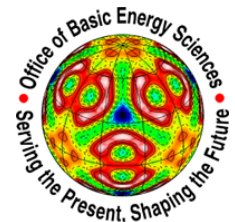


# QMC and GW Calculations in Fullerenes

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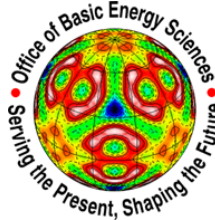
**Collaborators:** Murilo Tiago (ORNL), Paul R.C. Kent (ORNL)  
and Randy Q. Hood (LLNL)

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  - DOE
- Codes
  - PWSCF PARSEC & RGWBS DFT, GW-BSE, TDLDA
  - CASINO QMC
- Computational support
  - NCCS at ORNL
  - NERSC
  - LLNL
  - TACC



•Thanks A. J. Williamson, R. J. Needs, N. Drummond, M. Towler, J. Kim and M. Kalos

# DMC for fermions

$$\frac{\partial f}{\partial \tau} = \frac{\hbar^2}{2m} \left[ \sum_{i=1}^N \nabla_i^2 f - \nabla_i \left( f \nabla_i \ln |\psi_T|^2 \right) \right] - \left[ \frac{H \psi_T}{\psi_T} - \epsilon_0 \right] f$$

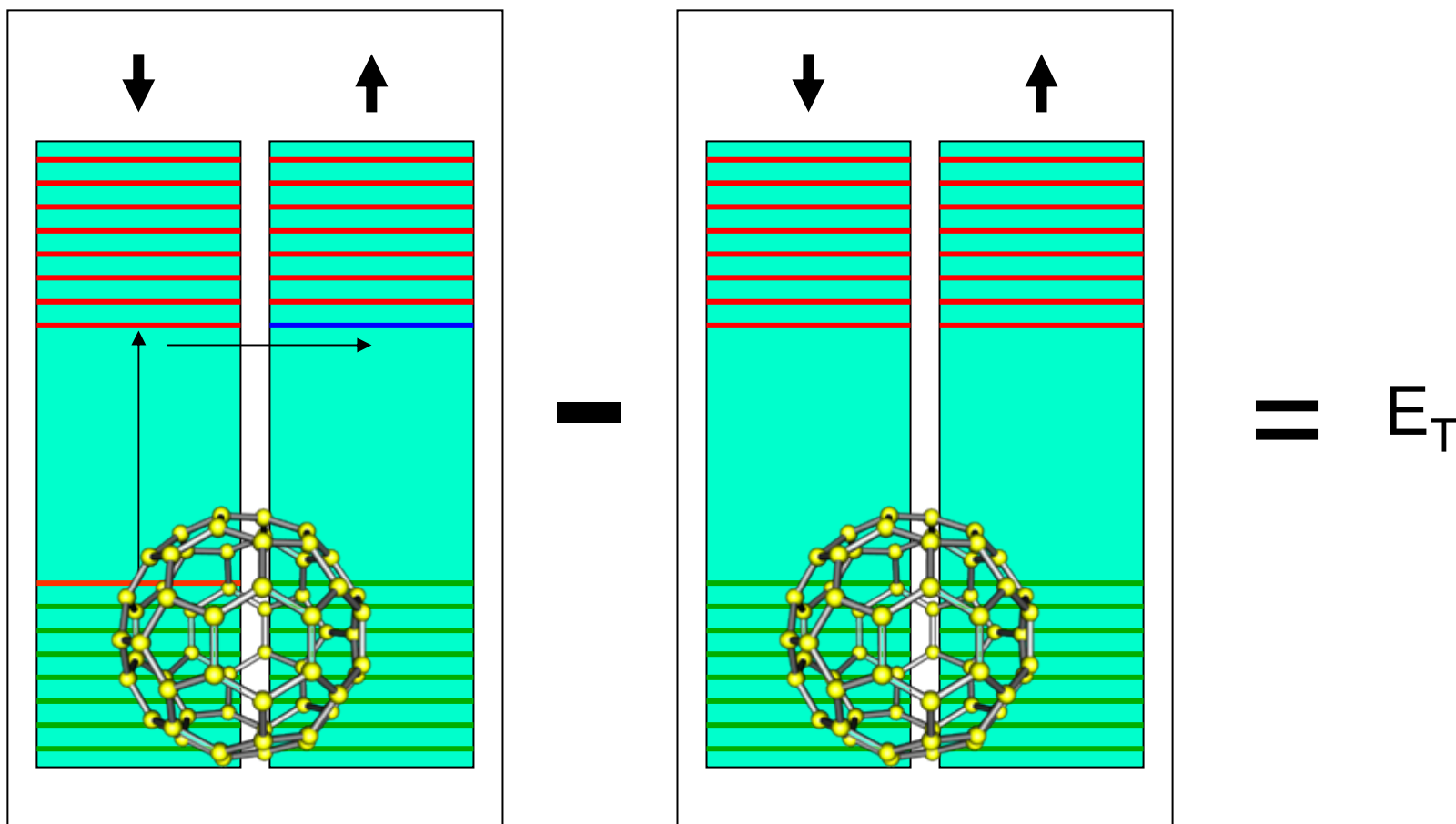
Ceperley Alder PRL 1980

$$H = - \sum_i \frac{\hbar^2 \nabla_i^2}{2m} + \sum_{i,j} \frac{e^2}{|\vec{r}_i - \vec{r}_j|} - \sum_{i,I} \frac{Z_I e^2}{|\vec{r}_i - \vec{R}_I|} + \sum_{J,I} \frac{Z_I Z_J e^2}{|\vec{R}_J - \vec{R}_I|}$$

$$\Psi = \underbrace{\begin{pmatrix} \phi_1(\mathbf{r}_1) & \phi_1(\mathbf{r}_2) & \dots \\ \phi_2(\mathbf{r}_1) & \phi_2(\mathbf{r}_2) & \\ \vdots & & \ddots \\ & & & \phi_N(\mathbf{r}_N) \end{pmatrix}}_{\substack{\text{Slater Determinant} \\ O(N^3)}} \cdot \underbrace{\exp \left[ \sum_i^N \chi(\mathbf{r}_{iI}) - \sum_i^N u(\mathbf{r}_{ij}) \right]}_{\substack{\text{Jastrow correlation function} \\ O(N^2)}}$$

We use CASINO 2.1  Cambridge Quantum Monte Carlo Code

# Approximations (1) pseudopotentials, (2) Fix Node (3) DFT structure



This is a very demanding test for DMC  
Total energy differences require cancelation of errors on 0.01% or more

# Levels of approximation of GW methods:

“DFT”

$$\Sigma = V_{xc}$$

Hybertsen & Louie (1985)

$$G_0(\mathbf{r}, \mathbf{r}'; E) = \sum_n \frac{\psi_n(\mathbf{r})\psi_n^*(\mathbf{r}')}{E - E_n}$$

$$W \approx \epsilon^{-1} \frac{e^2}{|\mathbf{r} - \mathbf{r}'|}$$

$G_0W_0$  approximation

$$\Sigma = iG_0W_0 \quad \text{Del Sole et al. (1994)}$$

Tiago et. al (2006)

$G_0W_0$  + vertex:  $G_0W_f$  approximation

$$\Sigma = iG_0W_0\Gamma_0$$

Self-consistent: GW approximation

$$\Sigma = iGW$$

Hedin's equations

$$\begin{cases} W = V + VPW \\ P = -iGG\Gamma \\ \Sigma = iG\Gamma W \\ \Gamma = 1 + \frac{\delta\Sigma}{\delta G}GG\Gamma \end{cases}$$

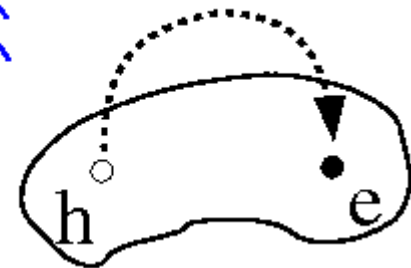
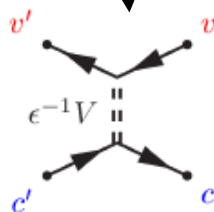
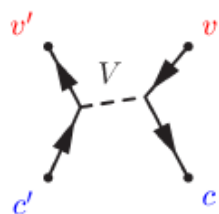
$G_0W_0$  and  $G_0W_f$  approximations rely on DFT as a “good starting point”.

# Optical Excitations: Bethe-Salpeter Equation

Eigenvalue problem:

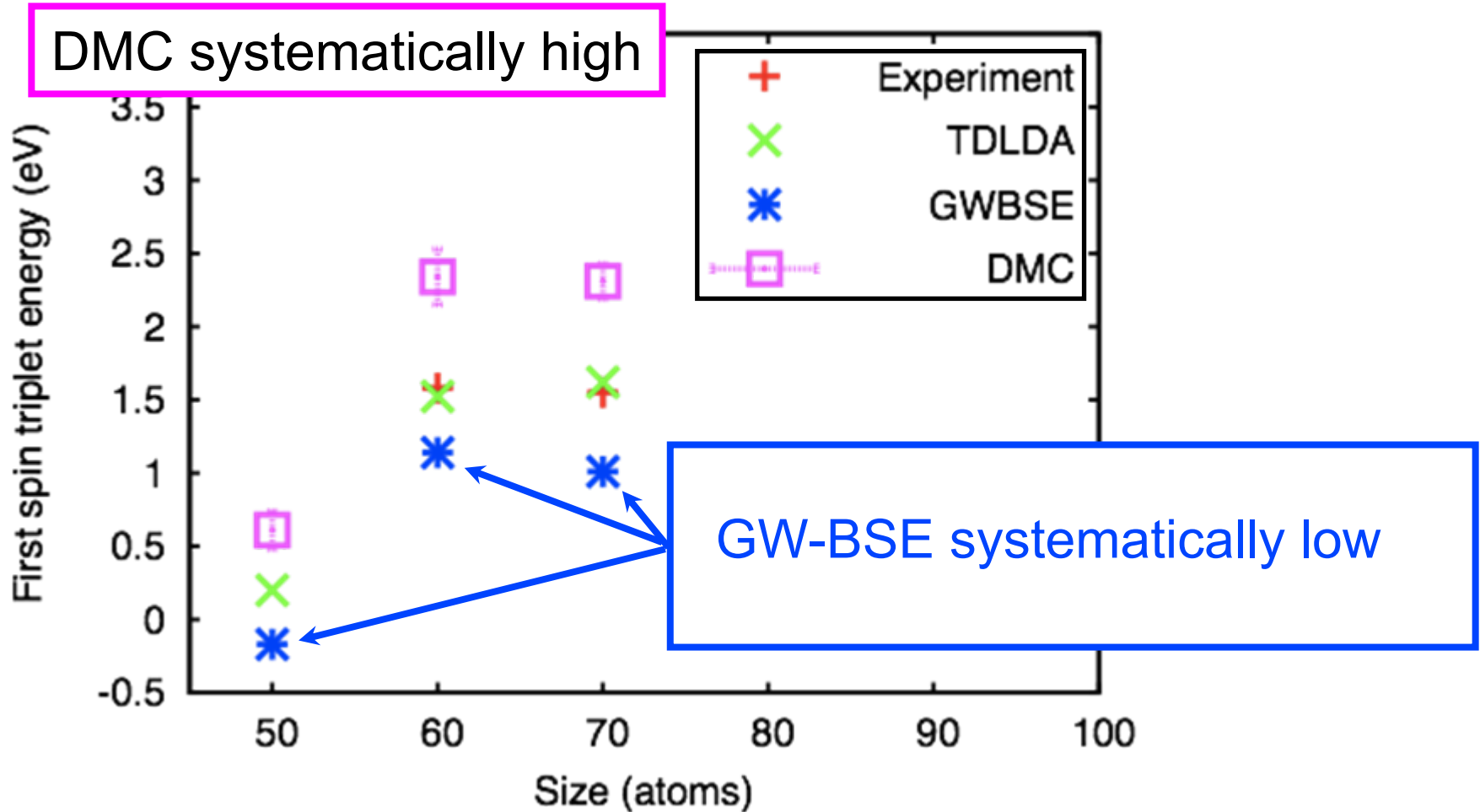
$$\left(\epsilon_c^{GW} - \epsilon_v^{GW}\right) A_{cv}^S + \sum_{c'v'} \langle vc | K^{BSE} | c'v' \rangle A_{c'v'}^S = \Omega^S A_{cv}^S$$

