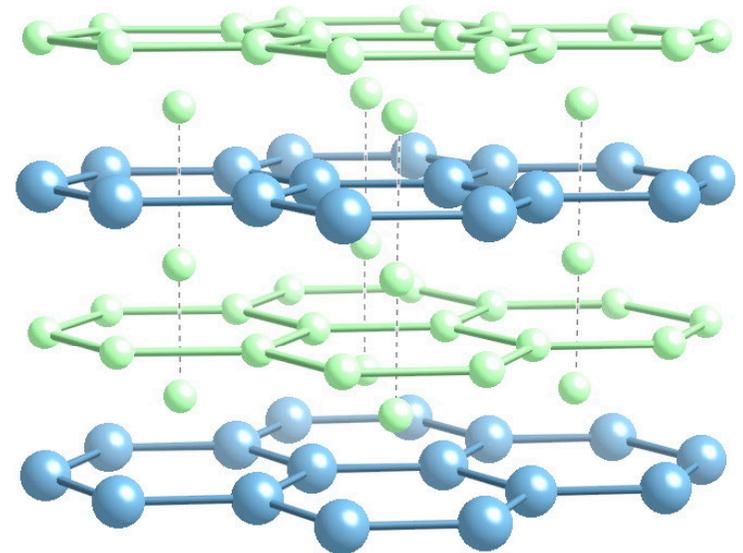


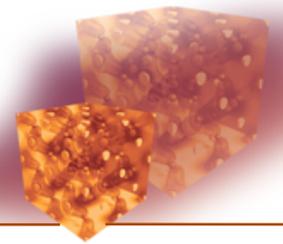
# Emergent 2d behaviour in a 3d electronic environment: Dense Be-Li alloys

**Richard G. Hennig, Ji Feng, Neil W. Ashcroft and Roald Hoffmann**

- Do Be and Li form alloys?
- What is their electronic structure?
- Can they have higher superconducting temperatures than pure Be and Li?



# The Beryllium Story



## Elemental beryllium

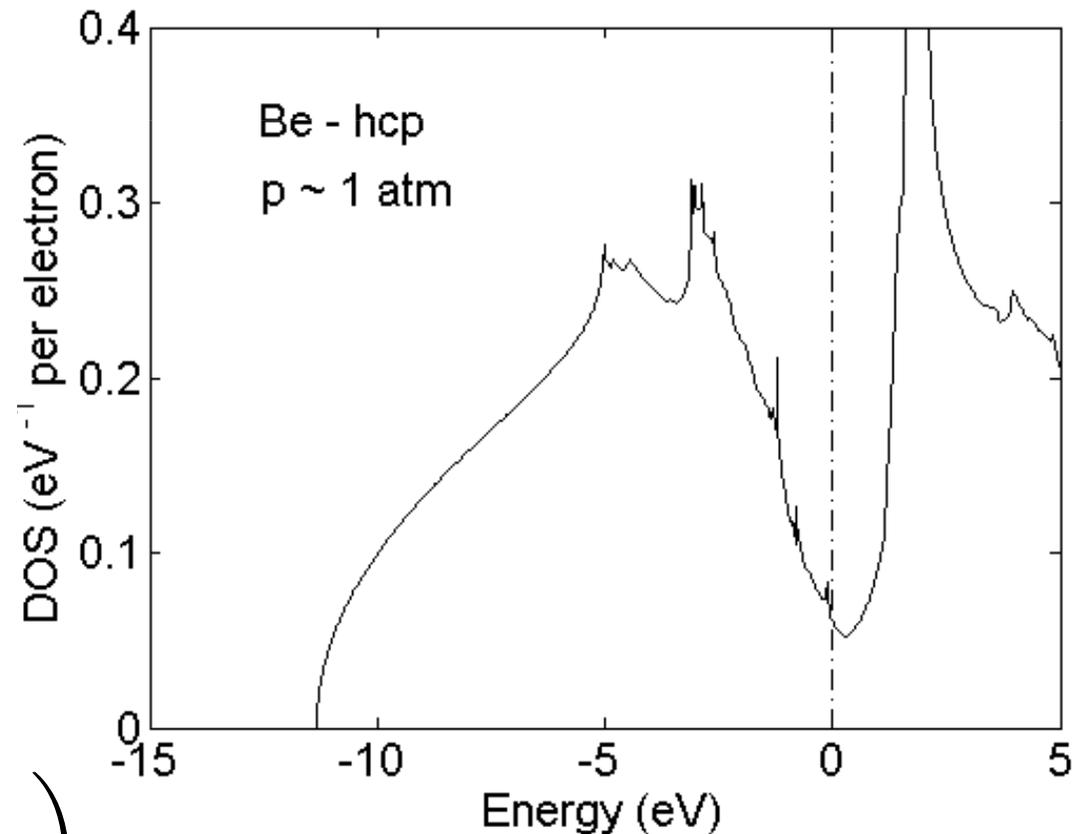
- Highest Debye temperature of all metallic elements:  $\Theta_D = 1100$  K
- Superconducting transition temperature of only  $T_c = 26$  mK
- Because Be is barely a metal

