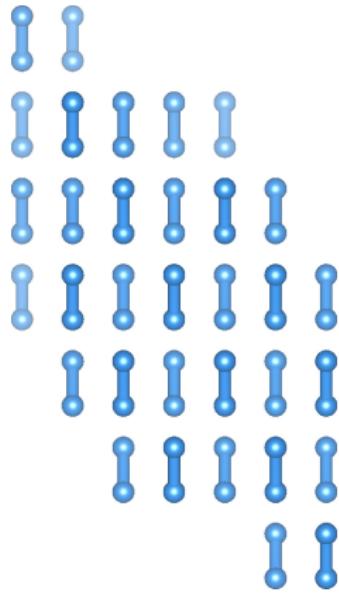
## Candidate structures - C<sub>2</sub>/c-24



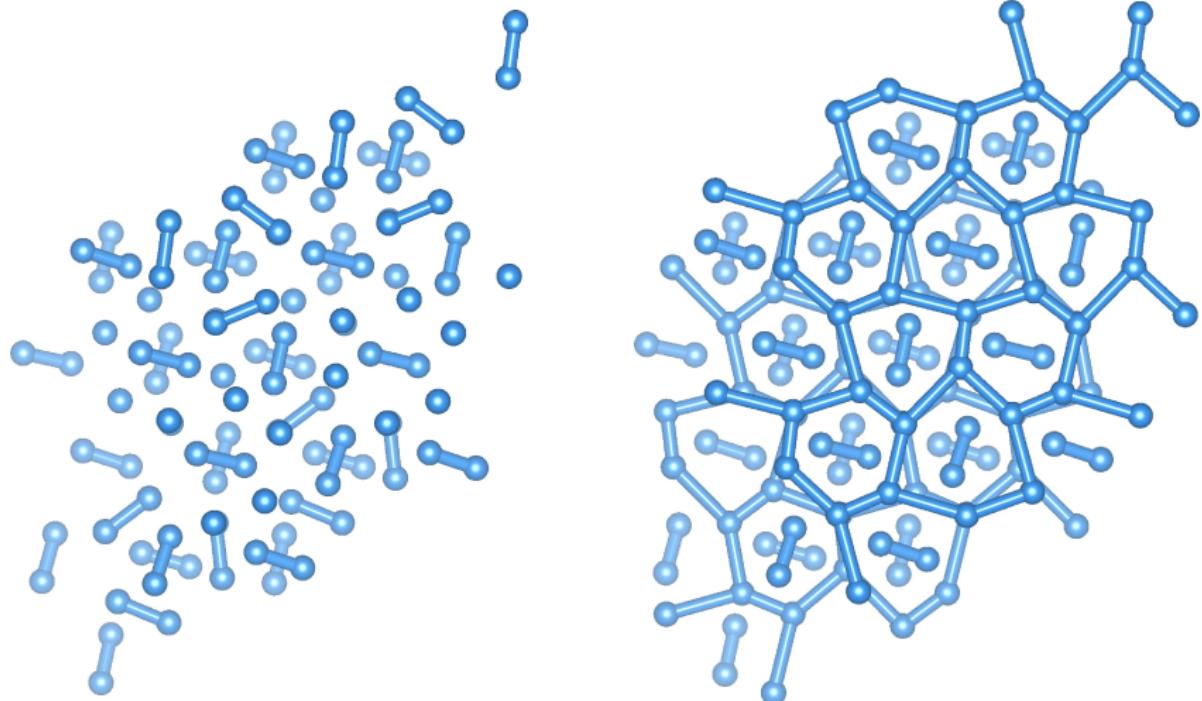
## Candidate structures - Cmca-12



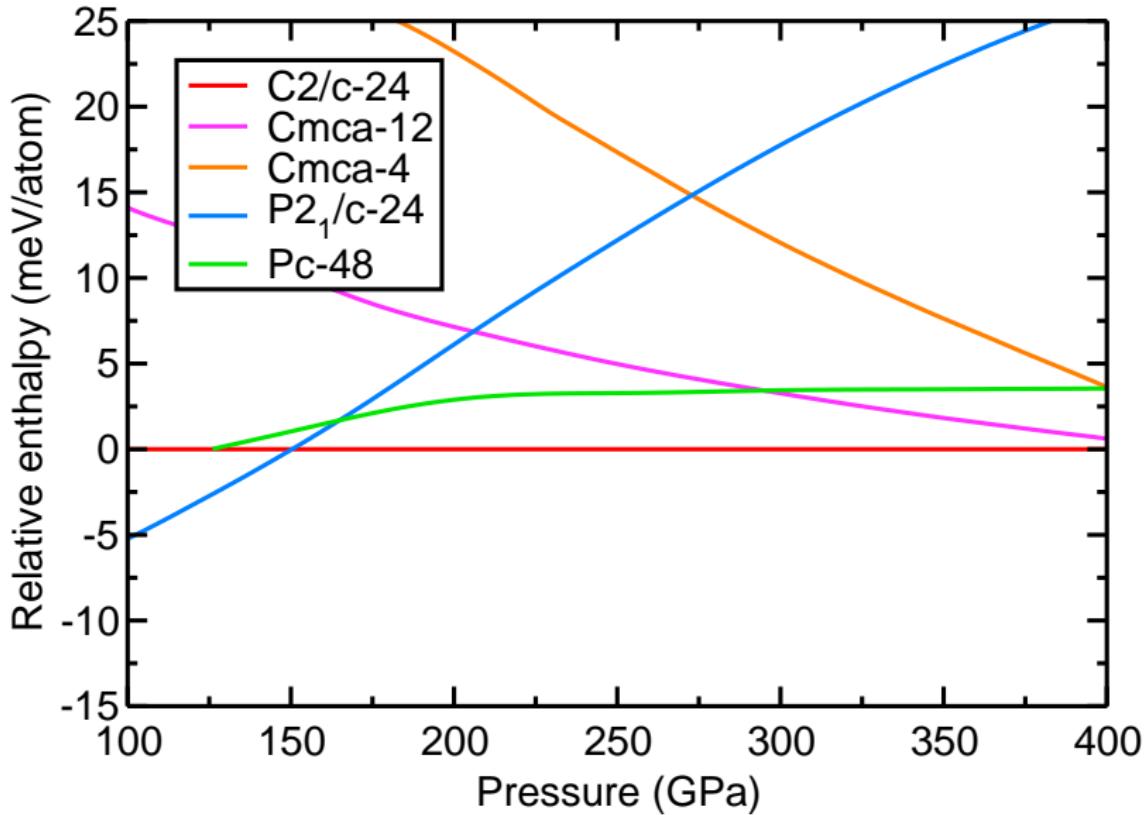
## Candidate structures - Cmca-4



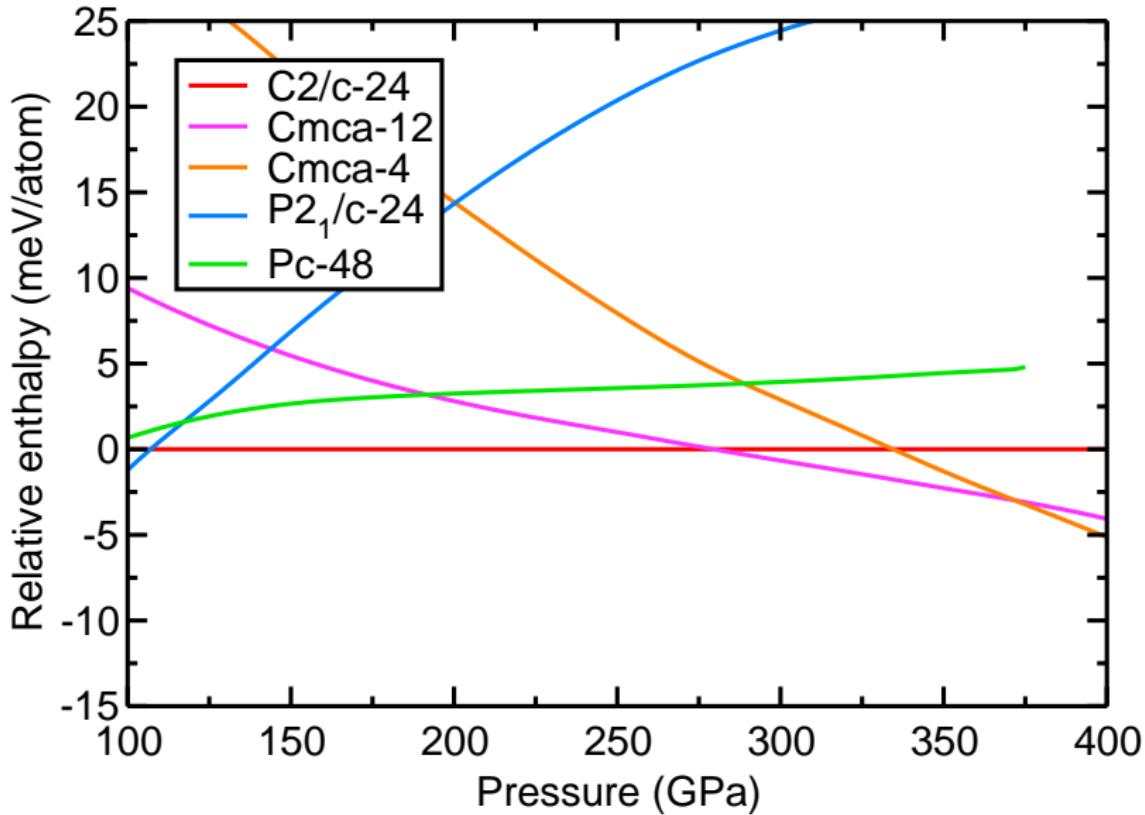