$$K^{BSE} = \frac{\delta}{\delta G} [1 \cdot V_{Coul} + \Sigma]$$



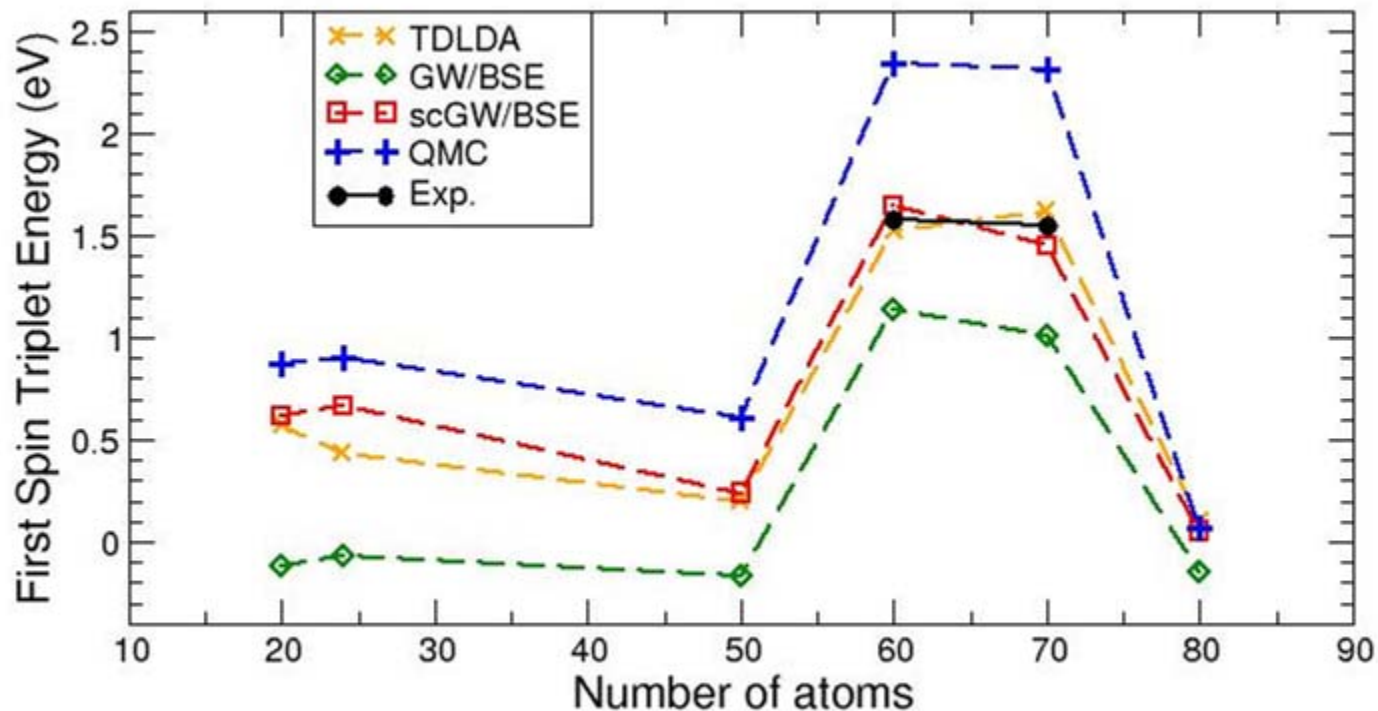
- Many-body expansion of the electron-hole propagator.
- Dynamics of electron-hole excitation obtained by solving the Bethe-Salpeter equation (BSE).
- Requires knowledge of quasiparticle orbitals (get from GW).

# Results: First spin-triplet



- Stoke's shifts estimated as max 0.2 eV from DFT not included in above data

# Experimental data suggest incorporating self-consistency

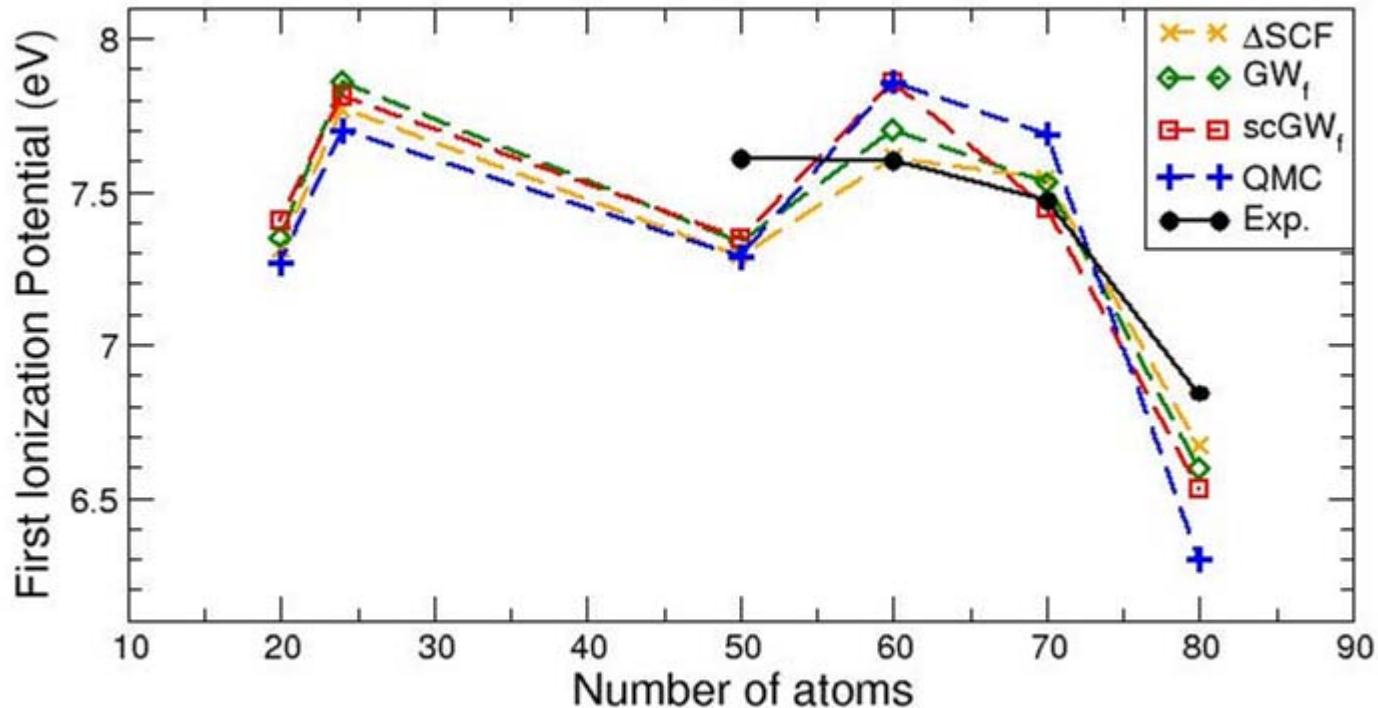


**Scissors (eigen-values) self-consistency seems essential in GW/BSE.**

**DMC gives higher (~0.8 eV) triplet energies**

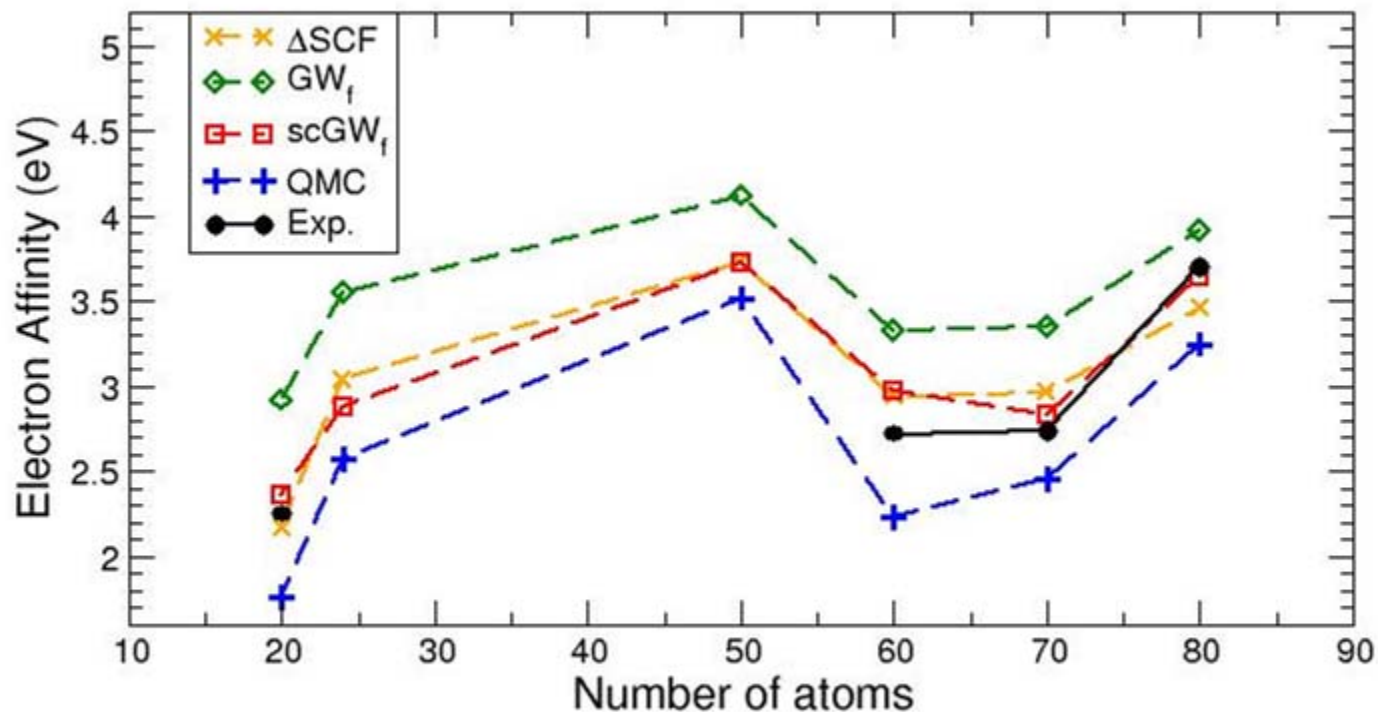


# First Ionization Potentials



We find good agreement for Ionization Potentials  $\Delta$ SCF (DFT) QMC and  $GW$  are similar.

