

# In Exploration of Solid Metallic Hydrogen

*Sam Azadi and Matthew Foulkes*

QMC Workshop  
Towler Institute  
August 2012

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## Beginning of the story 1935:

DECEMBER, 1935

JOURNAL OF CHEMICAL PHYSICS

VOLUME 3

### On the Possibility of a Metallic Modification of Hydrogen

E. WIGNER AND H. B. HUNTINGTON, *Princeton University*

(Received October 14, 1935)

At very high pressure solid molecular hydrogen would dissociate and form an atomic solid that is metallic

*Metallic Solid Hydrogen*

1968:

VOLUME 21, NUMBER 26

PHYSICAL REVIEW LETTERS

23 DECEMBER 1968

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METALLIC HYDROGEN: A HIGH-TEMPERATURE SUPERCONDUCTOR?

N. W. Ashcroft

Laboratory of Atomic and Solid State Physics, Cornell University, Ithaca, New York 14850

(Received 3 May 1968)

Application of the BCS theory to the proposed metallic modification of hydrogen suggests that it will be a high-temperature superconductor. This prediction has interesting astrophysical consequences, as well as implications for the possible development of a superconductor for use at elevated temperatures.

*High-Temperature Superconductivity*

2004:

## **letters to nature**

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# **A superconductor to superfluid phase transition in liquid metallic hydrogen**

**Egor Babaev<sup>1,2</sup>, Asle Sudbø<sup>2</sup> & N. W. Ashcroft<sup>1</sup>**

*<sup>1</sup>Laboratory of Atomic and Solid State Physics, Cornell University, Ithaca, New York, 14853-2501, USA*

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at high (but experimentally accessible) pressures, compressed hydrogen will adopt a **liquid state, even at low temperatures**. In reaching this phase, hydrogen is also projected to pass through an insulator-to-metal transition. This raises the possibility of new state of matter: a near **ground-state liquid metal**, and its ordered states in the quantum domain.

## Three phases predicted by experiment:

**Phase I (  $P < 110$  GPa):** freely rotating  $\text{H}_2$  molecules whose centers of mass are arranged in an *hcp* structure.

**Phase II (  $110 < P < 150$  GPa):** Also known as the broken symmetry phase. DFT calculations predict **orthorhombic  $\text{Cmc}2_1$ ,  $\text{Pca}2_1$ , and  $\text{P}2_1/\text{C}$**  structures.

**Phase III (  $P > 150$  GPa):** At 150 GPa, a large low-temperature discontinuity in the frequency of the Raman and a strong rise in the IR molecular vibrons mark the appearance of Phase III.

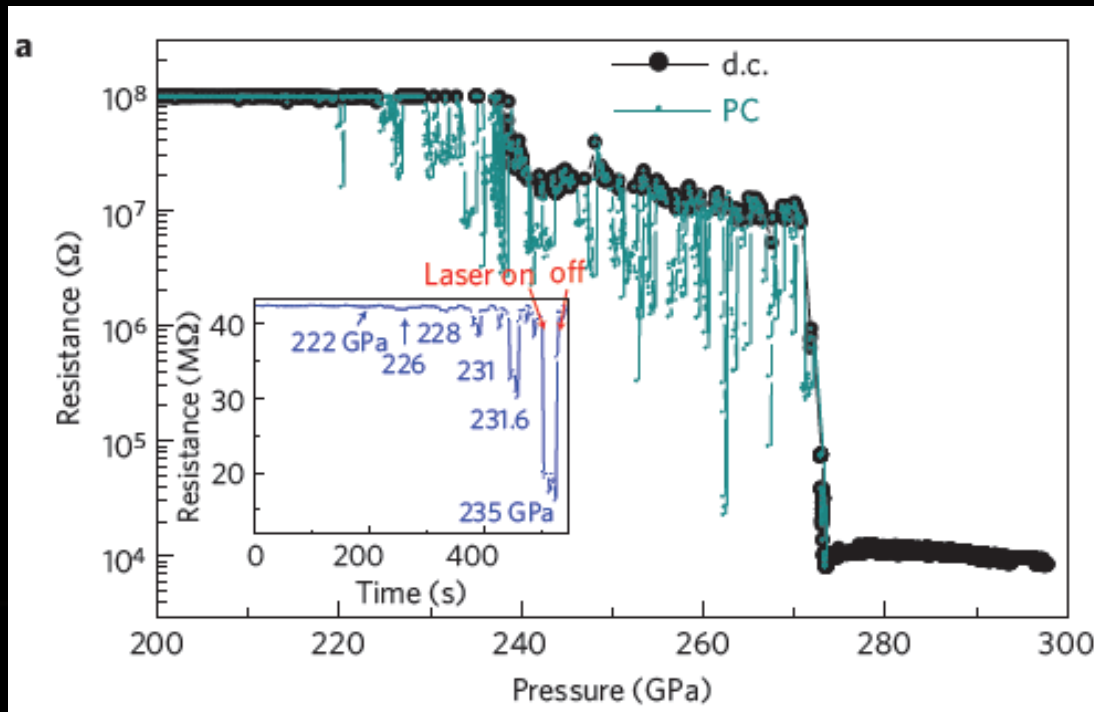
Recent X-ray results suggest ***hcp*** structure up to 185 GPa

Phys. Rev. B **82**, 060101(R) (2010)

## Recent Experimental Contradiction:

Above 220 GPa, hydrogen became opaque and electrically conductive. At 260–270 GPa, hydrogen transformed into a metal as the conductance of hydrogen sharply increased and changed little on further pressurizing up to 300 GPa or cooling to at least 30 K; and the sample reflected light well.