## Candidate structures - Pc-48



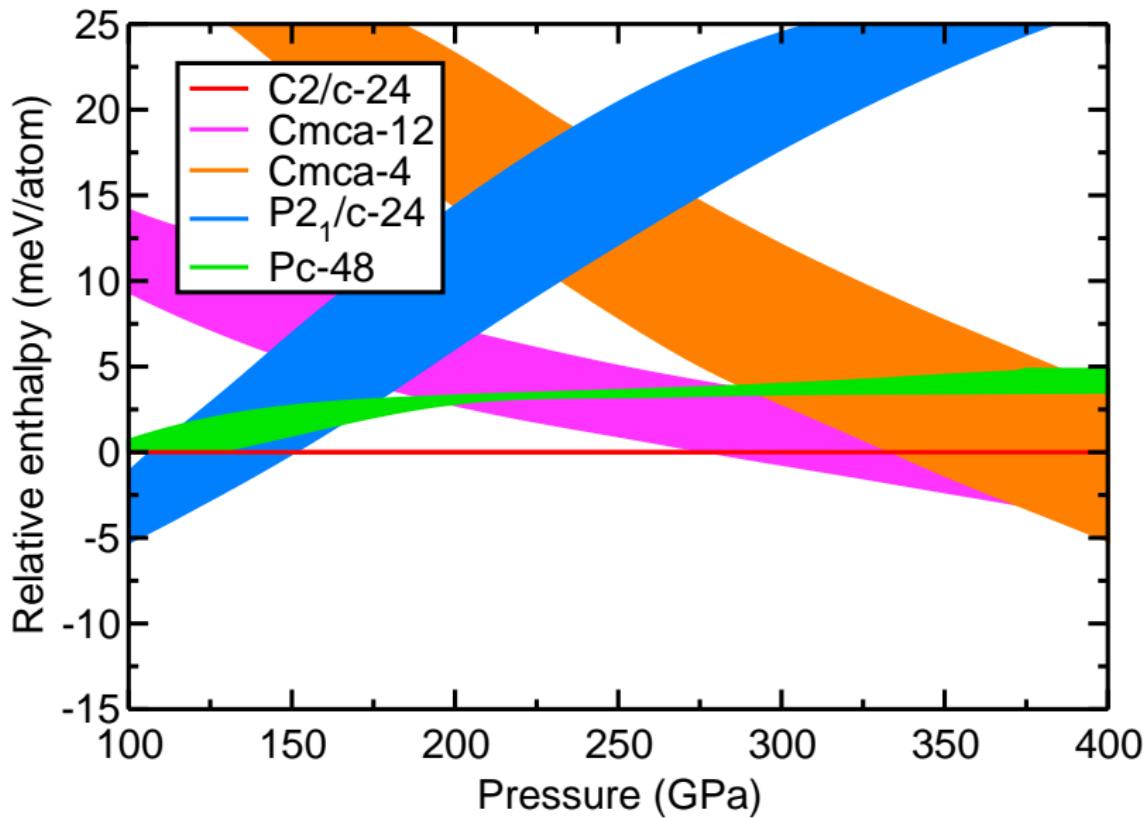
# DFT phase diagram - BLYP



# DFT phase diagram - PBE



## DFT phase diagram



## Brief digression - choice of supercell

- ▶ Superlattice basis vectors:

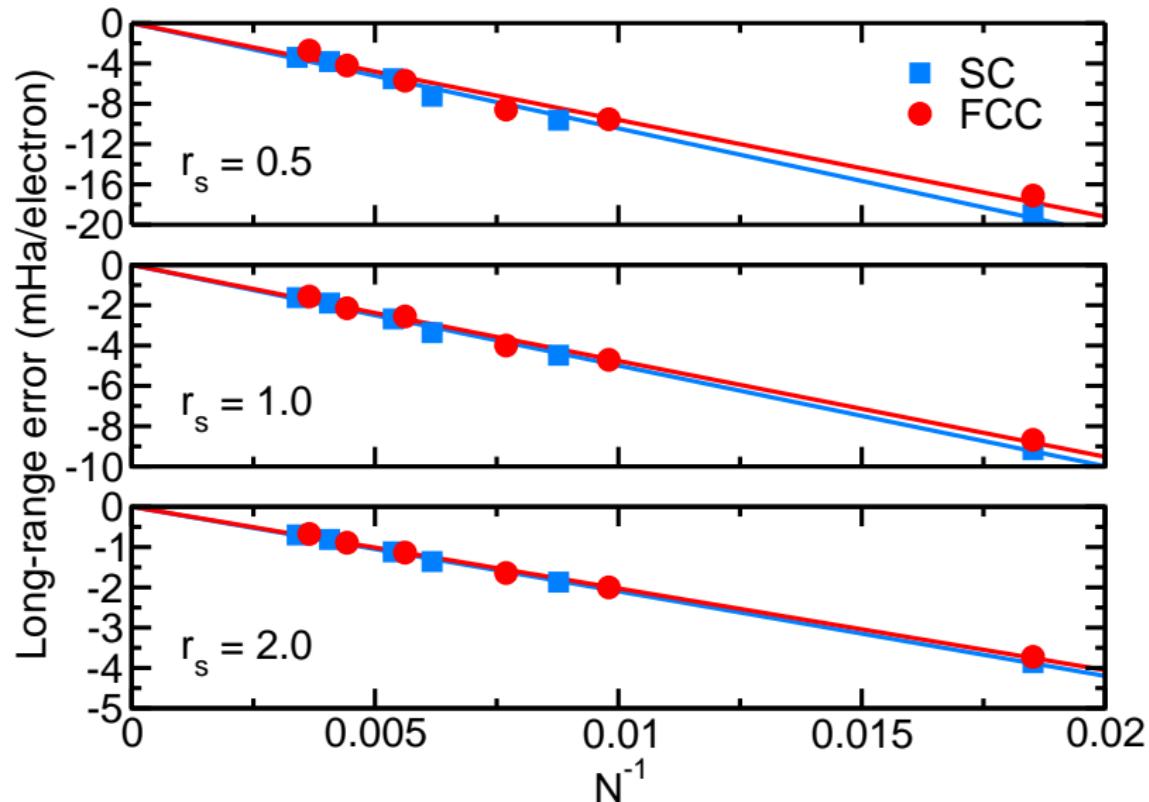
$$\begin{pmatrix} \mathbf{a}_{s_1} \\ \mathbf{a}_{s_2} \\ \mathbf{a}_{s_3} \end{pmatrix} = \begin{pmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{pmatrix} \begin{pmatrix} \mathbf{a}_{p_1} \\ \mathbf{a}_{p_2} \\ \mathbf{a}_{p_3} \end{pmatrix}$$