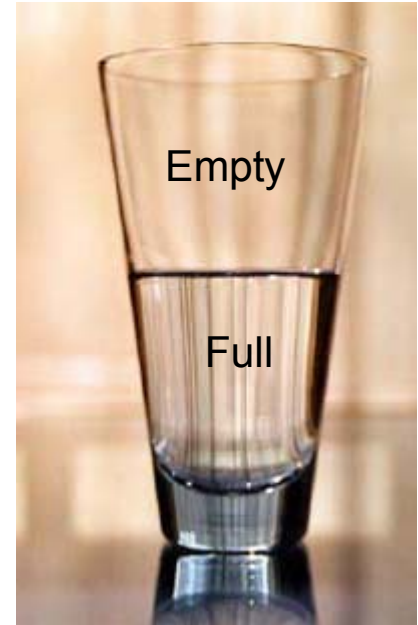
# Electron Affinities



**$\Delta$ SCF (DFT) is still not far from experimental data.  
GW requires self-consistency  
QMC systematically underestimates EA.**

# Is the glass full or empty?

- GW-BSE
  - Scissors self-consistent improves agreement with experiment
  - Has the approach predictive power in general?
- $\Delta$ SCF and TDLDA
  - Agree with experiment in fullerenes
  - They are known to disagree in carbon nanotubes
- QMC
  - Cancellations of the relative systematic errors  $< 10^{-5}$
  - Need compact multiconfigurational expansions &/or orbital optimization for large systems
  - Pseudopotential evaluation related errors are small
- Experiment
  - Experiments in single molecules



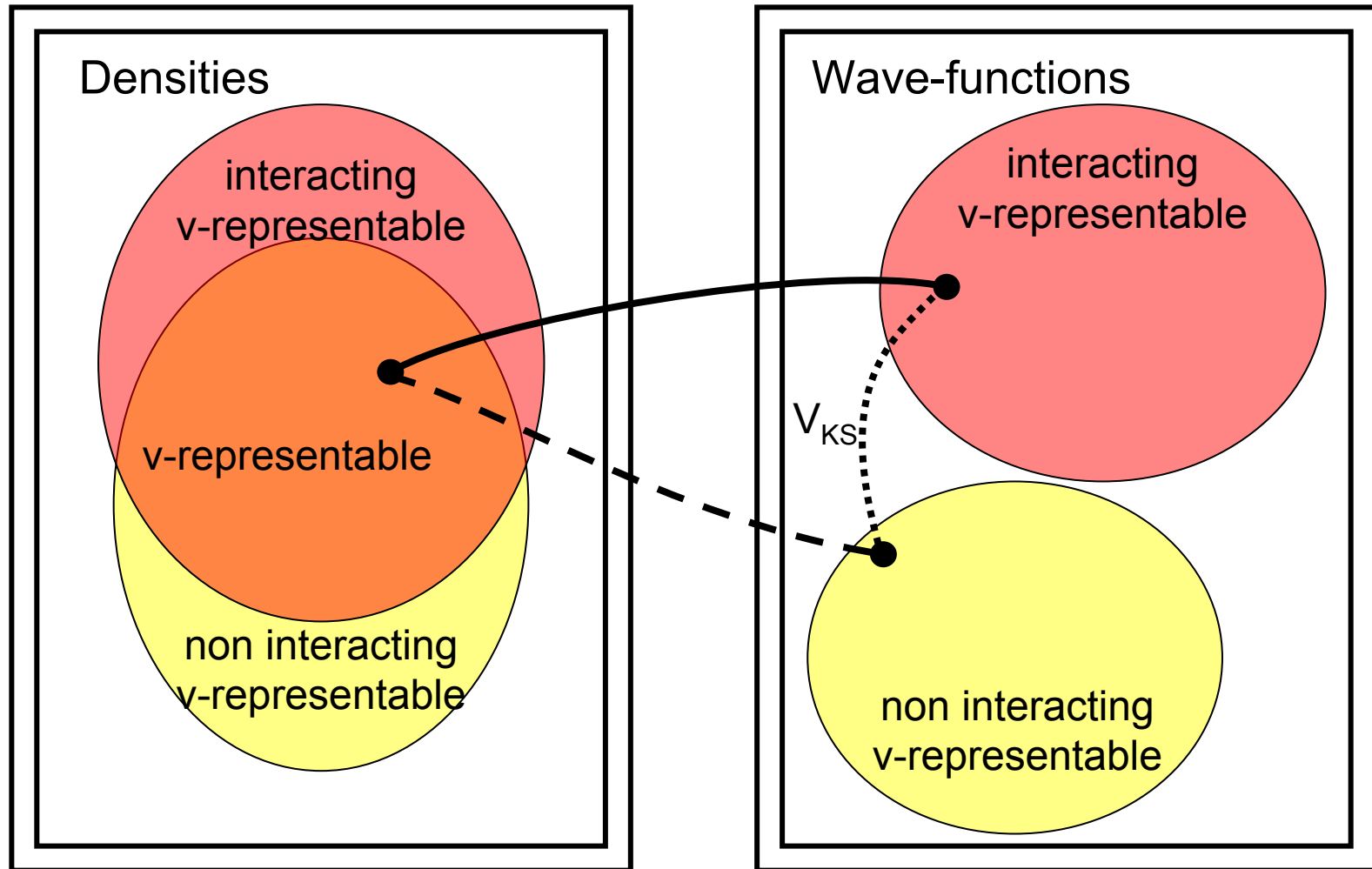
<http://arxiv.org/abs/0803.0560>

# Motivation

- The Kohn-Sham formulation of DFT establishes an equivalence between interacting and non interacting densities.
- No other property of the interacting ground state wave-function is in principle retained.
- However, Kohn-Sham wave-functions are used as starting point for methods that go beyond DFT such as QMC or GW-BSE etc.

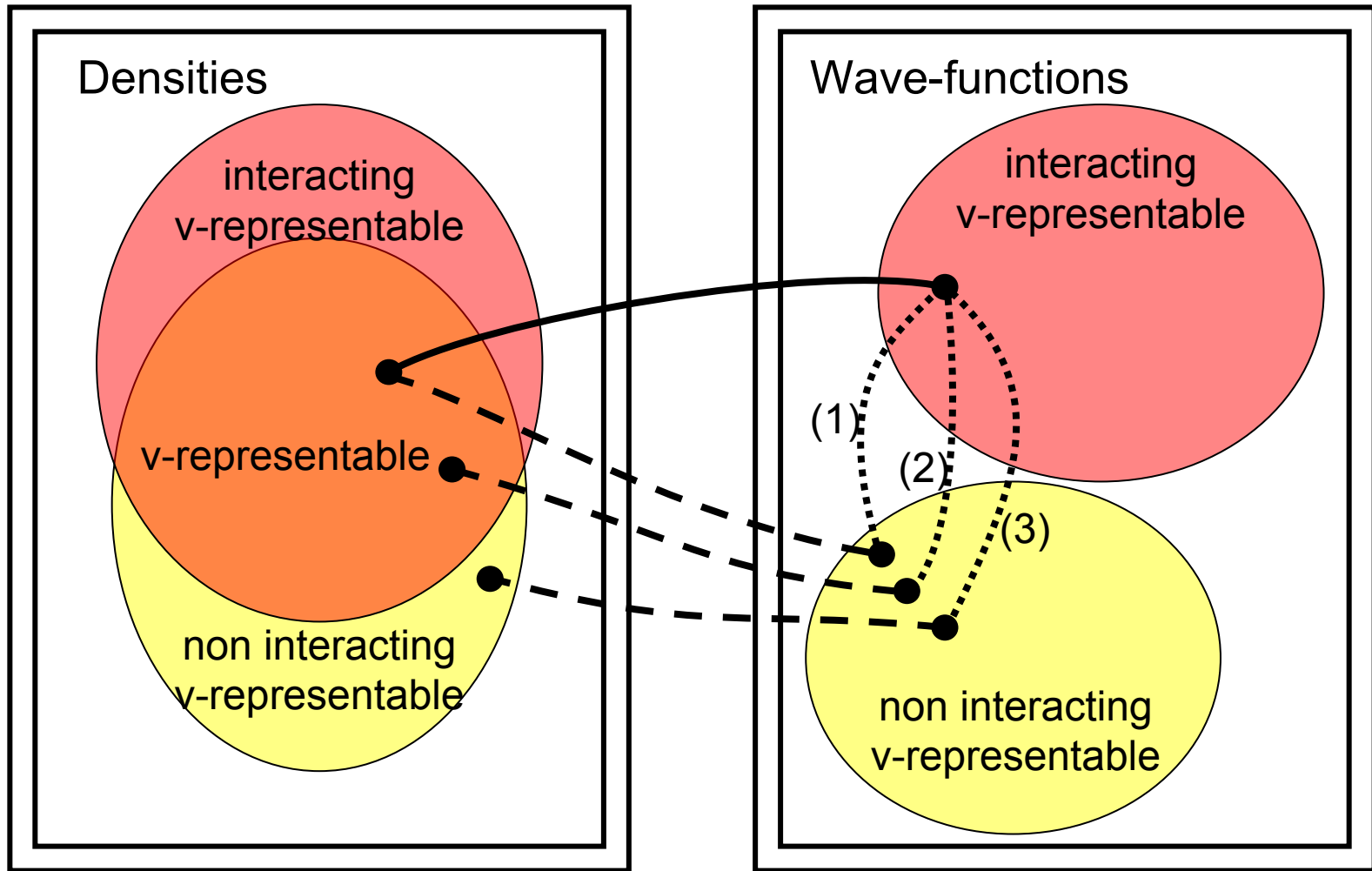
Are Kohn and Sham wave-functions any good?  
Do they retain any physical property beyond the  
density?

# Kohn-Sham correspondence between interacting and non-interacting densities



For non-degenerate systems there is at most one Kohn-Sham wave-function with the interacting density

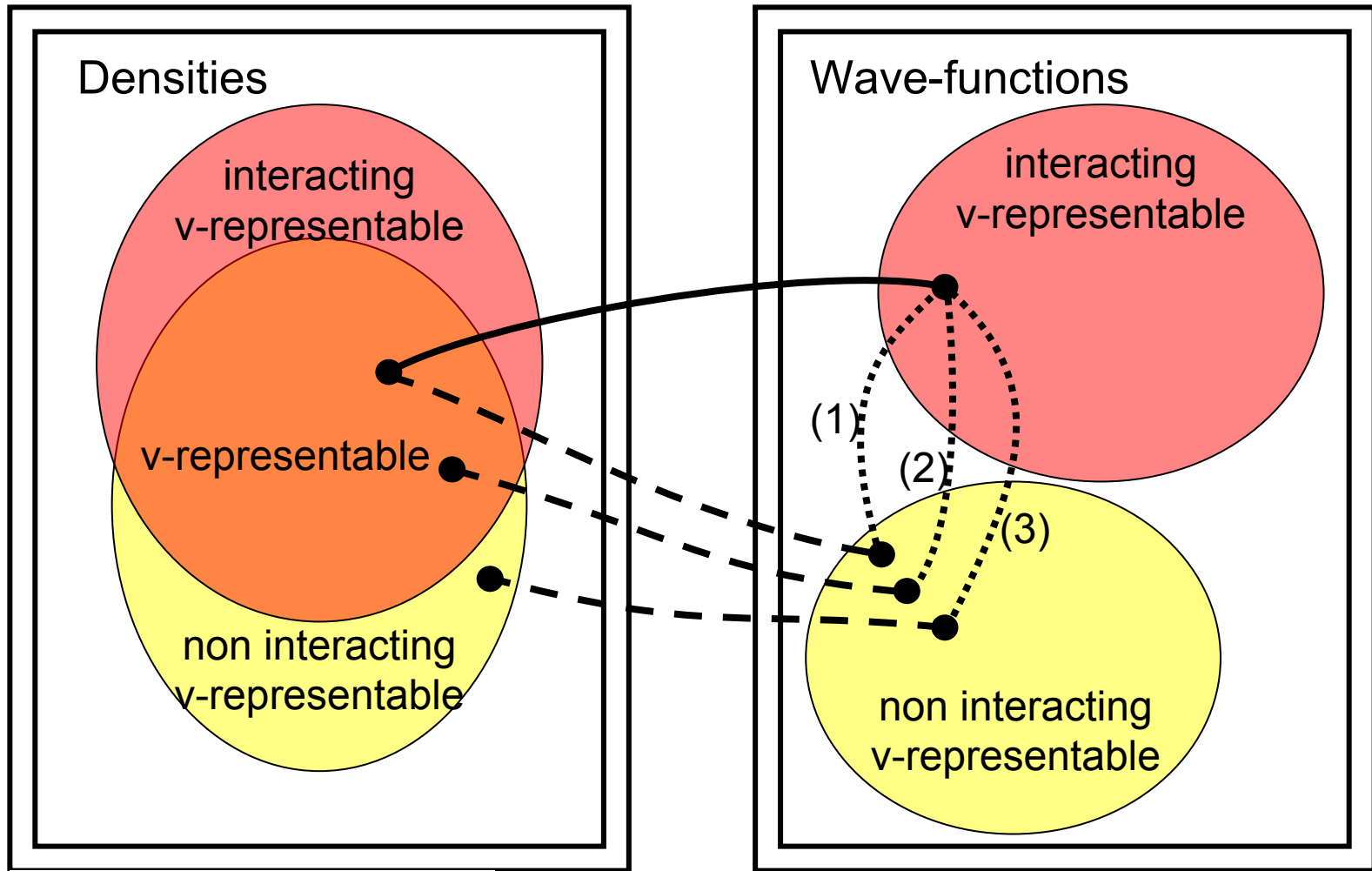
# Retaining other properties of the interacting ground state in the non interacting wave-function