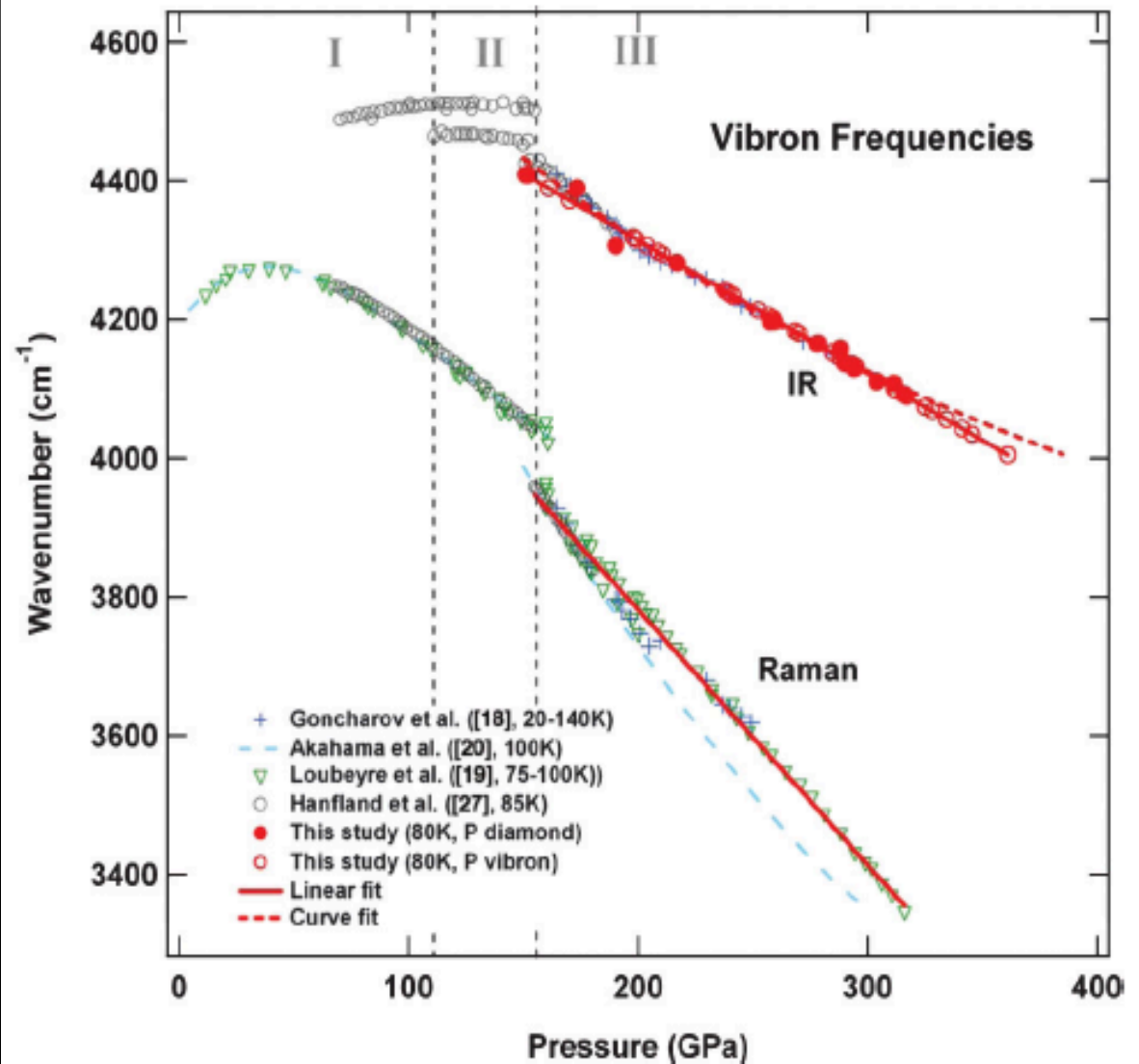
The metallic phase transformed back at 295 K into molecular hydrogen at 200 GPa. This significant hysteresis indicates that the transformation of molecular hydrogen into a metal is accompanied by a first-order structural transition presumably into a monatomic liquid state.



Eremets, Nature December 2011

The persistence of the strong infrared absorption of the vibron characteristic of phase III indicates the **stability of the paired state of hydrogen**. There is **no evidence for the predicted Metallic state (up to 360 GPa)** over these conditions, in contrast to recent reports, but electronic properties consistent with **semimetallic** behavior are observed.

Our measurements show **no evidence for the optical conductivity** expected for a metal over the entire range of temperatures to the highest pressures explored, and **no signatures of an atomic liquid** are observed.

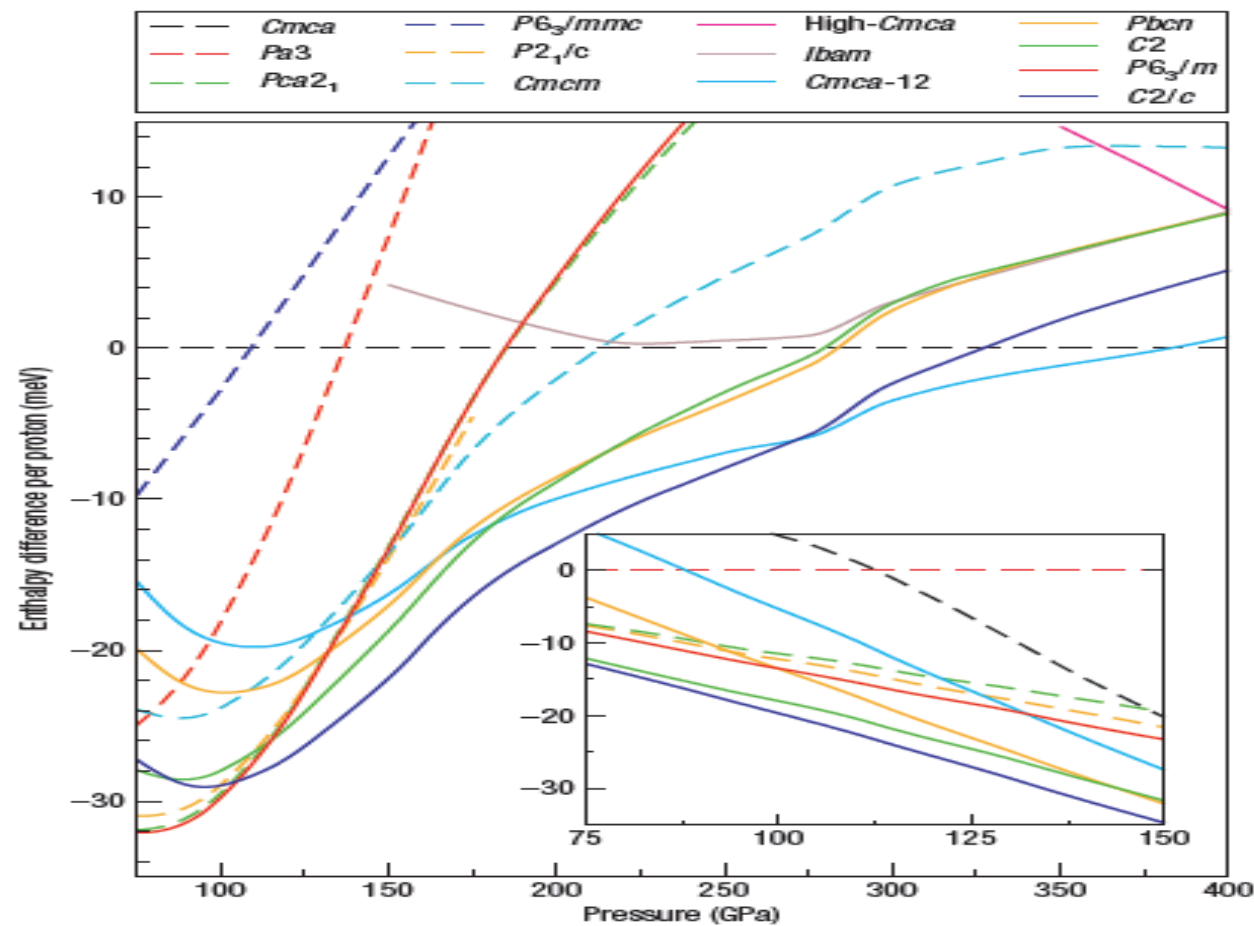


## Problems:

The phase diagram of hydrogen is poorly understood. Determining the stable structures of solid (?!) hydrogen is a tremendous experimental challenge, because hydrogen atoms scatter X-rays only weakly, leading to low-resolution diffraction patterns.