$$S_{ij} \in \mathbb{Z}$$

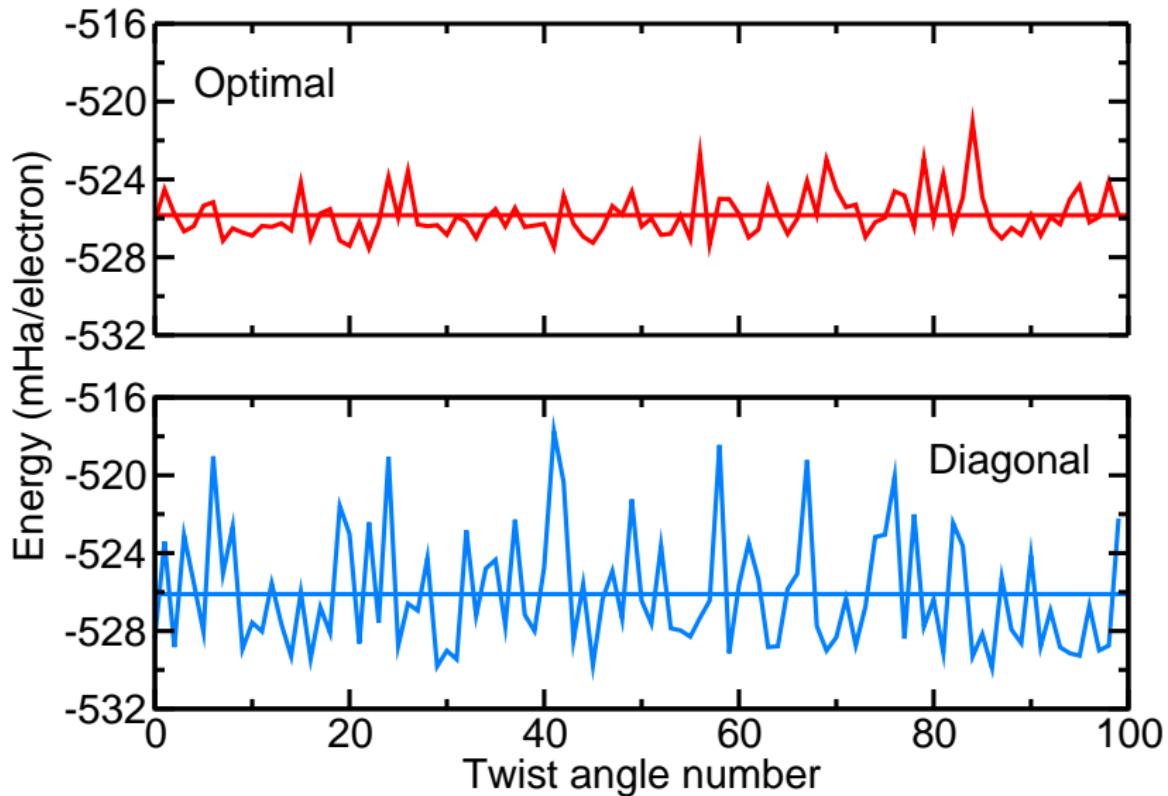
- ▶ Two different supercell matrices  $S$  and  $S'$  generate different bases for the same superlattice if  $S'$  can be reduced to  $S$  by integer row operations.
- ▶ The canonical form for such operations is the upper-triangular Hermite normal form:

$$\begin{pmatrix} a & b & d \\ 0 & c & e \\ 0 & 0 & f \end{pmatrix}, \quad 0 \leq b < c, \quad 0 \leq d, e < f.$$

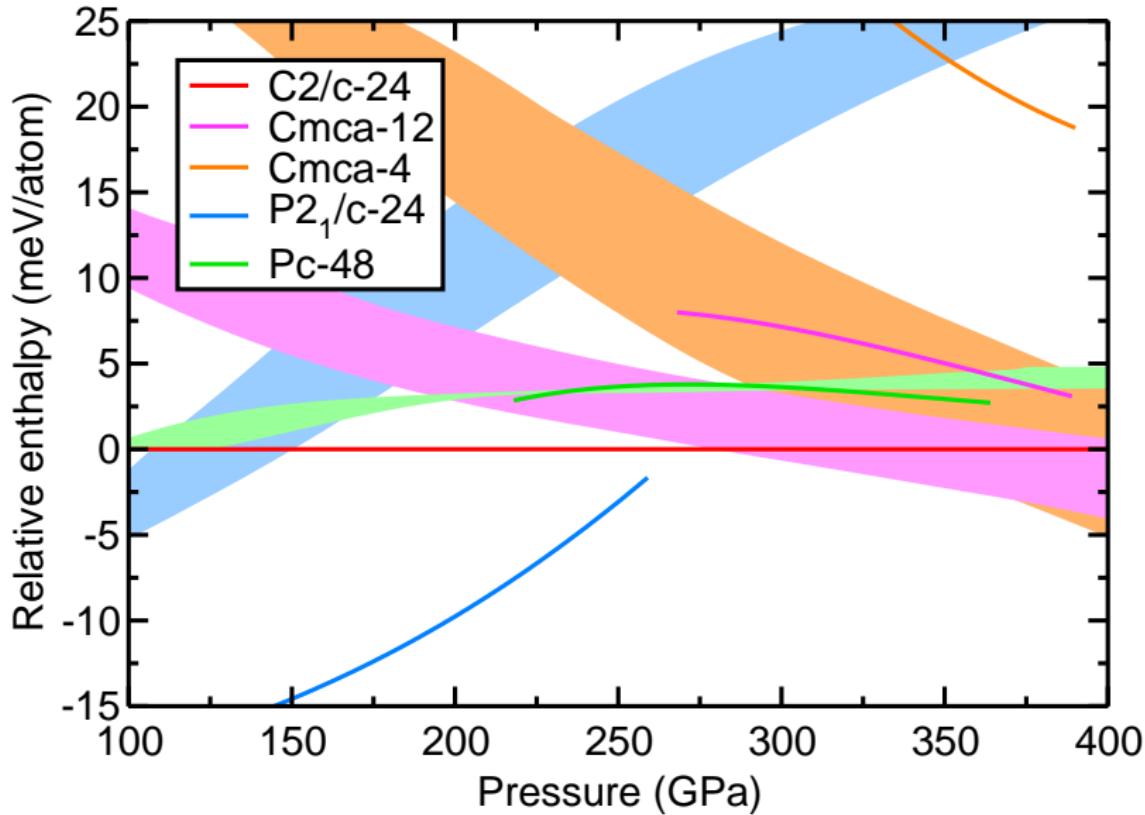
## Brief digression - long-range finite-size errors