For non-degenerate systems there is at most one Kohn-Sham wave-function with the interacting density

Optimizing other properties thus requires to change the density

# Retaining other properties of the interacting ground state A density-density functional transformation must be found



$$\bar{\rho}_K(\mathbf{r}) = U_K[\rho(\mathbf{r})]$$

Reboredo & Kent PRB (2008)

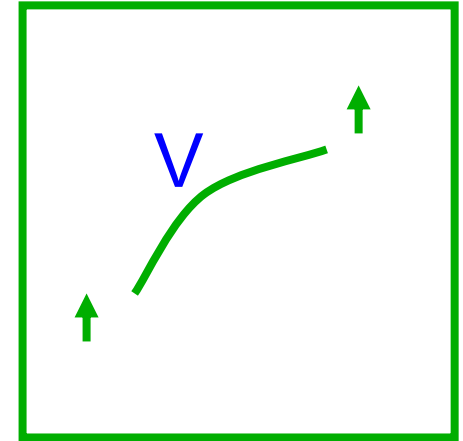
Optimizing other properties thus requires to change the density

# Two interacting particles in a square box

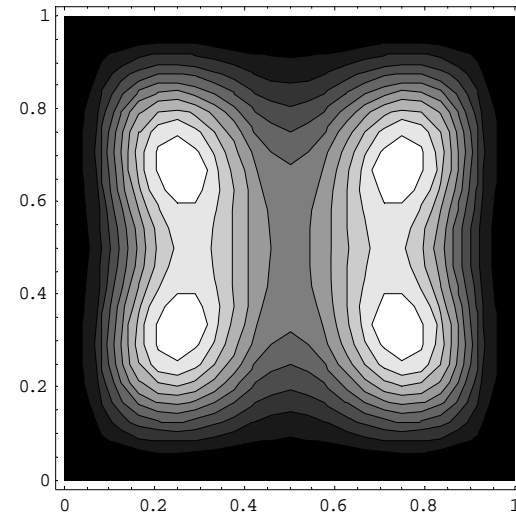
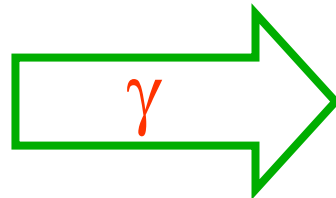
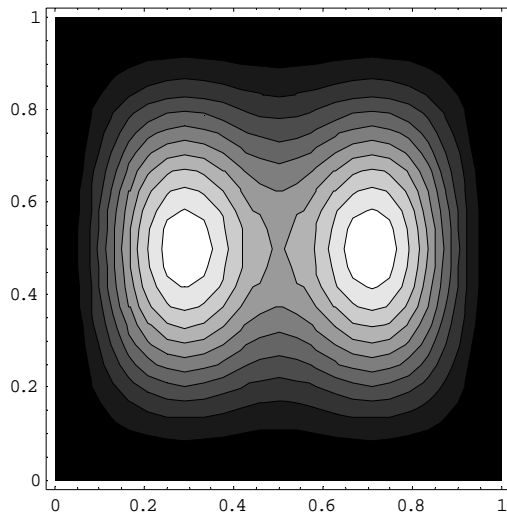
$$V = 8\gamma \cos[\alpha\pi(x-x')] \cos[\alpha\pi(y-y')]$$

$\alpha$  and  $\gamma$  control the shape and strength of  $V$

$V$  is **repulsive** for  $|\alpha| < 1/2$



H expanded in the first 300 non-interacting eigenfunctions with the ground state symmetry. All integrals done analytically. Converged results





# Minimization of cost functions in the v-representable set

$$K_\rho = \frac{1}{2} \int d\mathbf{r} [\bar{\rho}(\mathbf{r}) - \rho(\mathbf{r})]^2$$

Kohn-Sham DFT

$$\Delta \bar{V}_{K_\rho}(\mathbf{r}) = \int d\mathbf{r}' [\rho(\mathbf{r}') - \bar{\rho}(\mathbf{r}')] \frac{\delta V(\mathbf{r}')}{\delta \rho(\mathbf{r})}$$

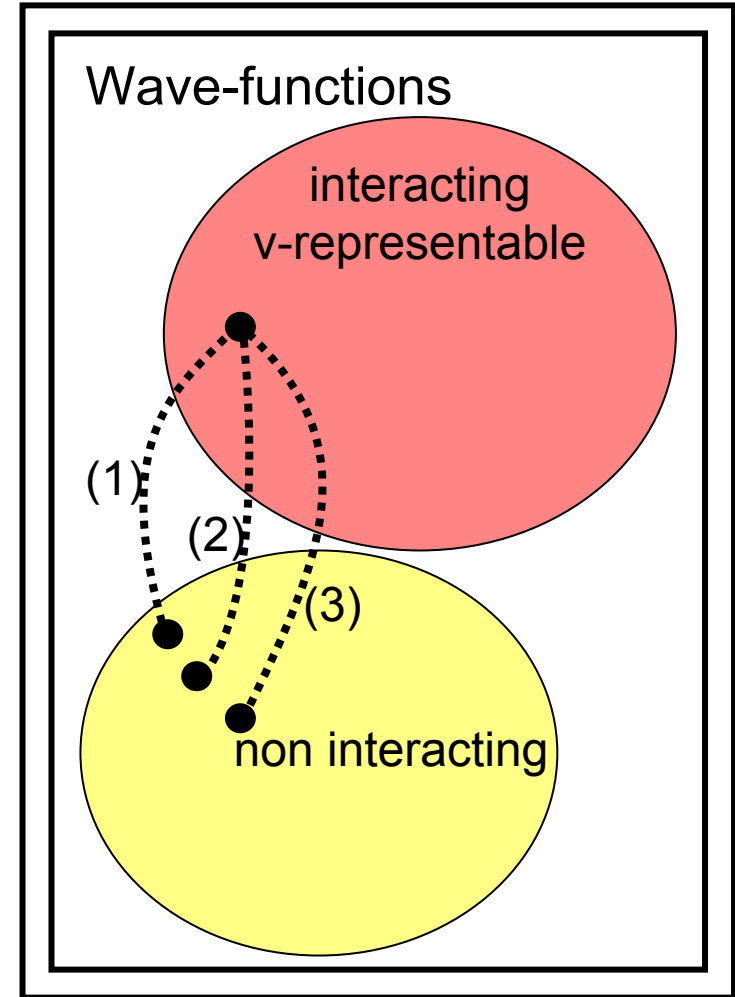
$$K_{Det} = -|\langle \Psi | \Phi_T \rangle|^2$$

$$\delta V_{K_{Det}}(\mathbf{r}) = \epsilon \langle \Psi | \Phi_T \rangle \sum_\nu^o \sum_n^u \langle \Psi | c_n^\dagger c_\nu | \Phi_T \rangle \frac{\phi_n(\mathbf{r}) \phi_{n\nu}(\mathbf{r})}{\epsilon_\nu - \epsilon_n}$$

$$K_{S_0} = \int_{S_0} dS |\Phi_T(\mathbf{R})|^2$$

$$\delta V_{K_{Det}}(\mathbf{r}) = \epsilon \sum_\nu^o \sum_n^u \int_{S_0} dS \Phi_T^{n,\nu}(\mathbf{R}) \Phi_T(\mathbf{R}) \frac{\phi_n(\mathbf{r}) \phi_{n\nu}(\mathbf{r})}{\epsilon_\nu - \epsilon_n}$$

$$\left[ -\frac{1}{2} \nabla^2 + \bar{V}(\mathbf{r}) \right] \phi_\nu(\mathbf{r}) = \epsilon_\nu \phi_\nu(\mathbf{r})$$



**Different properties imply different cost functions and different potentials**

# The density is depends strongly on the cost function

$$K_\rho = \frac{1}{2} \int d\mathbf{r} [\bar{\rho}(\mathbf{r}) - \rho(\mathbf{r})]^2$$

**Exact Kohn-Sham**

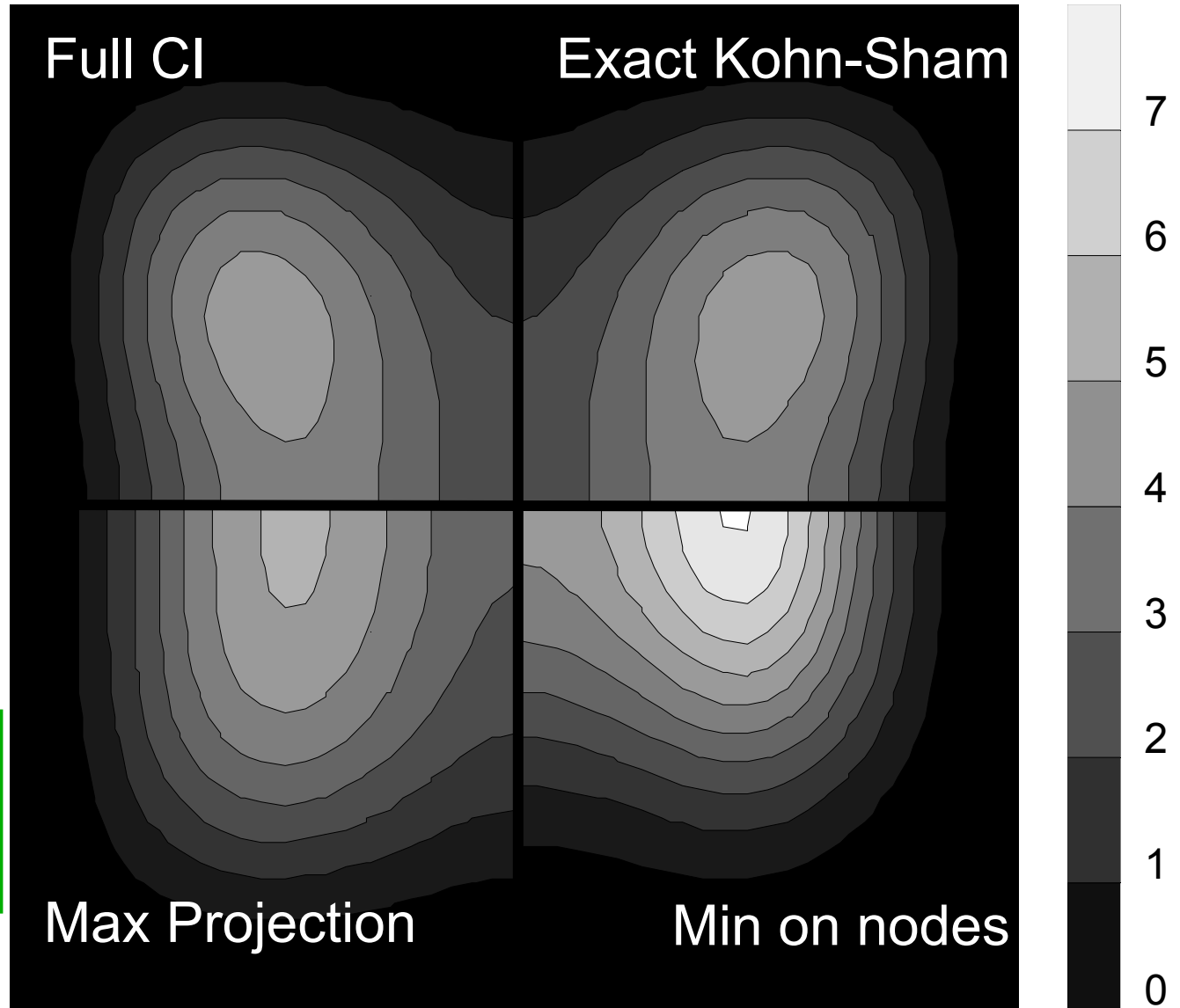
$$K_{Det} = -|\langle \Psi | \Phi_T \rangle|^2$$

**Max Projection**

$$K_{S_0} = \int_{S_0} dS |\Phi_T(\mathbf{R})|^2$$

**Min on nodes**

$$\bar{\rho}_K(\mathbf{r}) = U_K [\rho(\mathbf{r})]$$



**A density-density functional must be found to optimize other properties**

# Optimized effective potentials

$$K_\rho = \frac{1}{2} \int d\mathbf{r} [\bar{\rho}(\mathbf{r}) - \rho(\mathbf{r})]^2$$