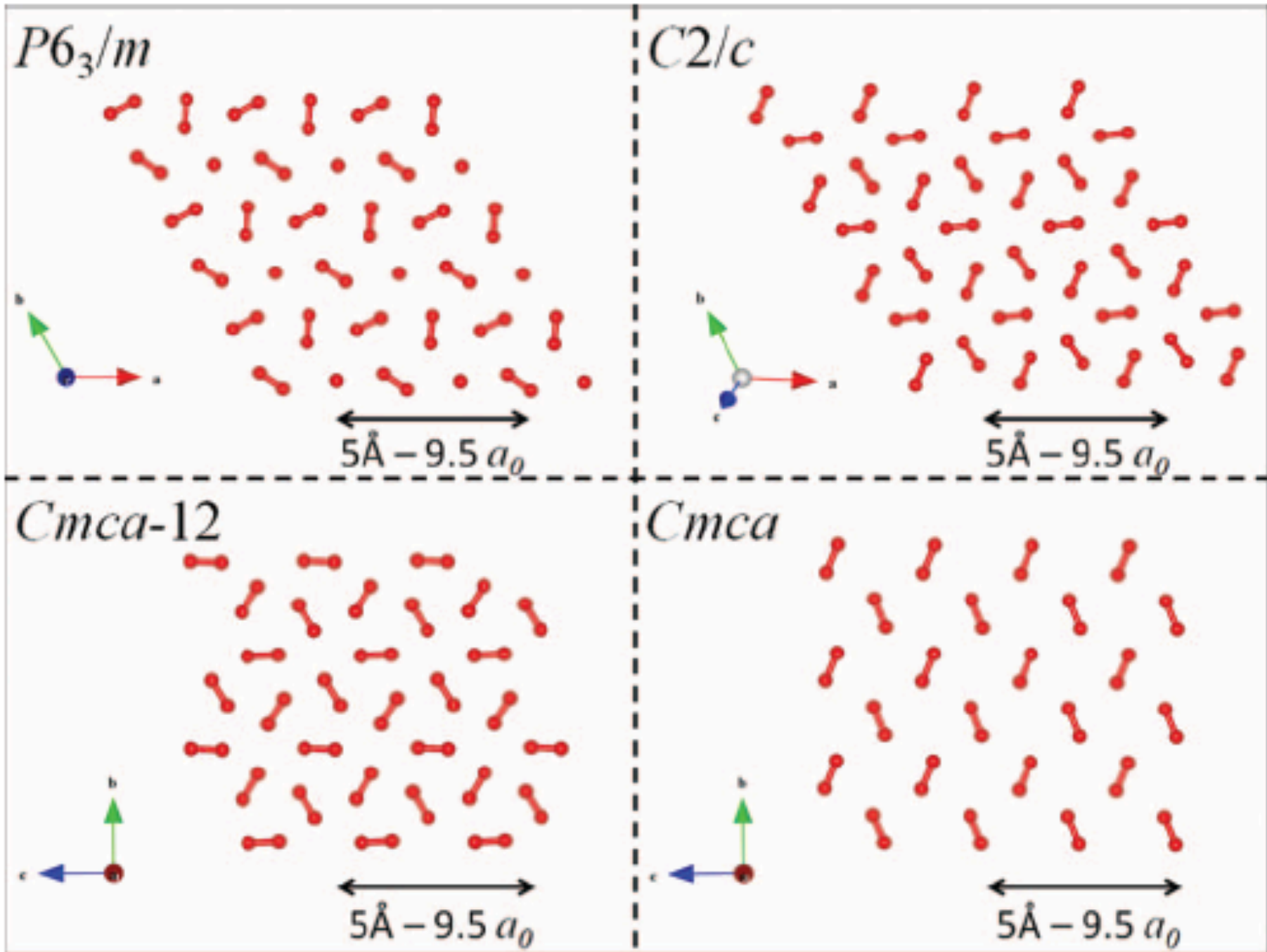
Theoretical studies encounter major difficulties owing to the small energy differences between structures and the importance of the zero-point motion of the protons.

# Density Functional Theory (DFT) phase diagram

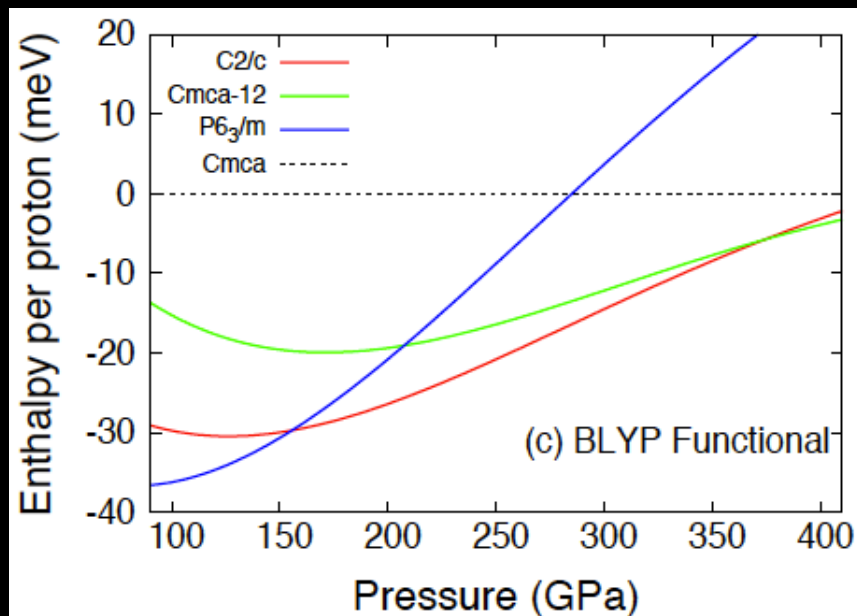
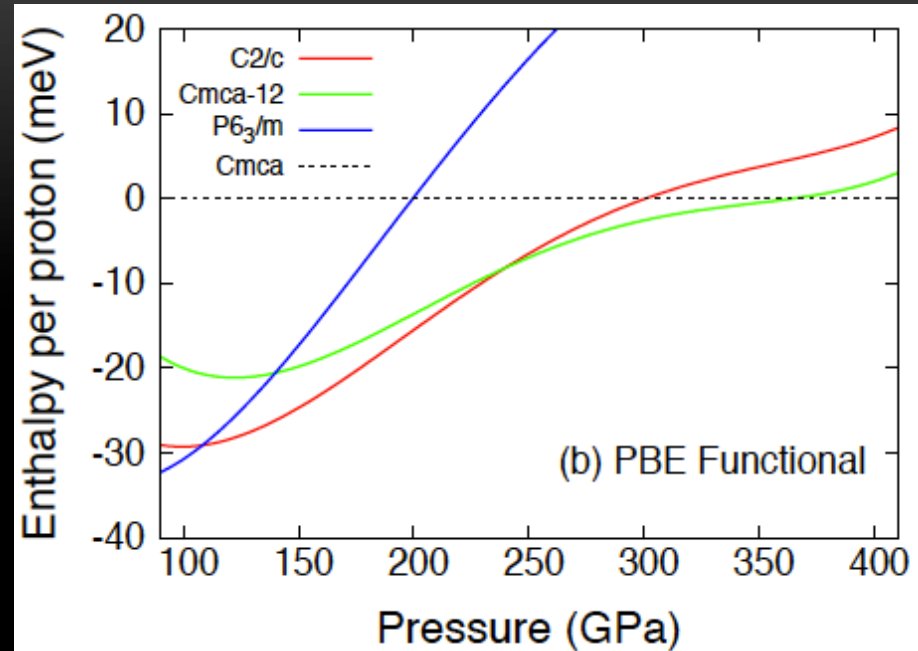
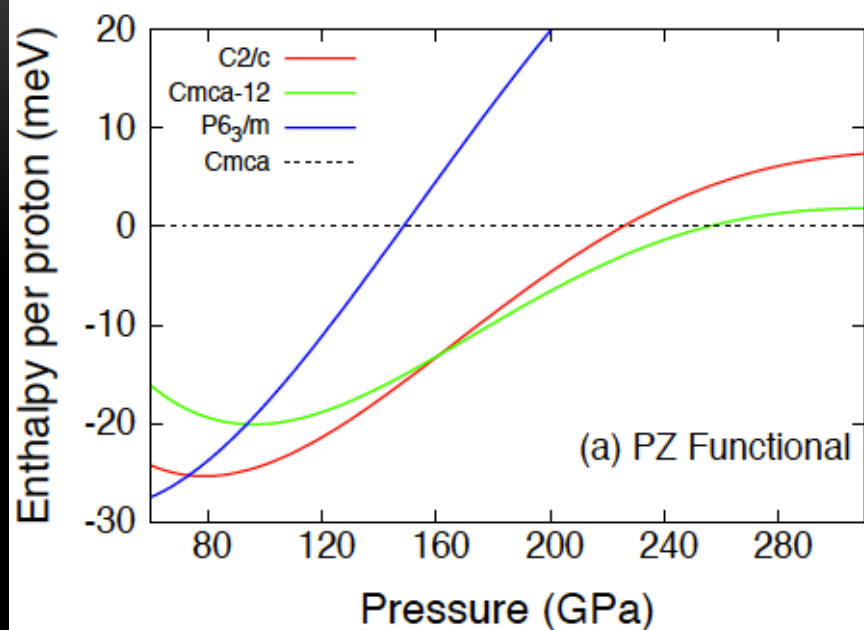