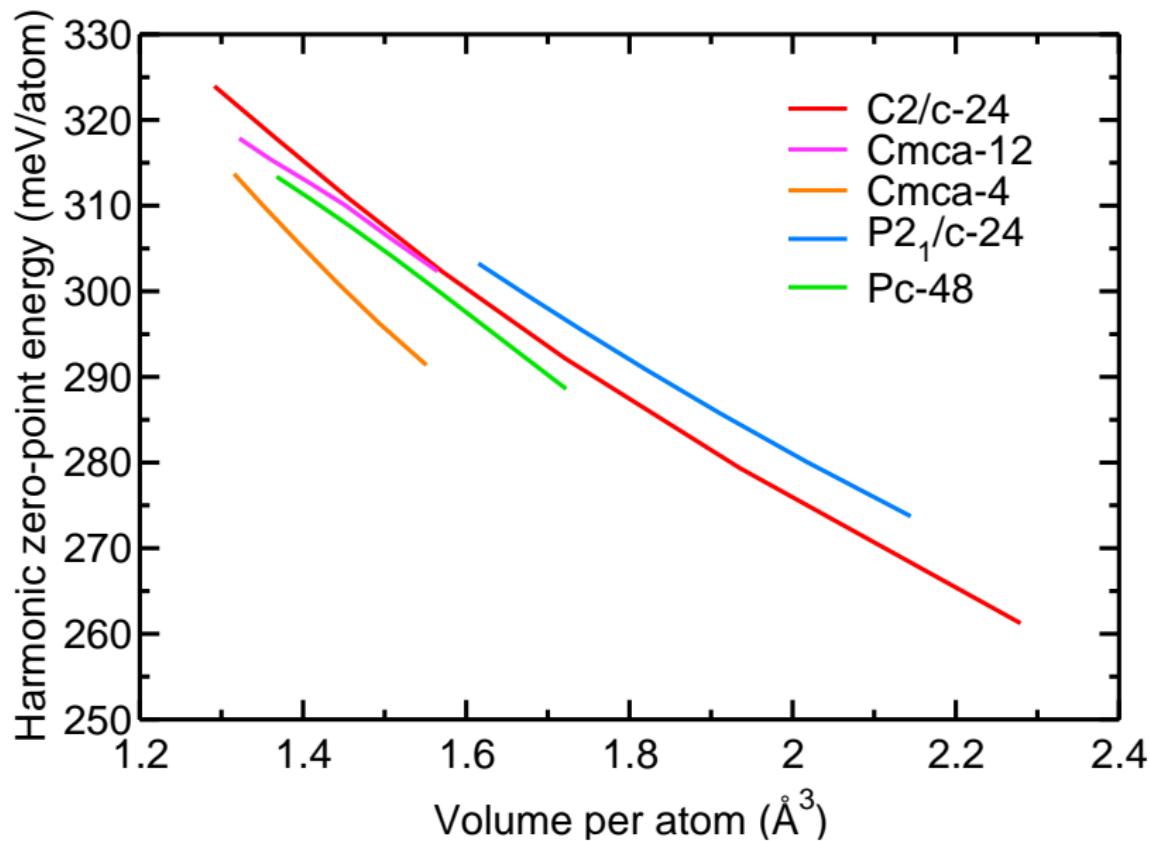
## Brief digression - single-particle fluctuations



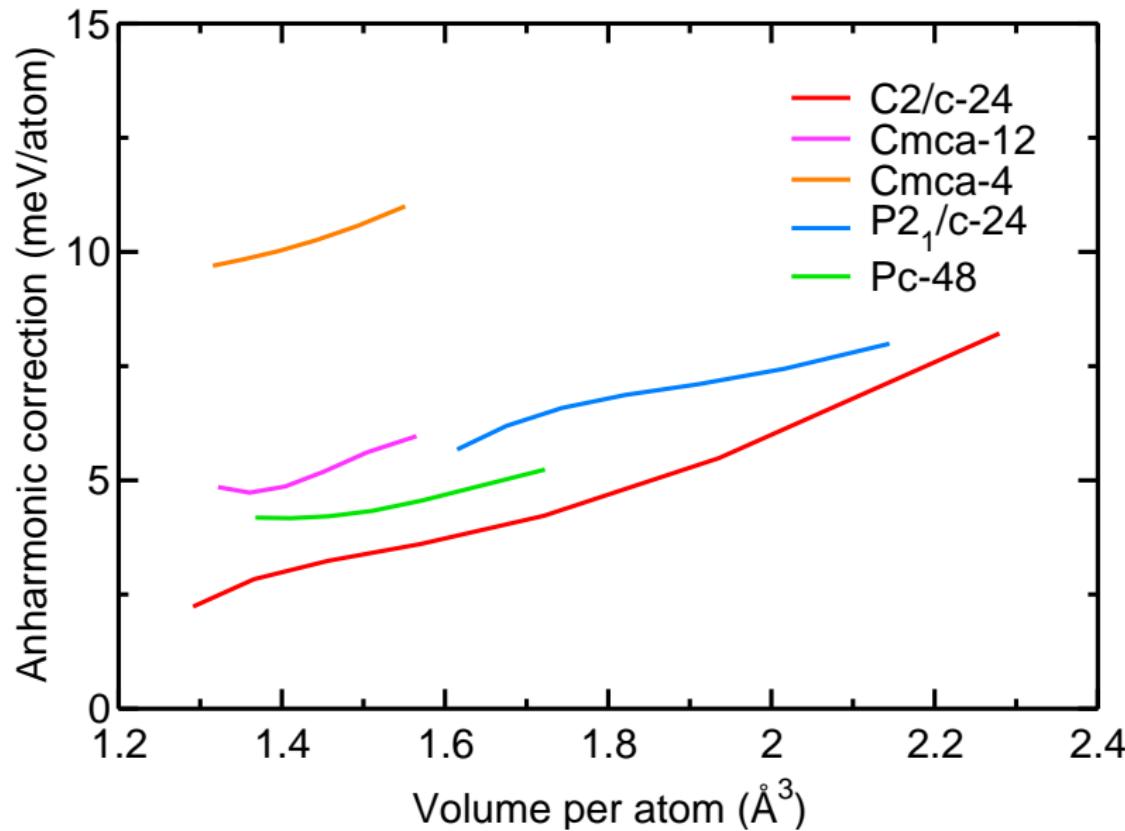
# Comparison of static lattice phase diagrams