**Exact Kohn-Sham**

$$K_{Det} = -|\langle \Psi | \Phi_T \rangle|^2$$

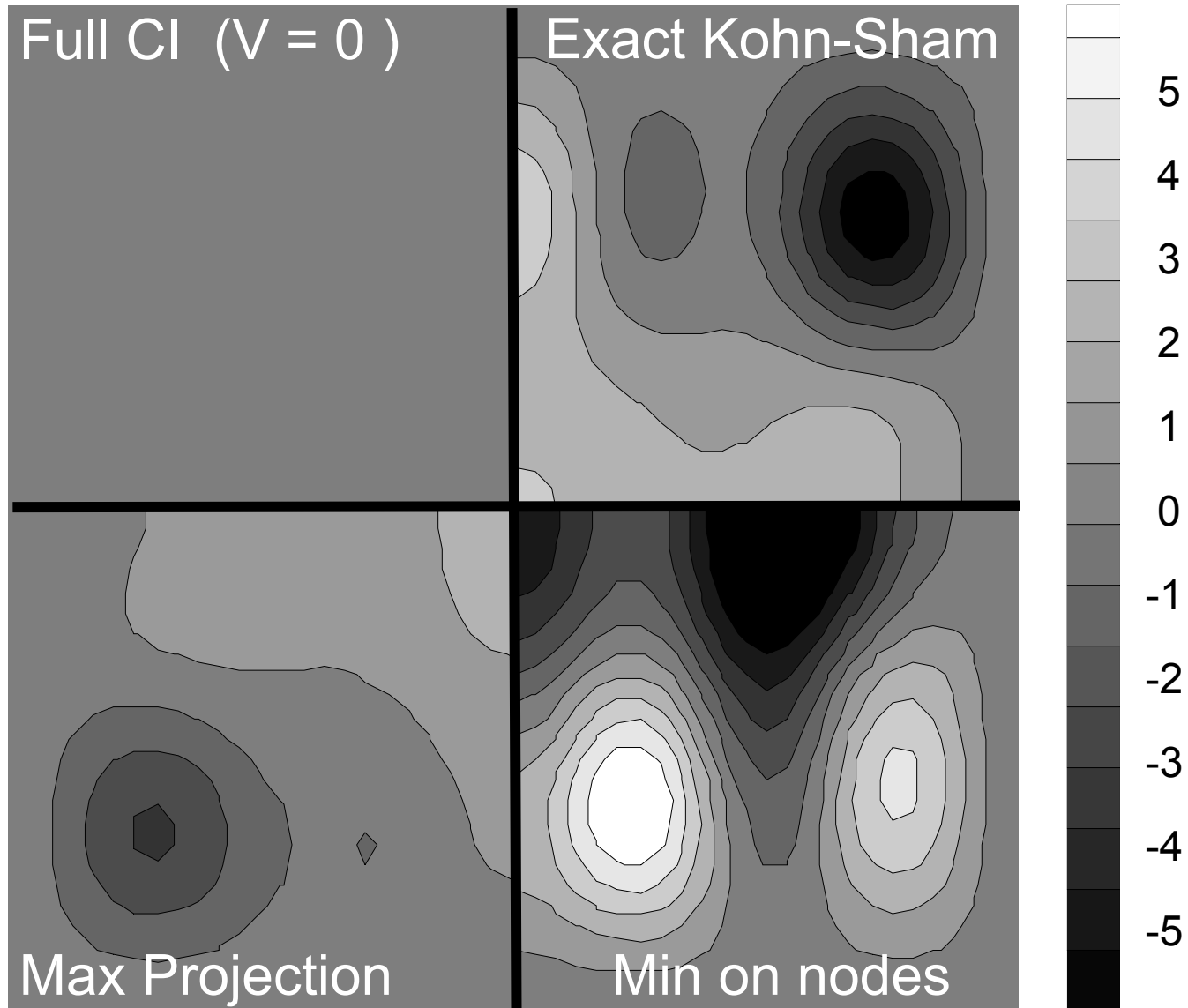
**Max Projection**

$$K_{S_0} = \int_{S_0} dS |\Phi_T(\mathbf{R})|^2$$

**Min on nodes**

Full CI ( $V = 0$ )

Exact Kohn-Sham



Max Projection

Min on nodes

**Effective potentials depend strongly on the many-body property retained**

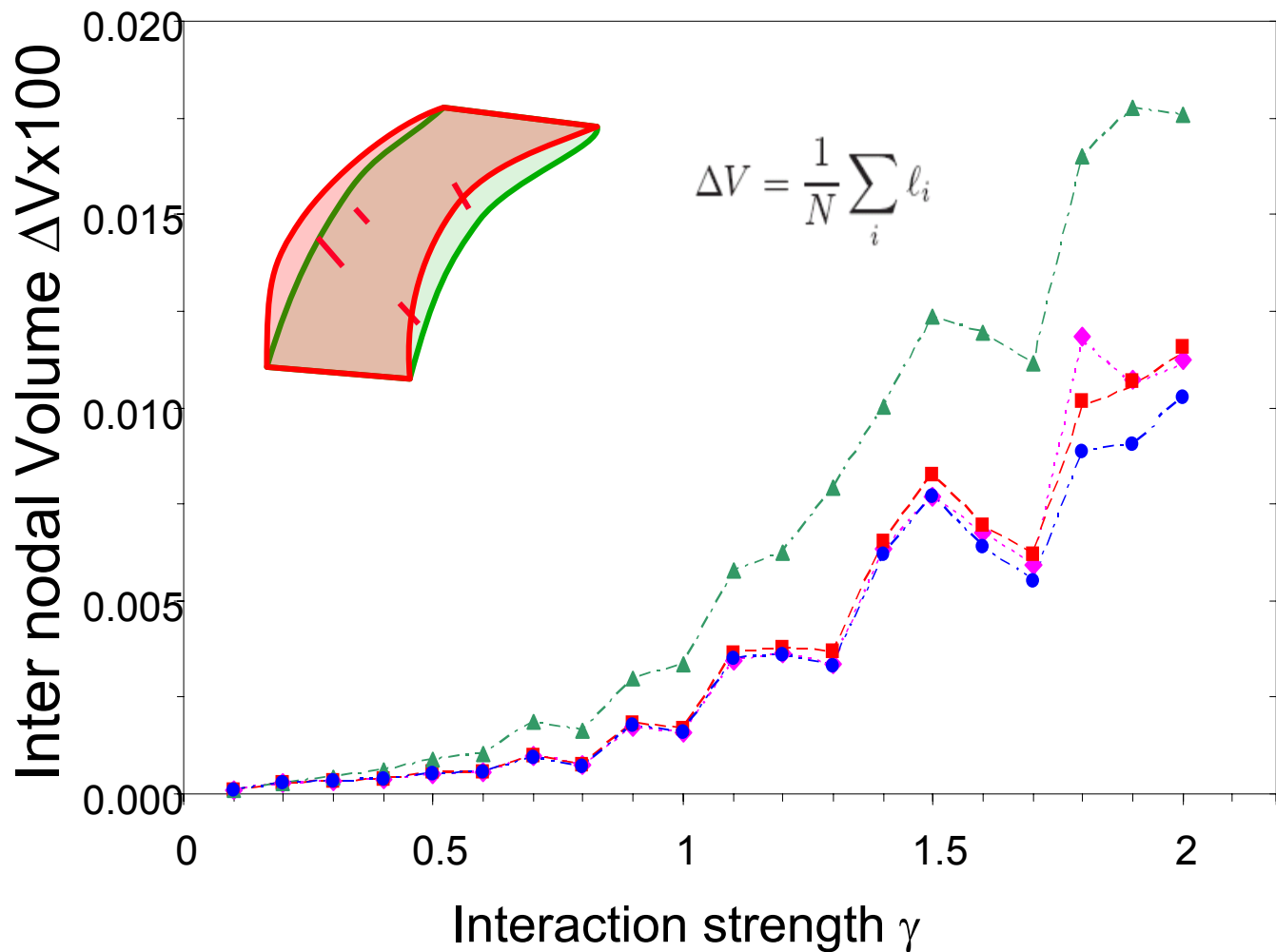
# Volume between interacting and non-interacting nodes