Their approach is to relax many random structures to minima in the enthalpy at fixed pressure

Nature **3**, 473 (2007)

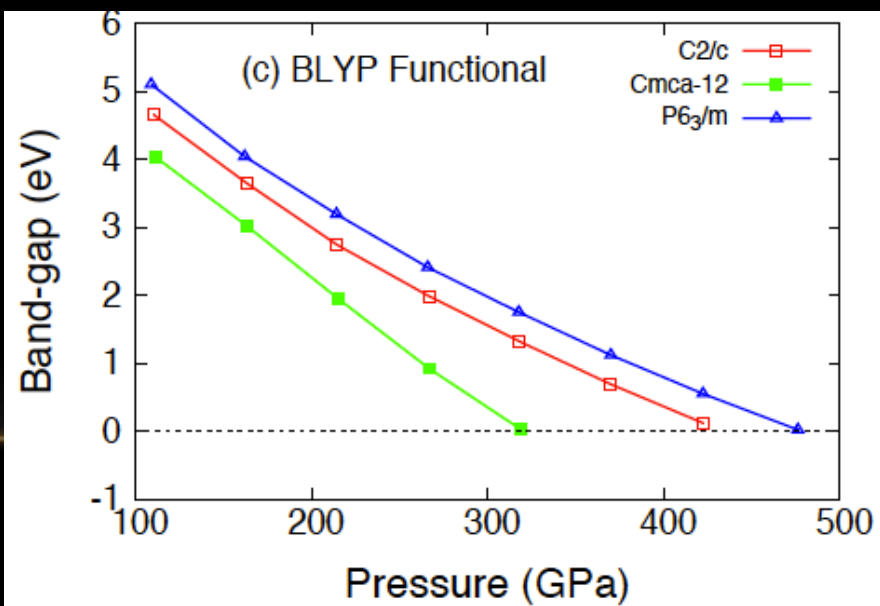
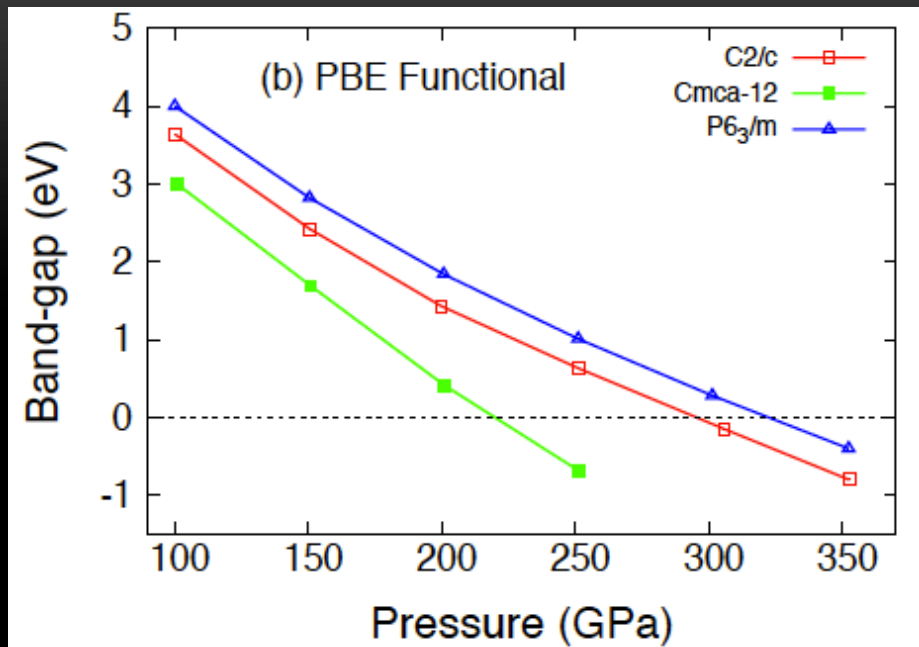
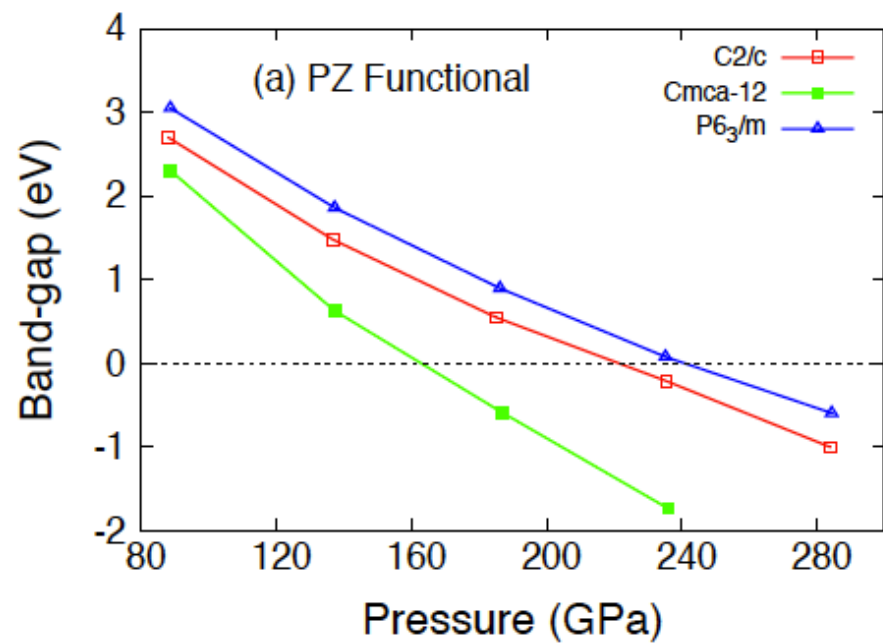


# Can we trust DFT phase diagram ?



At least DFT shows three structure transitions !