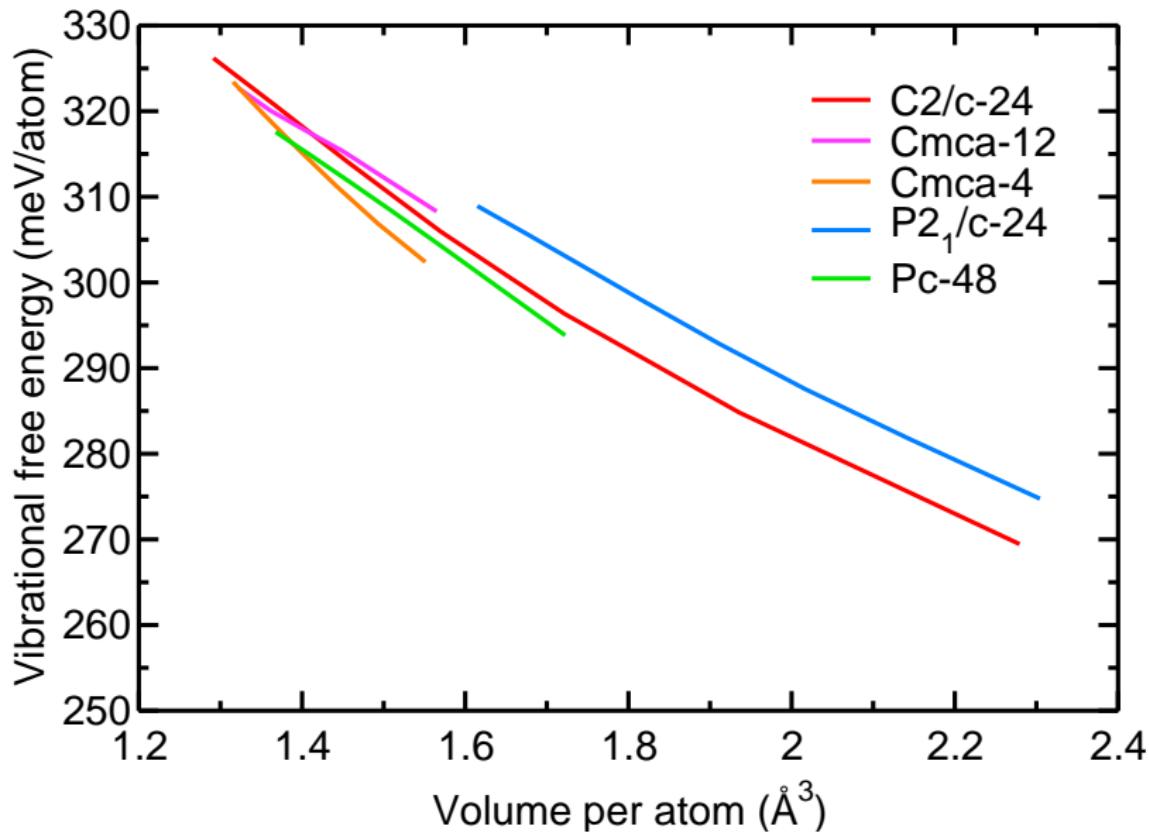
## Zero-point energies - harmonic approximation



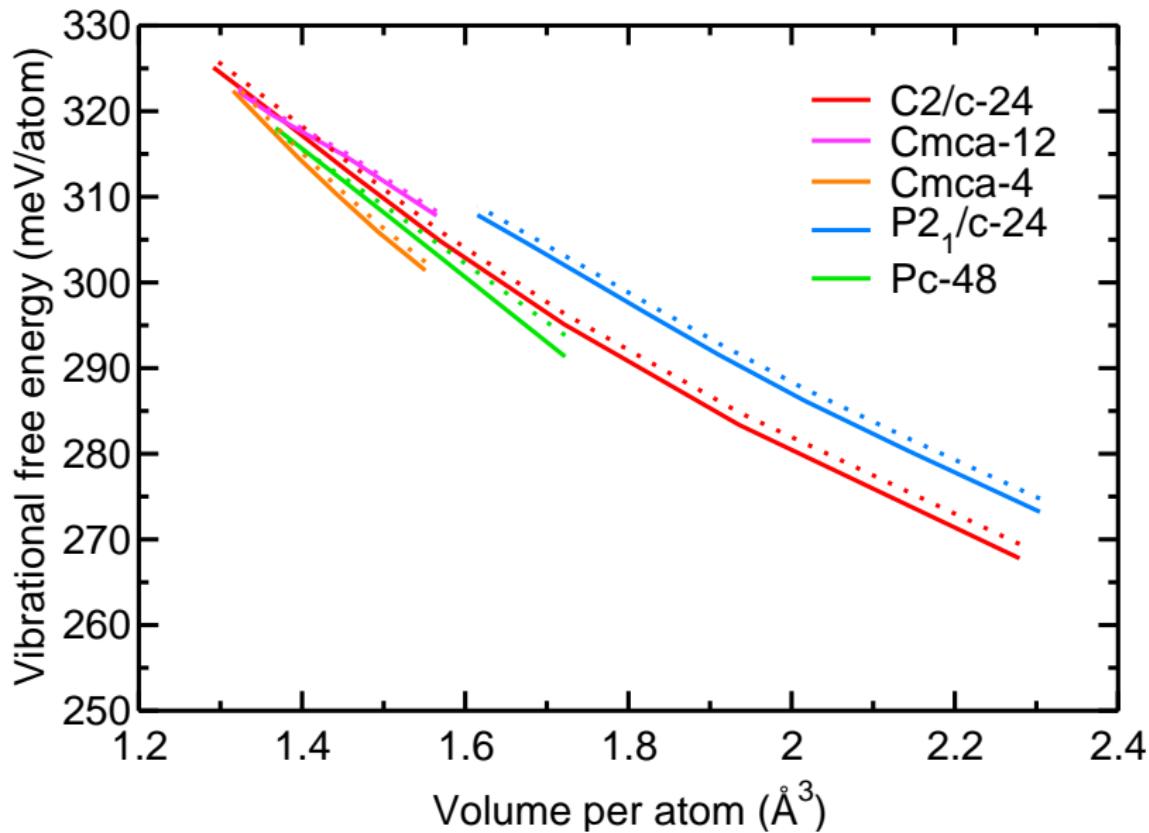
## Zero-point energies - anharmonic correction