Exact Kohn-Sham

Max Projection

Min on nodes

Non interacting



# Volume between interacting and non-interacting nodes



Exact Kohn-Sham



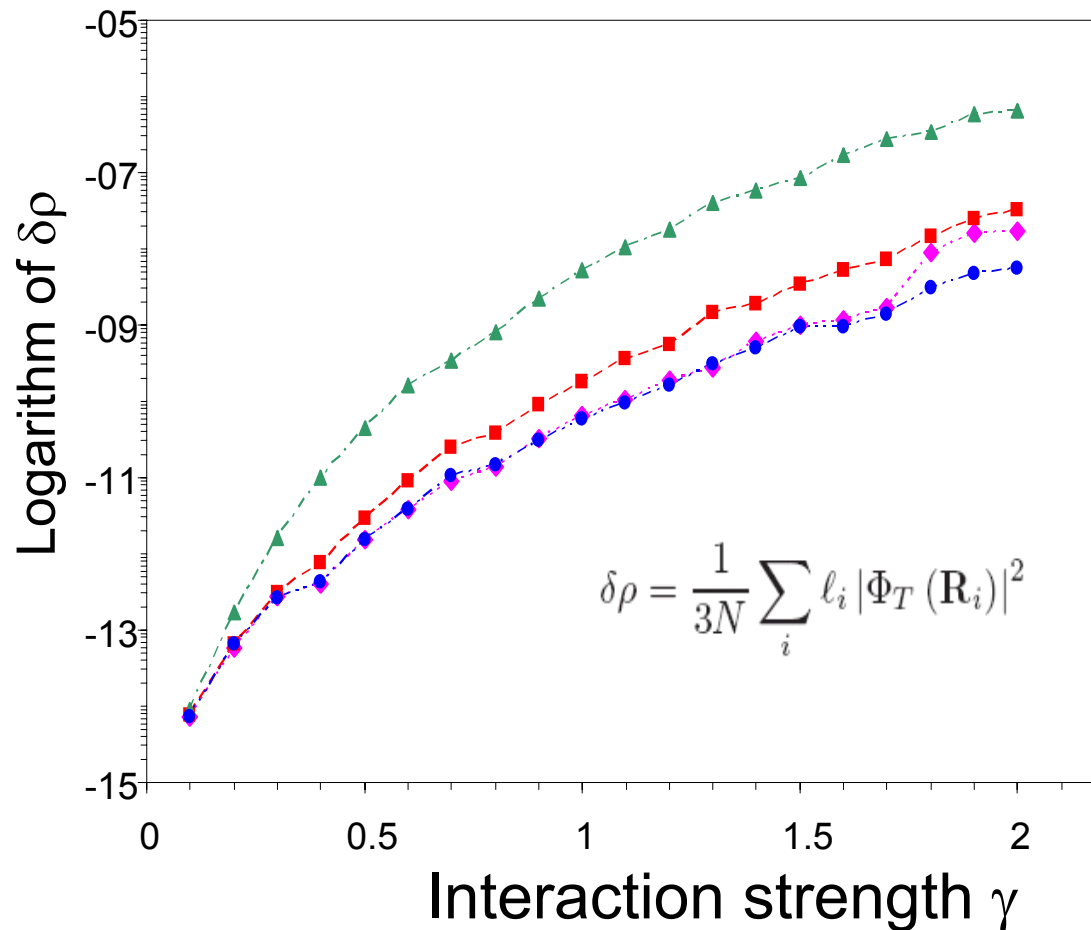
Max Projection



Min on nodes



Non interacting

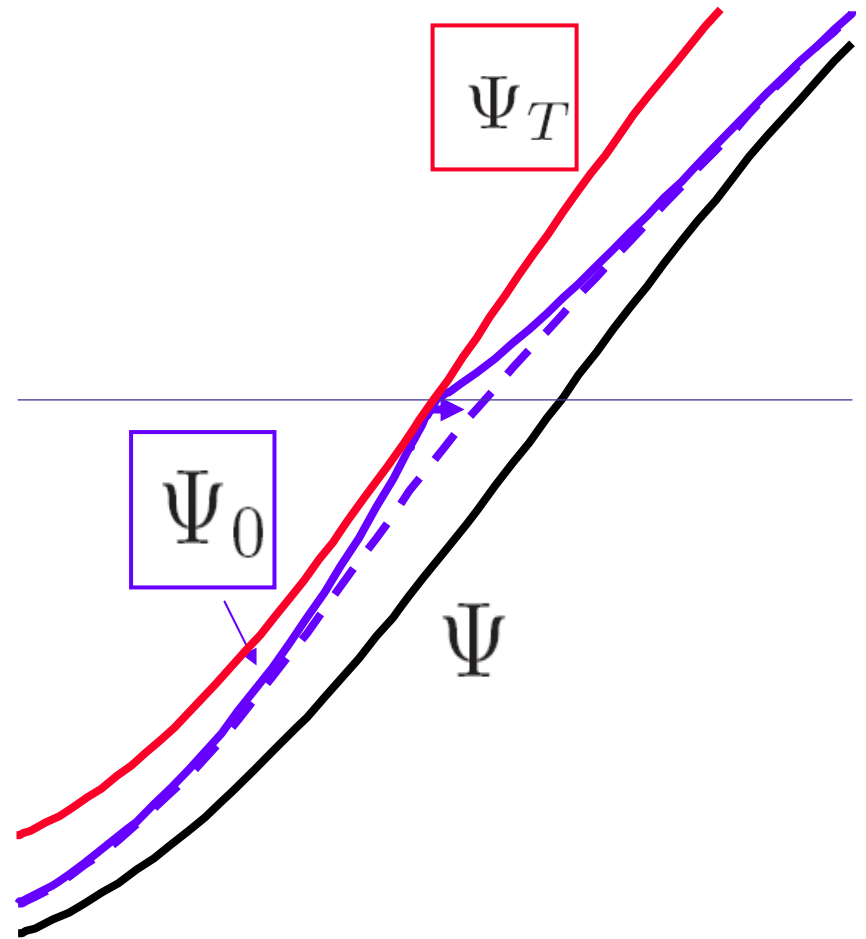


For this model exact DFT is the worst description of the nodes !

**How do we get good nodes for large systems?**

**How do we transform a complex wave-function  
With good properties into a simpler-function with  
the same properties ?**

# What could we do if we new the Fixed Node wave function?



Removing the kink in  $\Psi_T$  moves the node in the right direction

# Standard Importance Sampling Diffusion Monte Carlo Algorithm

$$f(\mathbf{R}) = \frac{N_c}{\nu_T} \Psi_0(\mathbf{R}) \Psi_T(\mathbf{R})$$

Number of configurations

Trial Wave-Function

Fixed-Node Ground State Wave-Function

Distribution of walkers

We know  $\Psi_T(\mathbf{R})$ ,  $N_c$  and  $f(\mathbf{R})$

We want to know  $\Psi_0(\mathbf{R})$  and  $\nu_T$

D. M. Ceperley and B. J. Alder, Phys. Rev. Lett. **45**, 566 (1980)

One equation with three known quantities and two unknowns could be soluble



# The Fixed Node Wave-Function...

$$\Psi_0(\mathbf{R}) = e^{-J(\mathbf{R})} \sum_n \lambda_n \Phi_n(\mathbf{R})$$

Fixed-Node Ground State

$$\xi_n(\mathbf{R}) = e^{2J(\mathbf{R})} \frac{\Phi_n(\mathbf{R})}{\Phi_T(\mathbf{R})}$$

Wave-Function Projectors

$$\langle \tilde{\lambda}_n \rangle = \sum \xi_n(\mathbf{R}_i) \gamma(\mathbf{R}_i)$$

DMC sampling

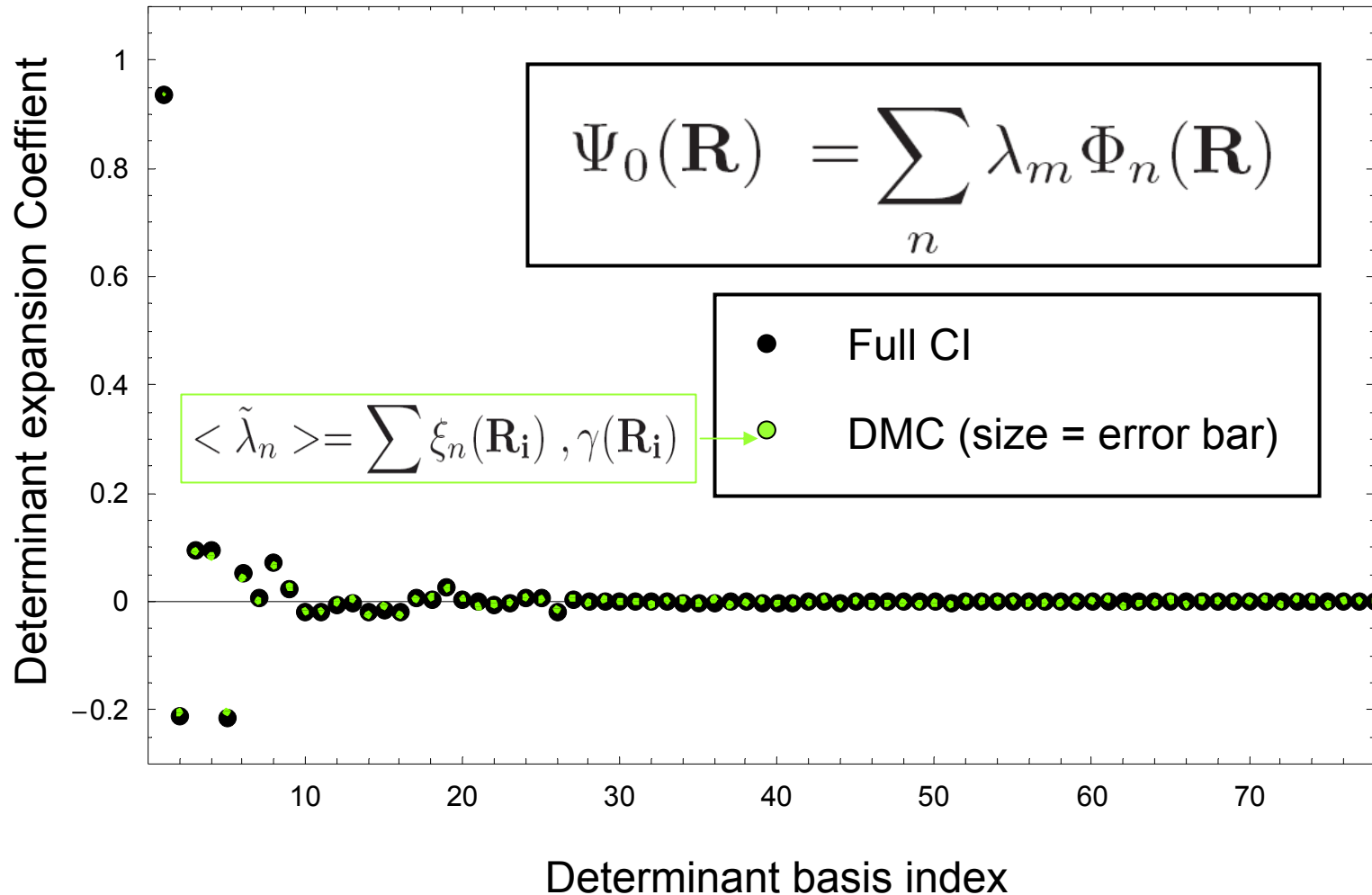
$$\langle \tilde{\lambda}_n^2 \rangle = \sum \xi_n(\mathbf{R}_i)^2 \gamma(\mathbf{R}_i)^2$$

$$\int d\mathbf{R} f(\mathbf{R}) \rightarrow \Sigma$$

....can be obtained directly by sampling over the walker distribution.

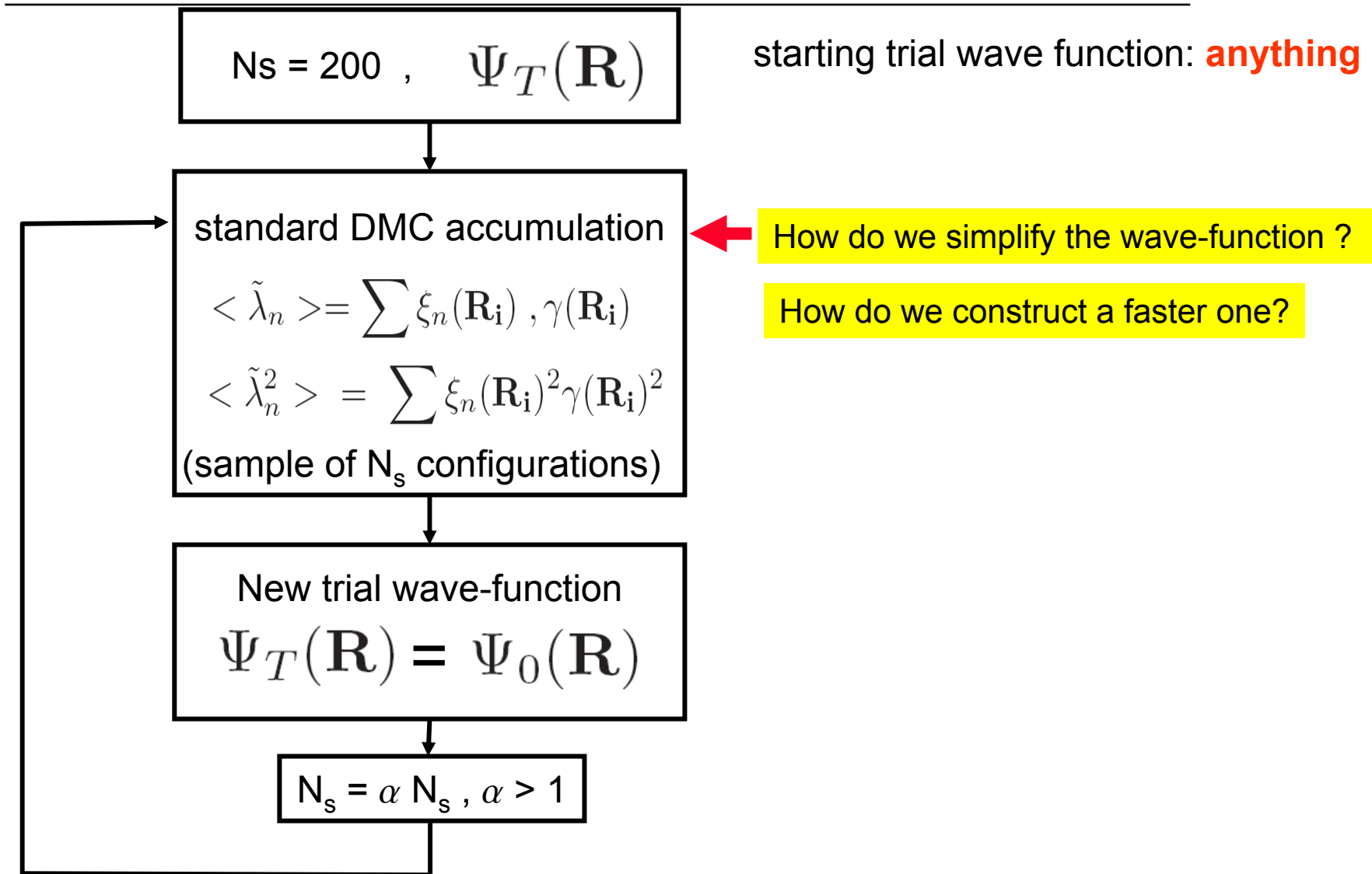
# A Simple Self-Healing DMC Algorithm: Results