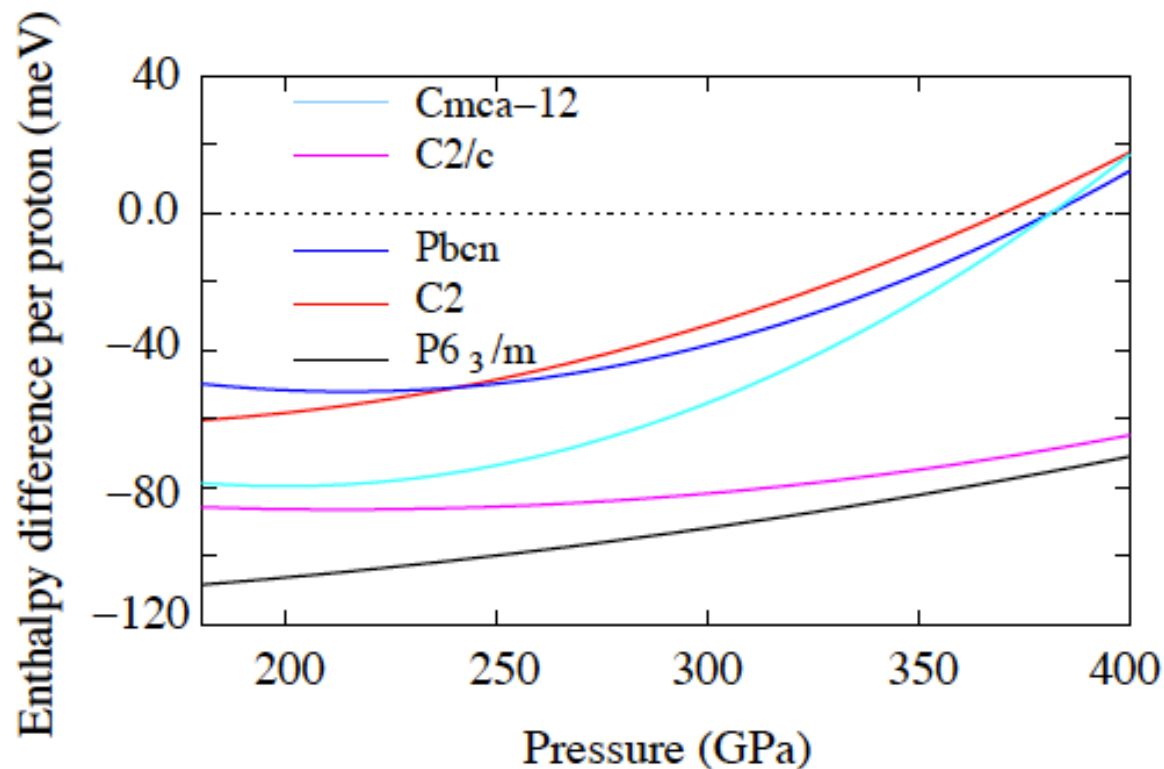
## DFT band-gap:



## Hybrid DFT phase diagram:

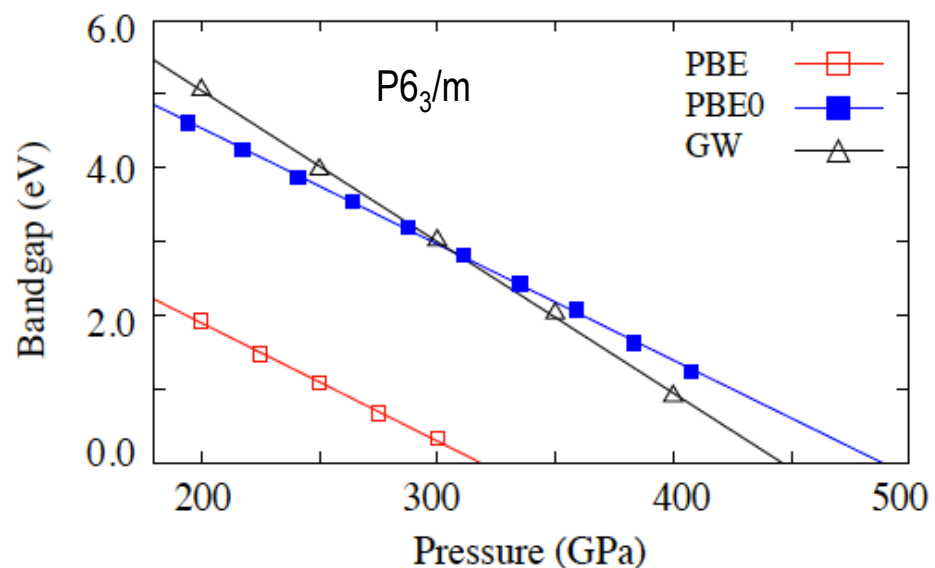
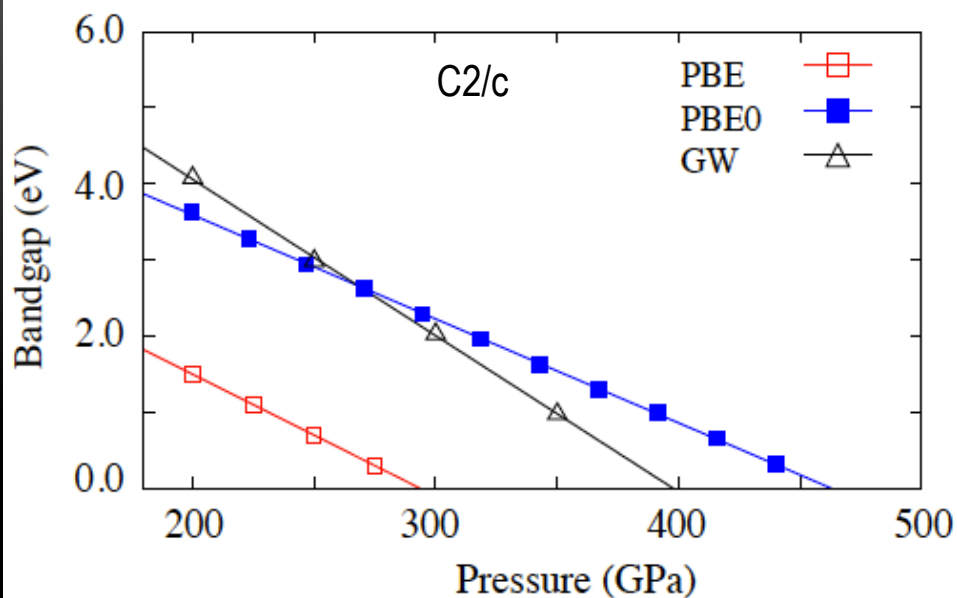
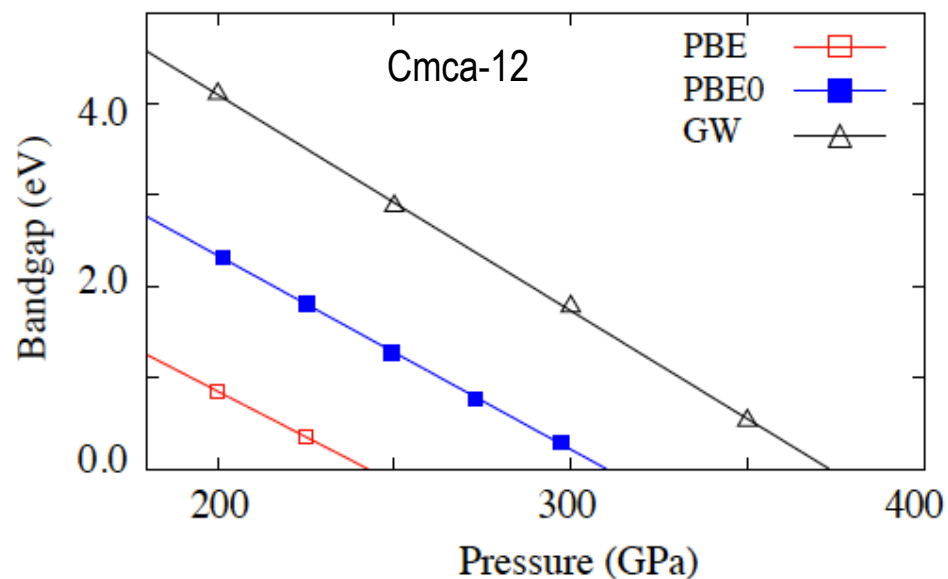
Hybrid DFT: DFT + small fraction of Hartree-Fock exchange