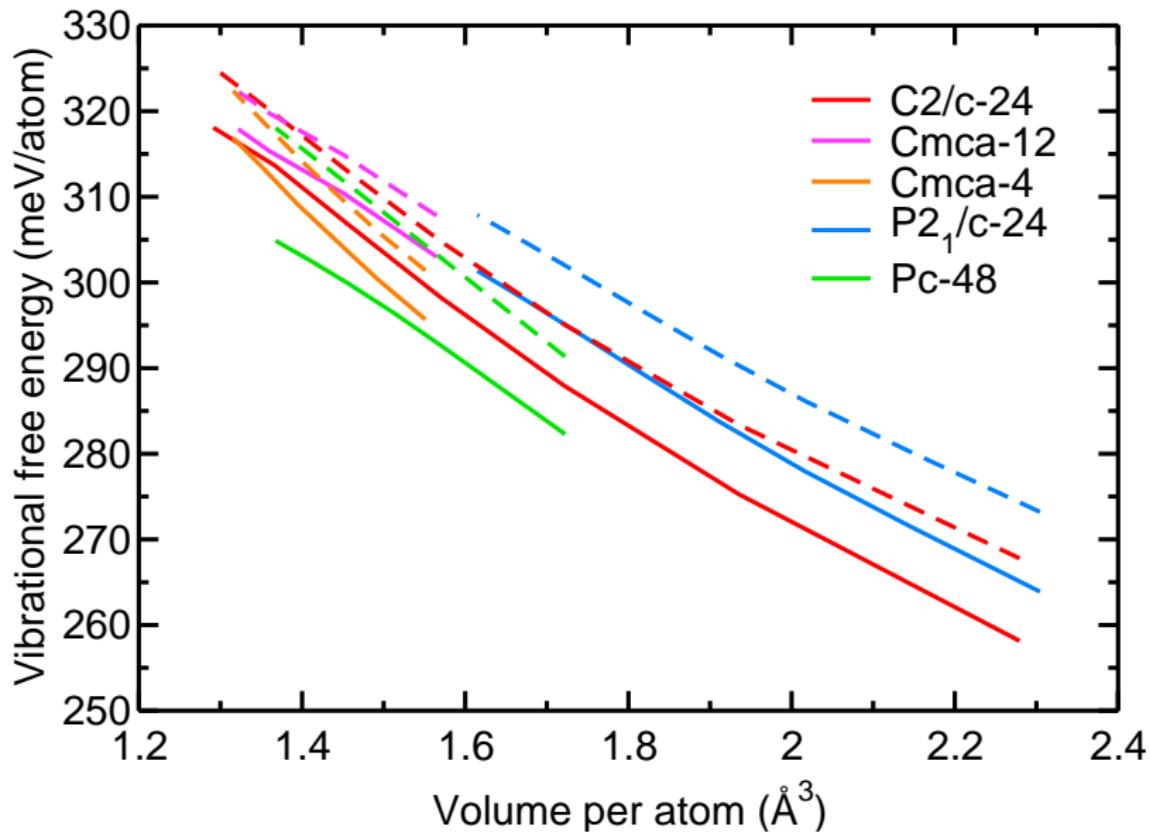
## Vibrational free energy - 0 K



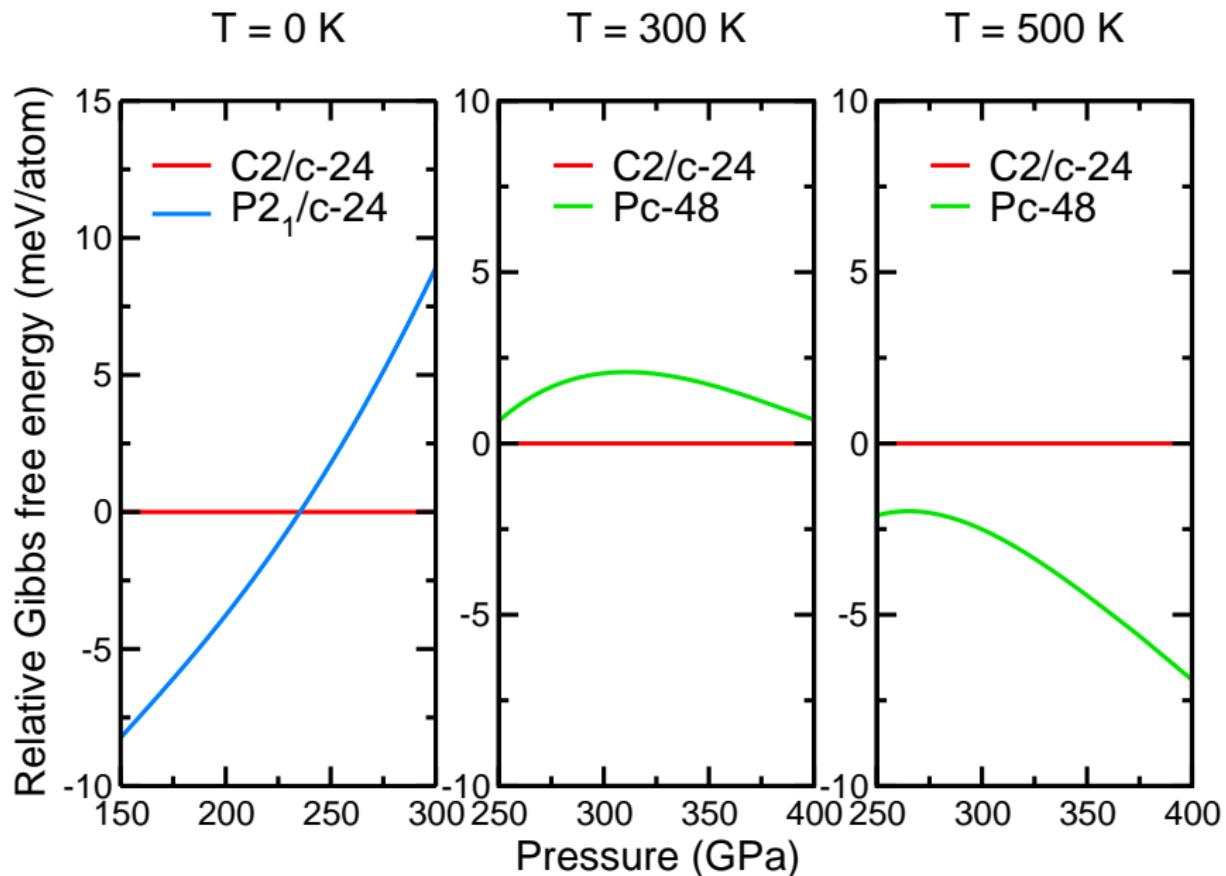
## Vibrational free energy - 300 K



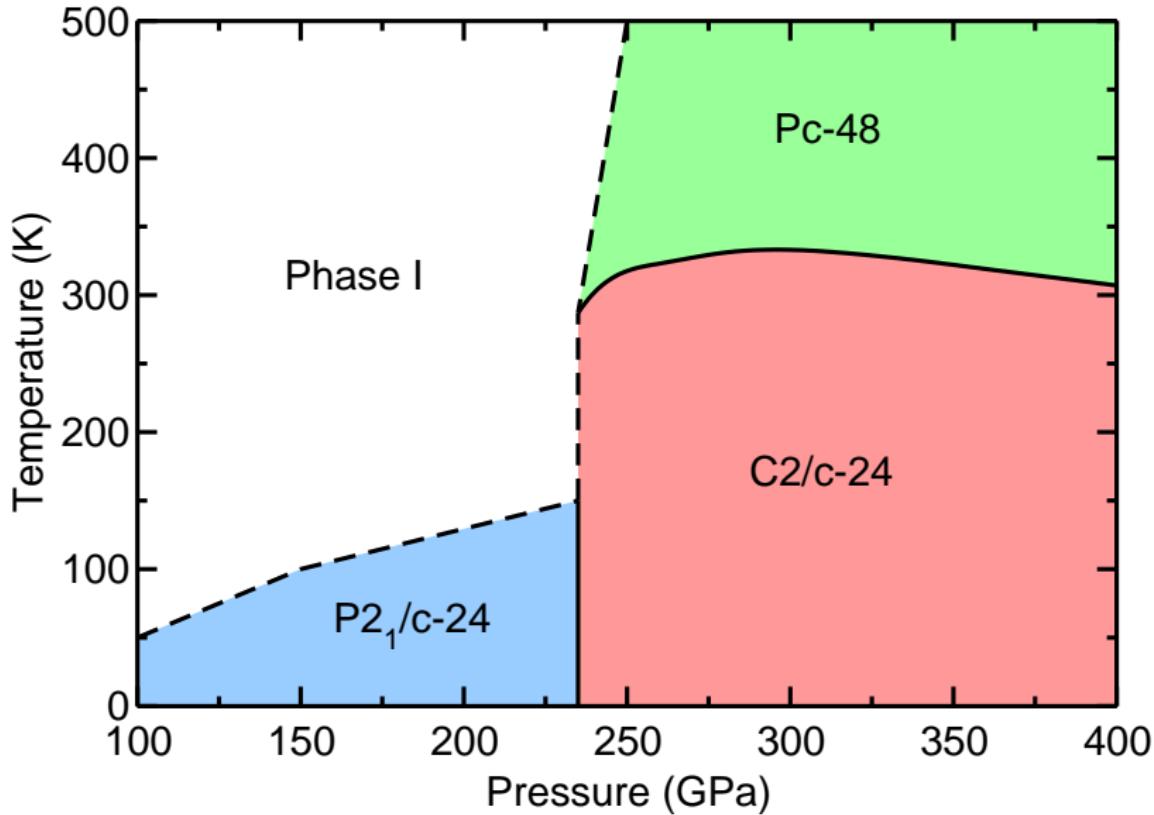
## Vibrational free energy - 500 K



## Phase transitions



## Phase diagram



## Conclusions

- ▶ Combination of DMC and anharmonicity removes Cmca-12 and Cmca-4 from the picture (up to 400 GPa).
- ▶ Calculated P2<sub>1</sub>/c-24 to C2/c-24 transition pressure is around 75 GPa higher than that observed for transition from phase II to phase III.
  - ▶ Nuclear spin effects important?
  - ▶ Better phase III structure?
- ▶ P<sub>c</sub>-48 is stabilized by temperature at pressures above 250 GPa, suggesting that it is a good structural model for phase IV.

# Acknowledgements



Bartomeu  
Monserrat



Pablo López  
Ríos



Neil  
Drummond



Richard Needs

---

**EPSRC**

Pioneering research  
and skills

The logo features a globe icon next to the text "U.S. DEPARTMENT OF ENERGY" above "INCITE" and "LEADERSHIP COMPUTING" below it.

U.S. DEPARTMENT OF ENERGY  
**INCITE**  
LEADERSHIP COMPUTING