## Expansion of the Ground-State Wave-Function

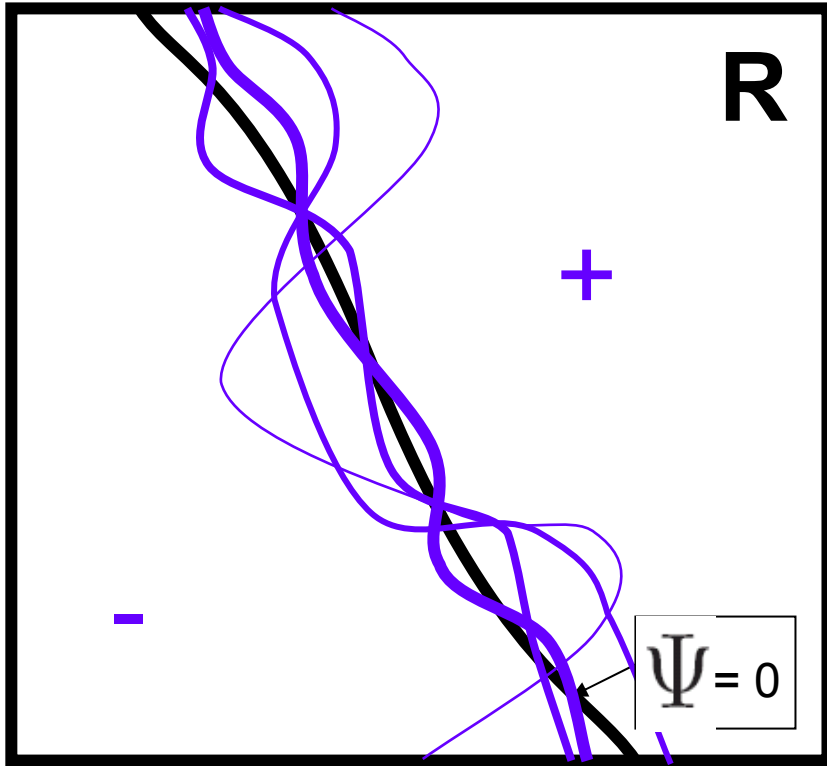


An expansion of the Ground-State Wave-Function can be obtained from DMC with full CI quality

# A Simple Self-Healing DMC Algorithm

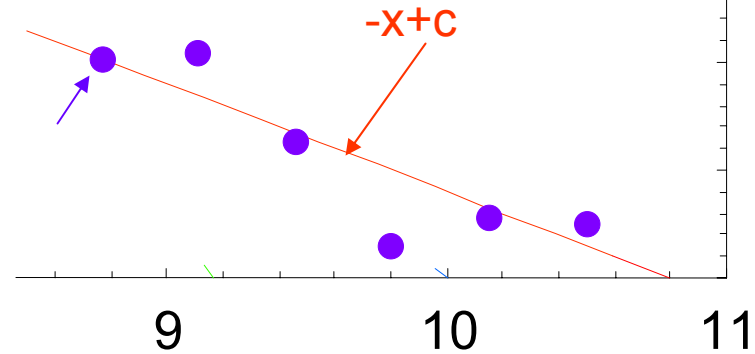


Conjecture (2) Noise in the coefficients plays the role of a temperature in a simulated annealing approach. Good fluctuations are reinforced bad ones are abandoned.



$$R_P = \log [1 - \langle \Psi | \Psi_T \rangle / (|\Psi| |\Psi_T|)]$$

Statistical Improvement  
(good nodes)



# Conjecture (3) Removing the high energy contribution removes the kink at the node and improves it

1) 
$$\Psi_0(\mathbf{R}) = e^{-J(\mathbf{R})} \sum_n \lambda_n \Phi_n(\mathbf{R}) \quad \boxed{\int d\mathbf{R} \Phi_n(\mathbf{R}) \Phi_m(\mathbf{R}) = \delta_{n,m}}$$

$\Phi_n(\mathbf{R})$  Are non-interacting eigen-functions:  
 $\Rightarrow$  increasing kinetic energy

2) 
$$\lambda_n = 0 \quad \text{if} \quad \left| \langle \tilde{\lambda}_n \rangle \right| < \frac{\langle \tilde{\sigma}_n \rangle}{\sqrt{N_c - 1}} \times 4$$
  
else  $\lambda_n = \langle \tilde{\lambda}_n \rangle$

3) We set 
$$\Psi_T(\mathbf{R}) = \Psi_0(\mathbf{R})$$

# A cost function for optimization of trial wave functions for Importance Sampling

Chi-Squared distribution

$$\chi^2 = \sum_i^M \frac{[n_i - N_c \Omega_i \bar{p}(\mathbf{R}_i)]^2}{N_c \Omega_i \bar{p}(\mathbf{R}_i)}$$

Relative errors have the same Impact for every bin

How close are two probability densities?

$$K_{pq} = \int d\mathbf{r} \frac{[q(\mathbf{R}) - p(\mathbf{R})]^2}{p(\mathbf{R})}$$

$$K_{DMC} = \int d\mathbf{R} \frac{\left[ \frac{N_c}{\tilde{\nu}} \tilde{\Psi}_T(\mathbf{R}) \Psi_T(\mathbf{R}) - f(\mathbf{R}) \right]^2}{\left| \frac{N_c}{\tilde{\nu}} \tilde{\Psi}_T(\mathbf{R}) \Psi_T(\mathbf{R}) \right|^\alpha} \times \theta[f(\mathbf{R}) - \eta]$$

This cost function extended to the full Hilbert space by the  $||$  and the  $\theta[ ]$

# A cost function for optimization of trial wave functions for Importance Sampling

$$K_{DMC} = \int d\mathbf{R} \frac{\left[ \frac{N_c}{\tilde{\nu}} \tilde{\Psi}_T(\mathbf{R}) \Psi_T(\mathbf{R}) - f(\mathbf{R}) \right]^2}{\left| \frac{N_c}{\tilde{\nu}} \tilde{\Psi}_T(\mathbf{R}) \Psi_T(\mathbf{R}) \right|} \times \theta[f(\mathbf{R}) - \eta]$$

$$f(\mathbf{R}) = \frac{N_c}{\nu_T} \Psi_0(\mathbf{R}) \Psi_T(\mathbf{R}) \quad \tilde{\nu} = \int d\mathbf{R} \tilde{\Psi}_T(\mathbf{R}) \Psi_T(\mathbf{R})$$

Any expression of the Ground State Wave-Function

- Single determinant
- Multi determinant
- Back flow
- Pfaffian
- etc

$$\delta V_K(\mathbf{r}) = -\epsilon \sum_{\nu}^{occ} \int d\mathbf{r}' \frac{\delta K}{\delta \phi_{\nu}(\mathbf{r}')} \frac{\delta \phi_{\nu}(\mathbf{r}')}{\delta \bar{V}(\mathbf{r})}$$

$$\left[ -\frac{1}{2} \nabla^2 + \bar{V}(\mathbf{r}) \right] \phi_{\nu}(\mathbf{r}) = \epsilon_{\nu} \phi_{\nu}(\mathbf{r})$$

wave functions can be optimized  
via an effective field potential ( $\nu$ -representable set)

# A cost function for optimization of trial wave functions via an effective field DFT-like potential

$$K_{DMC} = \int d\mathbf{R} \frac{\left[ \frac{N_c}{\tilde{\nu}} \tilde{\Psi}_T(\mathbf{R}) \Psi_T(\mathbf{R}) - f(\mathbf{R}) \right]^2}{\left| \frac{N_c}{\tilde{\nu}} \tilde{\Psi}_T(\mathbf{R}) \Psi_T(\mathbf{R}) \right|} \times \theta[f(\mathbf{R}) - \eta]$$

$$f(\mathbf{R}) = \frac{N_c}{\nu_T} \Psi_0(\mathbf{R}) \Psi_T(\mathbf{R}) \quad \tilde{\nu} = \int d\mathbf{R} \tilde{\Psi}_T(\mathbf{R}) \Psi_T(\mathbf{R})$$

$$\delta V_K(\mathbf{r}) = -\epsilon \sum_{\nu}^{occ} \int d\mathbf{r}' \frac{\delta K}{\delta \phi_{\nu}(\mathbf{r}')} \frac{\delta \phi_{\nu}(\mathbf{r}')}{\delta \bar{V}(\mathbf{r})}$$

$$\frac{\delta K_{DMC}}{\delta \phi_{\nu}(\mathbf{r}')} = \int d\mathbf{R} W(\mathbf{R}) e^{-\tilde{J}(\mathbf{R})} \frac{\delta \tilde{\Phi}(\mathbf{R})}{\delta \phi_{\nu}(\mathbf{r}')}$$

$$W(\mathbf{R}) = \frac{\delta K_{DMC}}{\delta \tilde{\Psi}(\mathbf{R})}$$



# A cost function for optimization of trial wave functions via an effective field DFT-like potential

$$K_{DMC} = \int d\mathbf{R} \frac{\left[ \frac{N_c}{\tilde{\nu}} \tilde{\Psi}_T(\mathbf{R}) \Psi_T(\mathbf{R}) - f(\mathbf{R}) \right]^2}{\left| \frac{N_c}{\tilde{\nu}} \tilde{\Psi}_T(\mathbf{R}) \Psi_T(\mathbf{R}) \right|} \times \theta[f(\mathbf{R}) - \eta]$$

$$f(\mathbf{R}) = \frac{N_c}{\nu_T} \Psi_0(\mathbf{R}) \Psi_T(\mathbf{R}) \qquad \tilde{\nu} = \int d\mathbf{R} \tilde{\Psi}_T(\mathbf{R}) \Psi_T(\mathbf{R})$$

---


$$\delta V_{K_{DMC}}(\mathbf{r}) = \epsilon \sum_{\nu}^o \sum_n^u \frac{\phi_n(\mathbf{r}) \phi_{n\nu}(\mathbf{r})}{\epsilon_{\nu} - \epsilon_n} \beta_{\nu}^n + c.c.$$

$$\beta_{\nu}^n = \int d\mathbf{R} W(\mathbf{R}) e^{-\tilde{J}(\mathbf{R})} \tilde{\Phi}_{\nu}^n(\mathbf{R}),$$

$$W(\mathbf{R}) = \frac{2\Delta(\mathbf{R})\alpha(\mathbf{R}) - \Delta(\mathbf{R})^2}{|\alpha(\mathbf{R})|^2} \text{sign}(\alpha(\mathbf{R}))$$

$$\times [1 - \alpha(\mathbf{R})] \psi_T(\mathbf{R}) \frac{N_c}{\tilde{\nu}} \theta[f(\mathbf{R}) - \eta]$$

$$\Delta(\mathbf{R}) = f(\mathbf{R}) - \alpha(\mathbf{R})$$

$$\alpha(\mathbf{R}) = \frac{N_c}{\tilde{\nu}} \tilde{\Psi}_T(\mathbf{R}) \Psi_T(\mathbf{R})$$

# Optimization of the Jastrow Parameters directly from the DMC run

$$K_{DMC} = \int d\mathbf{R} \frac{\left[ \frac{N_c}{\tilde{\nu}} \tilde{\Psi}_T(\mathbf{R}) \Psi_T(\mathbf{R}) - f(\mathbf{R}) \right]^2}{\left| \frac{N_c}{\tilde{\nu}} \tilde{\Psi}_T(\mathbf{R}) \Psi_T(\mathbf{R}) \right|} \times \theta[f(\mathbf{R}) - \eta]$$

$$f(\mathbf{R}) = \frac{N_c}{\nu_T} \Psi_0(\mathbf{R}) \Psi_T(\mathbf{R}) \quad \tilde{\nu} = \int d\mathbf{R} \tilde{\Psi}_T(\mathbf{R}) \Psi_T(\mathbf{R})$$