Typically yields much improved **band gap** that are often in close agreement with experiment



Oops ! No Metallization,  
No structure transition !

The lower the energy  
The wider the band-gap  
Hemley, PRL **67** 1138 (1991)



Hybrid functional metallization pressures are larger than GW ones ~ 50 GPa

GW predicts metallization should happen before 400 GPa

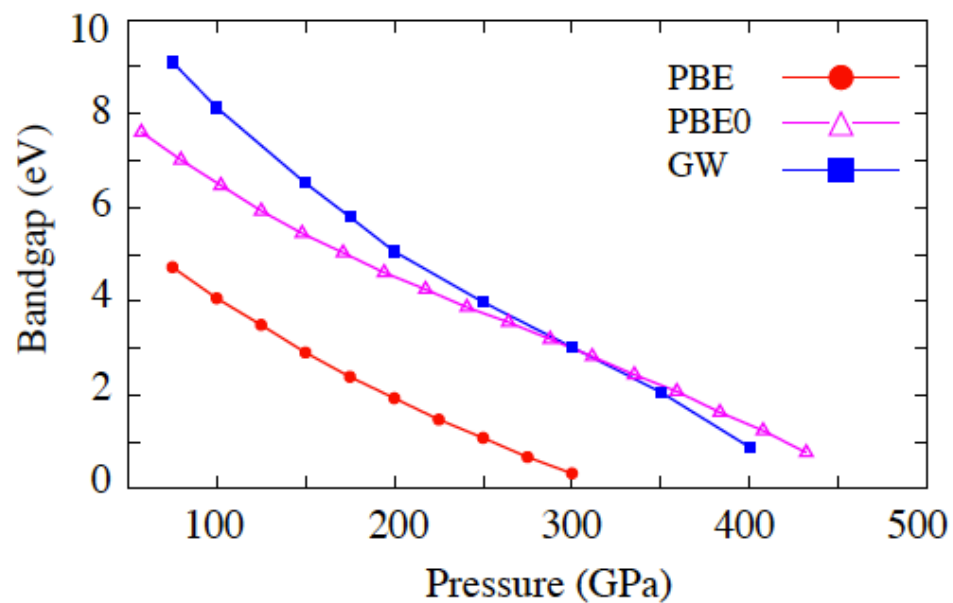


FIG. 4: (Colors online) Band-gap of  $P6_3/m$  hydrogen crystal structure as functions of pressure in the range of phase I,II, and III obtained by PBE,PBE0 and GW calculations.

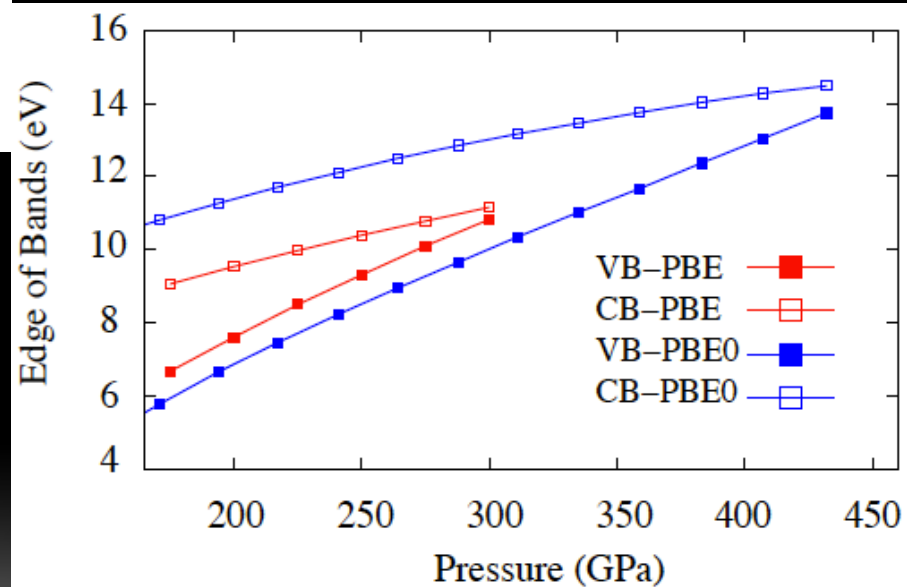


FIG. 5: (Colors online) VB-maxima and CB-minima of  $P6_3/m$  phase as functions of pressure obtained by PBE and PBE0 calculations. The significant influence of PBE0 on increasing of band-gap is due to the increasing of CB-minima.

## QMC phase diagram:

Contrary to standard DFT methods, QMC calculations have to be performed on a super-cell. Therefore, finite-size (FS) effects can be a relevant source of error in QMC calculations.

One-body term (independent particle source), arises from the kinetic and Hartree term

Two-body term, related to the FS effects of exchange and correlation

*Hold on! How DFT functionals change QMC energy ?*

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TABLE I. VMC and DMC energies of the  $P6_3/m$  phase using PBE and PBE0 single particle orbitals. Energies and pressures are in eV per proton and GPa, respectively.

Pressure	PBE-VMC	PBE0-VMC	PBE-DMC	PBE0-DMC
100	-15.1335(4)	-15.1397(4)	-15.2327(3)	-15.2338(3)
200	-14.5898(4)	-14.5979(4)	-14.6856(3)	-14.6867(3)
300	-14.0473(4)	-14.0524(4)	-14.2184(3)	-14.2193(3)

This demonstrates that, in the case of solid molecular hydrogen, DMC energies calculated using PBE and PBE0 orbitals differ negligibly, even though the change of functional has a large effect on the DFT phase diagram. This contrasts with the case of 3d transition metal compounds [[Phys. Rev. B \*\*82\*\* 115108 \(2010\)](#)], where changing the functional does affect the DMC results.

## QMC+DFT to calculate FS correction:

**DMC -TABC:** *Twist averaging* means taking the average of expectation values over all simulation-cell Bloch vectors  $\mathbf{k}_s$  in the first Brillouin zone of the simulation cell, i.e., over all offsets to the grid of  $\mathbf{k}$  vectors

**KZK functional:** the exchange and correlation energy functional is replaced by the LDA functional parameterized for a finite system, which keeps an explicit dependence on the number of particles ([Phys. Rev. Lett. 100, 126404 \(2008\)](#))

## Static Phase Diagram:

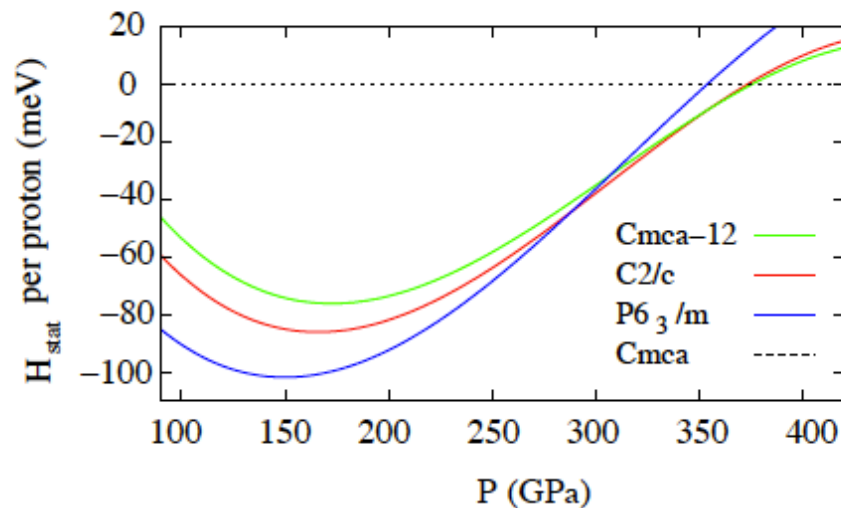
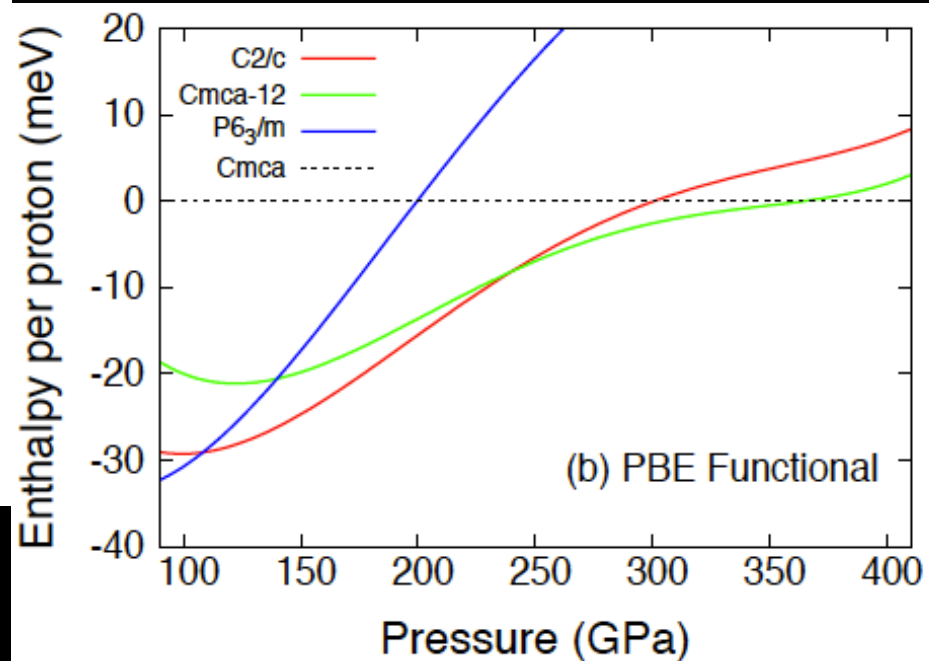
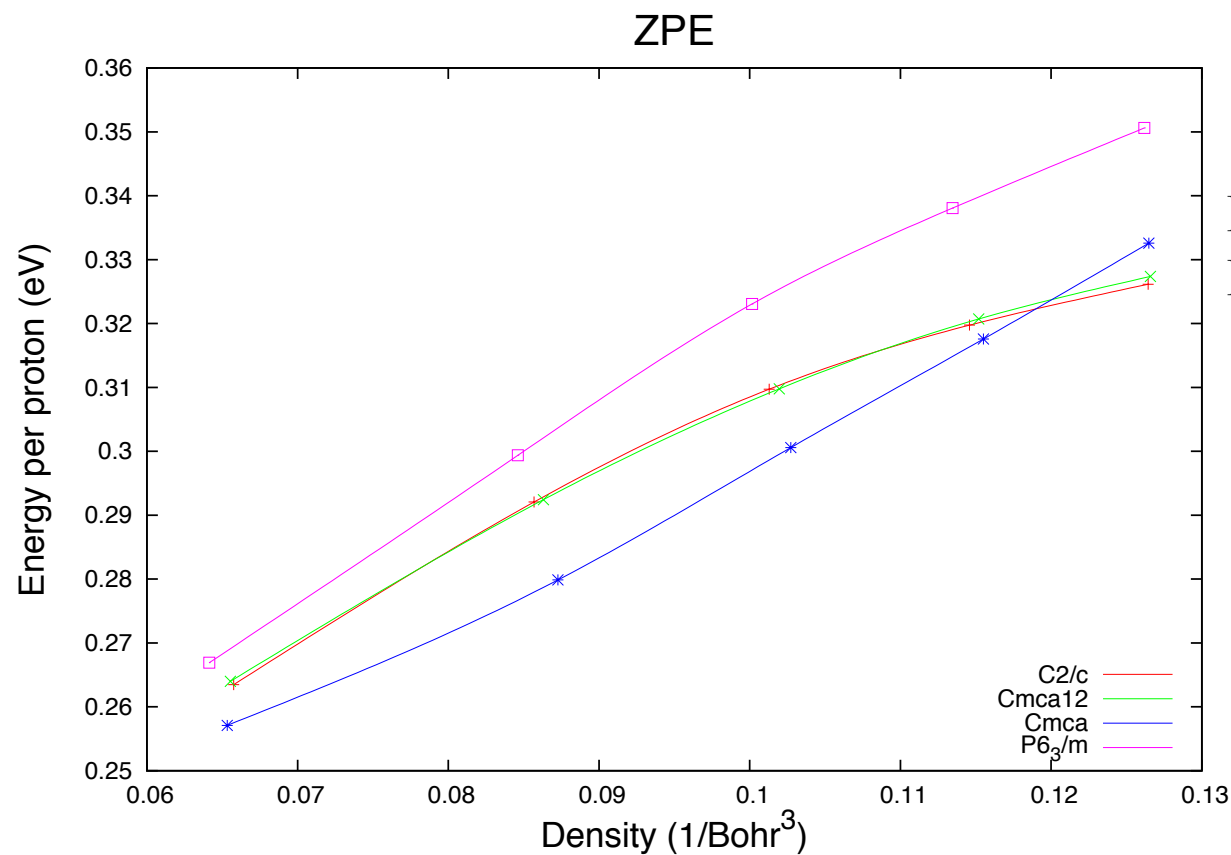


FIG. 1. (color online). Enthalpy as a function of pressure calculated using the DMC-TABC method in the static approximation.



Our  $GW$  results show that the band gaps of the  $P6_3/m$ , C2/c, and Cmca-12 structures vanish at the static pressures of 446, 398, and 373 GPa, respectively.