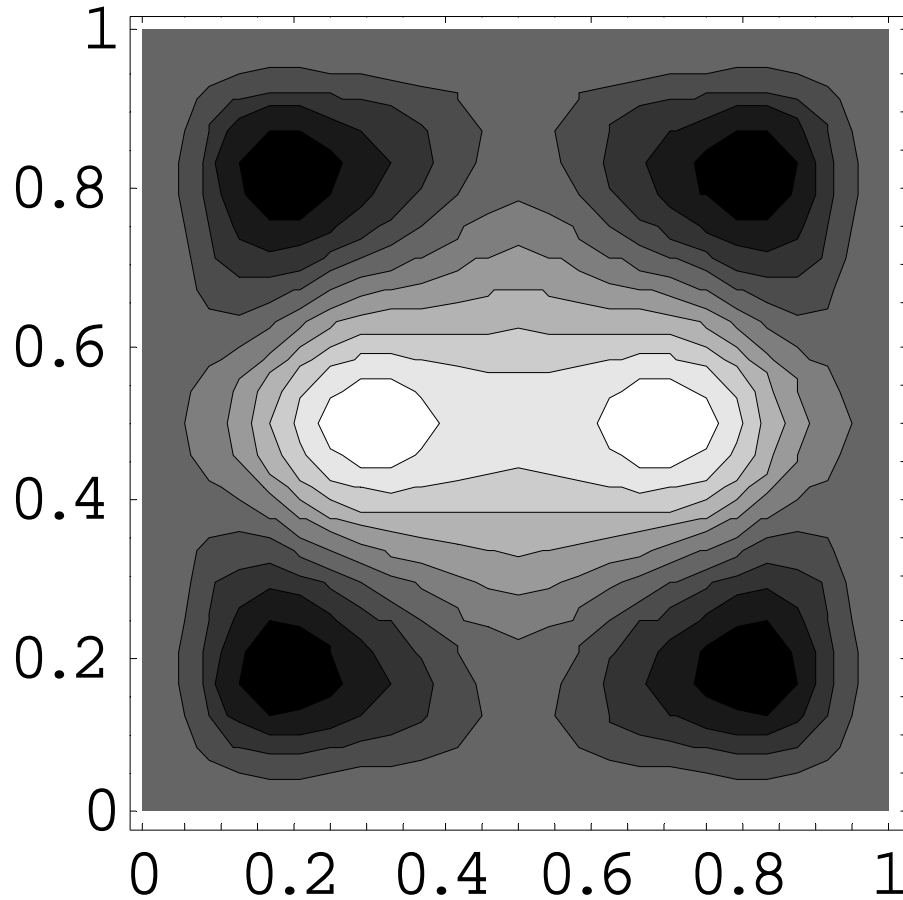
$$\frac{dK_{DMC}}{d\gamma_n} = - \int d\mathbf{R} W(\mathbf{R}) e^{-\tilde{J}(\mathbf{R})} \tilde{\Phi}_T(\mathbf{R}) \frac{dJ(\mathbf{R})}{d\gamma_n}$$

$$W(\mathbf{R}) = \frac{2\Delta(\mathbf{R})\alpha(\mathbf{R}) - \Delta(\mathbf{R})^2}{|\alpha(\mathbf{R})|^2} \text{sign}(\alpha(\mathbf{R})) \\ \times [1 - \alpha(\mathbf{R})] \psi_T(\mathbf{R}) \frac{N_c}{\tilde{\nu}} \theta[f(\mathbf{R}) - \eta]$$

$$\Delta(\mathbf{R}) = f(\mathbf{R}) - \alpha(\mathbf{R})$$

$$\alpha(\mathbf{R}) = \frac{N_c}{\tilde{\nu}} \tilde{\Psi}_T(\mathbf{R}) \Psi_T(\mathbf{R})$$

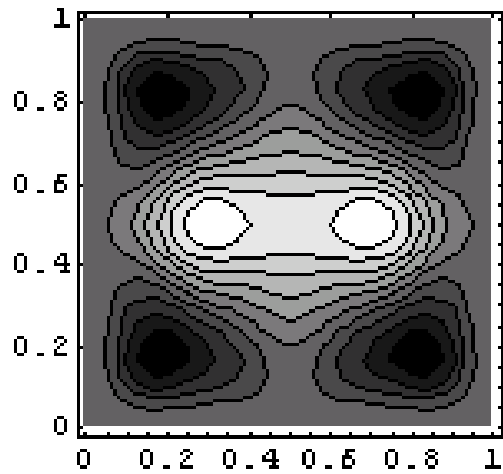
# Nodal Effective potential



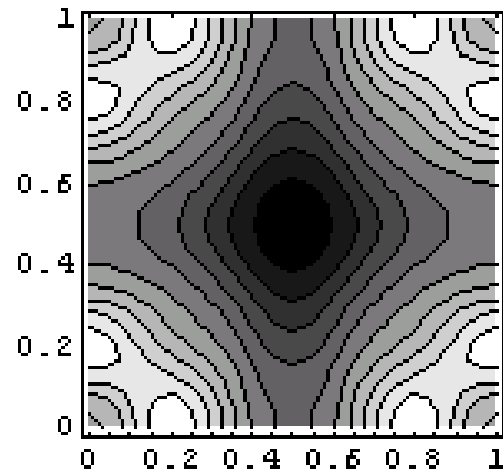
V-representable nodal surfaces

# The wave function can be optimized directly from the DMC run

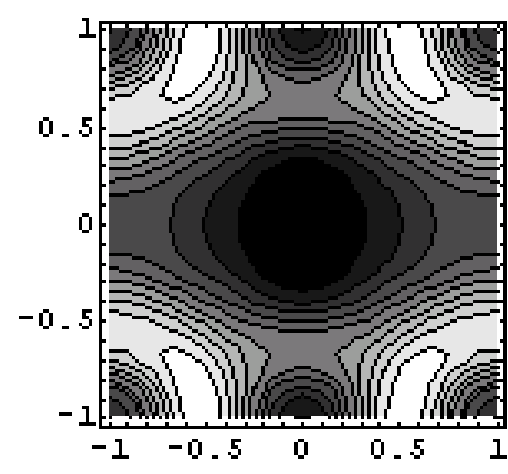
$V(r)$



$\chi(r)$



$J^2(r)$



# Optimization of **Back Flaw Parameters** directly from the DMC run

$$K_{DMC} = \int d\mathbf{R} \frac{\left[ \frac{N_c}{\tilde{\nu}} \tilde{\Psi}_T(\mathbf{R}) \Psi_T(\mathbf{R}) - f(\mathbf{R}) \right]^2}{\left| \frac{N_c}{\tilde{\nu}} \tilde{\Psi}_T(\mathbf{R}) \Psi_T(\mathbf{R}) \right|} \times \theta[f(\mathbf{R}) - \eta]$$

$$\tilde{\nu} = \int d\mathbf{R} \tilde{\Psi}_T(\mathbf{R}) \Psi_T(\mathbf{R}) \quad f(\mathbf{R}) = \frac{N_c}{\nu_T} \Psi_0(\mathbf{R}) \Psi_T(\mathbf{R})$$

$$\frac{dK_{DMC}}{da_n} = - \int d\mathbf{R} W(\mathbf{R}) e^{-\tilde{J}(\mathbf{R})} \sum_{\nu}^{occ} \frac{\delta \tilde{\Phi}(\mathbf{R})}{\delta \phi_{\nu}} \nabla \phi_{\nu}(\mathbf{r}) \cdot (\delta \mathbf{r} / da_n |_{\mathbf{r}})$$

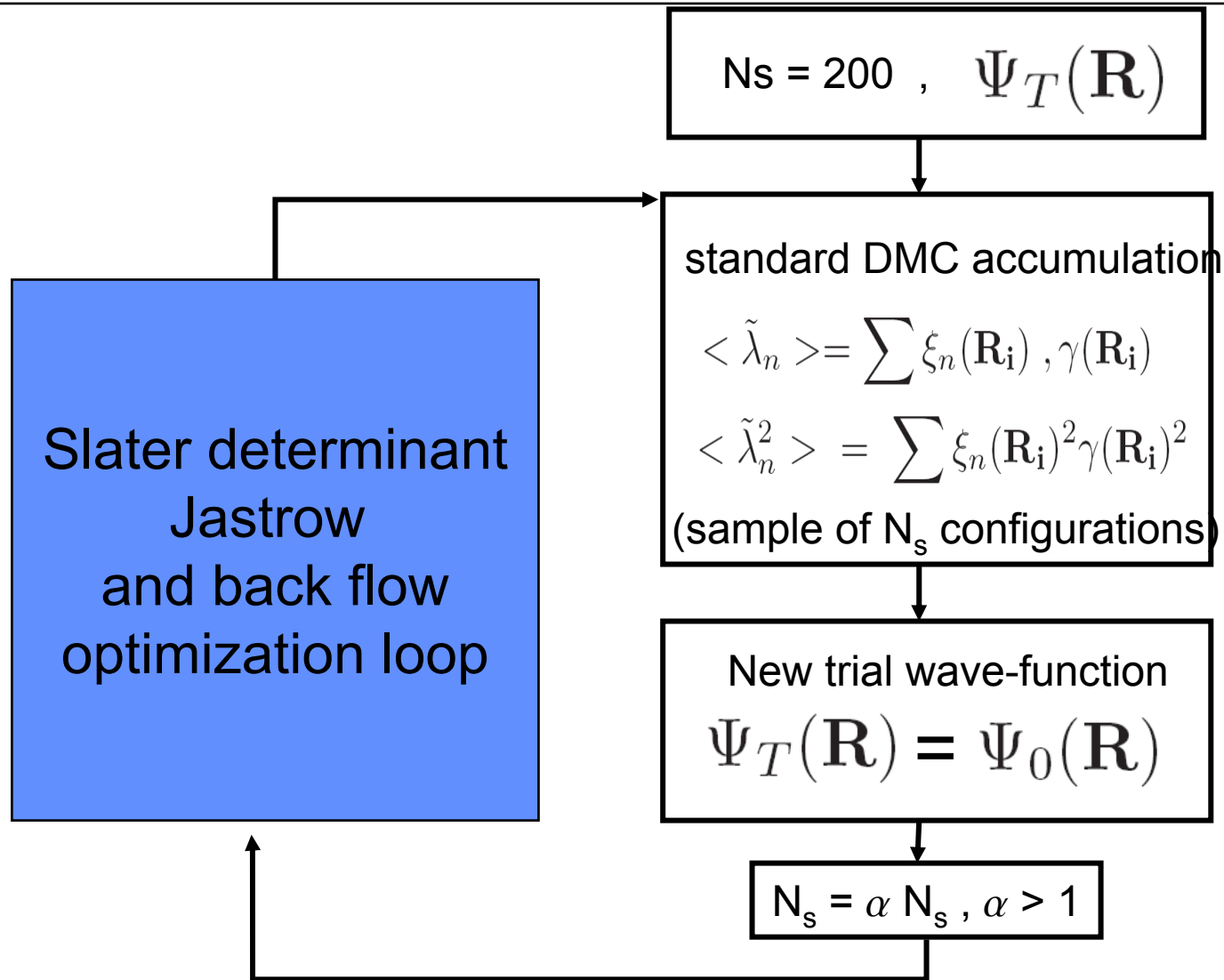
$$W(\mathbf{R}) = \frac{2\Delta(\mathbf{R})\alpha(\mathbf{R}) - \Delta(\mathbf{R})^2}{|\alpha(\mathbf{R})|^2} \text{sign}(\alpha(\mathbf{R}))$$

$$\times [1 - \alpha(\mathbf{R})] \psi_T(\mathbf{R}) \frac{N_c}{\tilde{\nu}} \theta[f(\mathbf{R}) - \eta]$$

$$\Delta(\mathbf{R}) = f(\mathbf{R}) - \alpha(\mathbf{R})$$

$$\alpha(\mathbf{R}) = \frac{N_c}{\tilde{\nu}} \tilde{\Psi}_T(\mathbf{R}) \Psi_T(\mathbf{R})$$

# The Complete Self-Healing DMC Algorithm



# Conclusions

- The fixed node ground state wave-function can be obtained directly by sampling over the walker distribution of a DMC run
- Single determinant Wave functions with backflow can be optimized directly from the DMC run reducing the impact of non-polynomial costs
- Conjecture (1) Removing the kink in the Fixed node wave-function always improves the nodes globally
- Conjecture (2) Random errors in the wave-function expansion improve the nodes in a simulating annealing-like process
- Conjecture (3) Removing the high energy contribution removes the kink at the node and improves it
- Provided that conjectures (1) to (3) are true, the sign problem can be circumvented accepting a finite error
- The cost of reducing this error grows exponentially with the number of particles

Work supported by DOE-BES