# Zero Point Energy (ZPE) :



Protons ZPE is estimated using  
Harmonic approximation

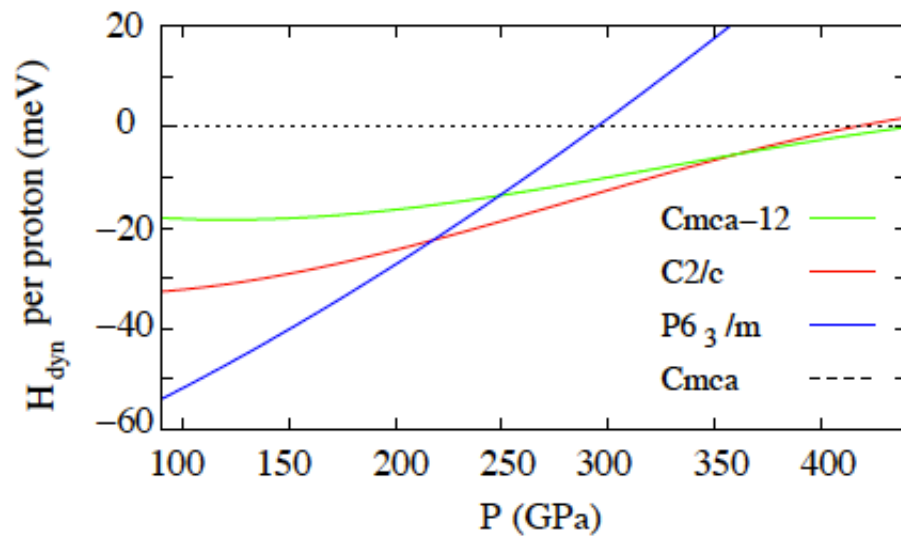


FIG. 2. (Colours online) Dynamic enthalpy obtained by DMC-TABC calculation including proton ZPE as a function of pressure calculated by DFT method

Semimetalic ?!

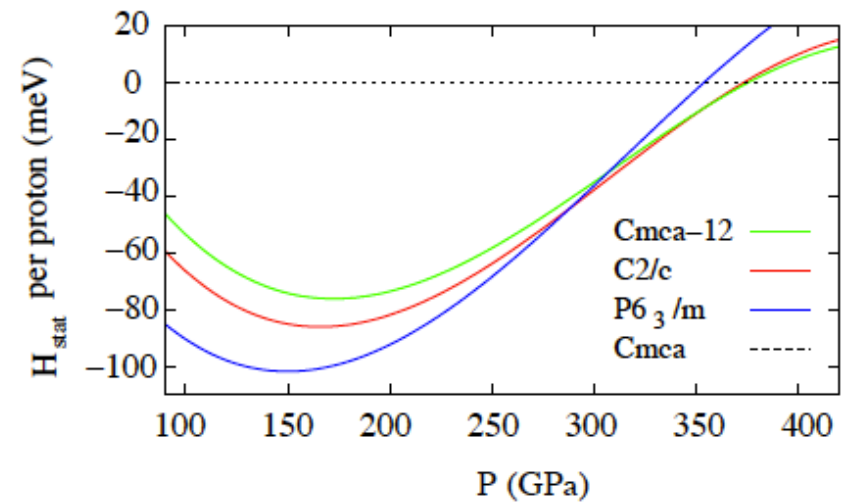
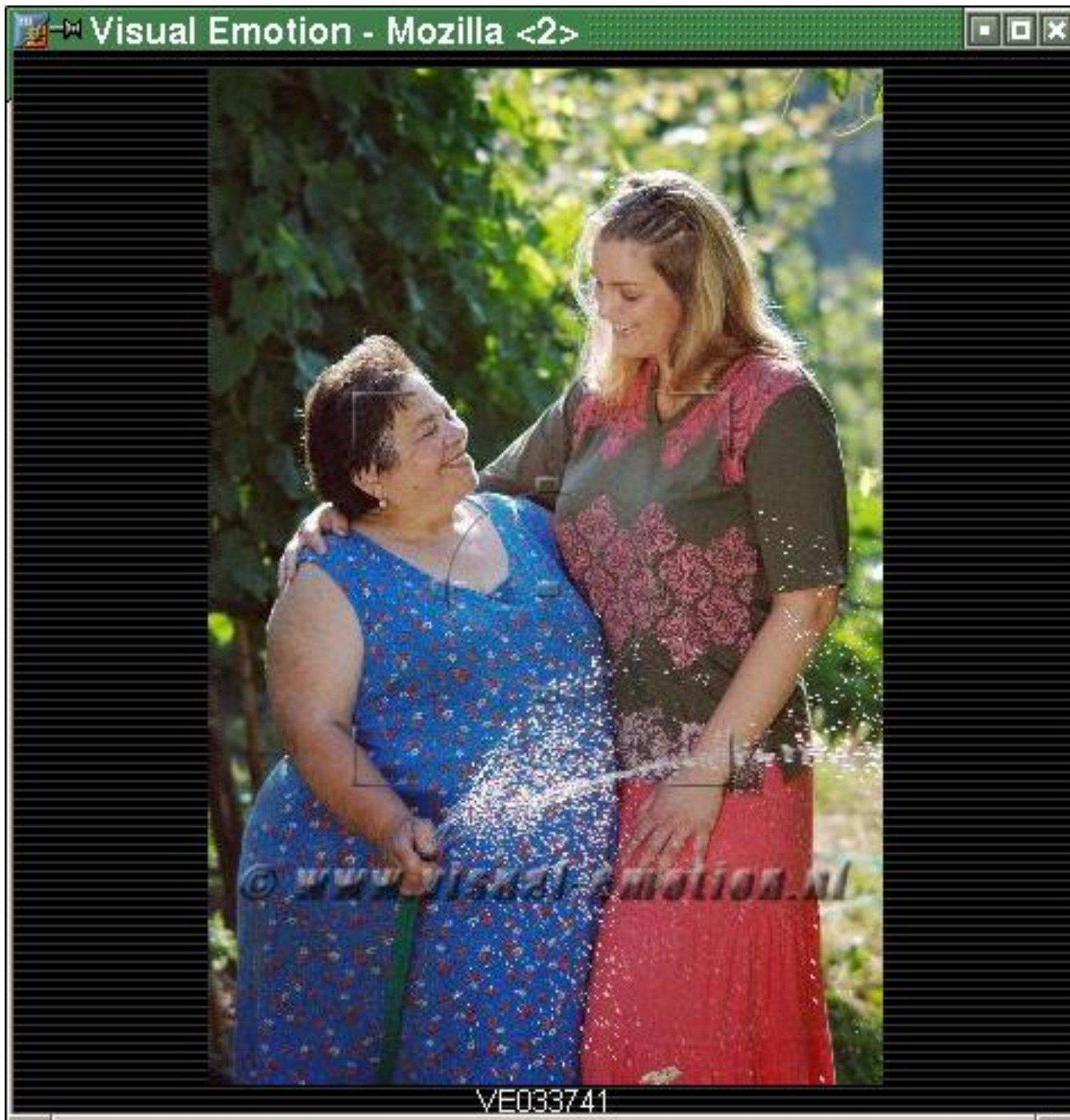


FIG. 1. (color online). Enthalpy as a function of pressure calculated using the DMC-TABC method in the static approximation.

Summary:

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*TTI Fashion  
Shoot !  
Thank you*