## Improve $T_c$ by alloying

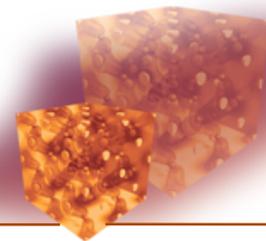
- BCS formula

$$T_c = 1.13 \cdot \theta_D \exp\left(-\frac{1}{g_0 \cdot V}\right)$$

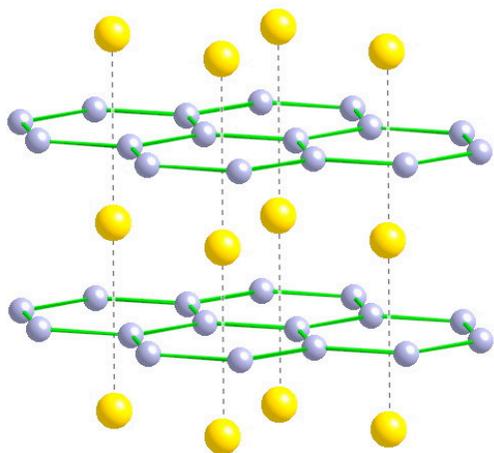
- If  $g_0V$  **increases**  $g'_0 \cdot V' = \alpha \cdot g_0 \cdot V$   $T'_c = T_c \cdot \frac{(2.3 \cdot 10^{-5})^{1/\alpha}}{2.3 \cdot 10^{-5}}$
- Alloying with light elements: Light, metallic, electropositive



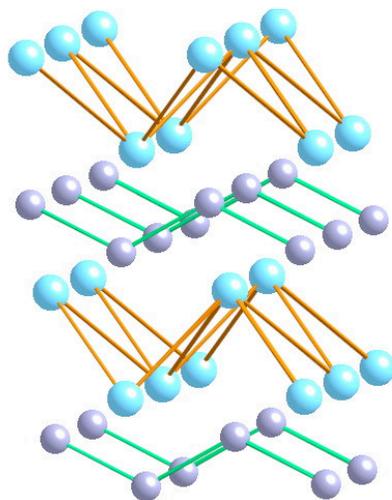
# Structural Prediction



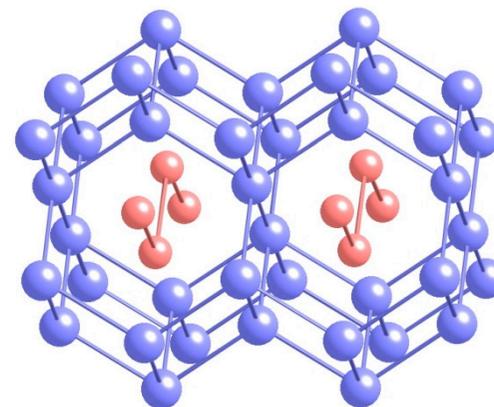
- Do we understand the structures of intermetallic compounds?
- Can we predict the structure of compounds given the stoichiometry?



MgB<sub>2</sub>



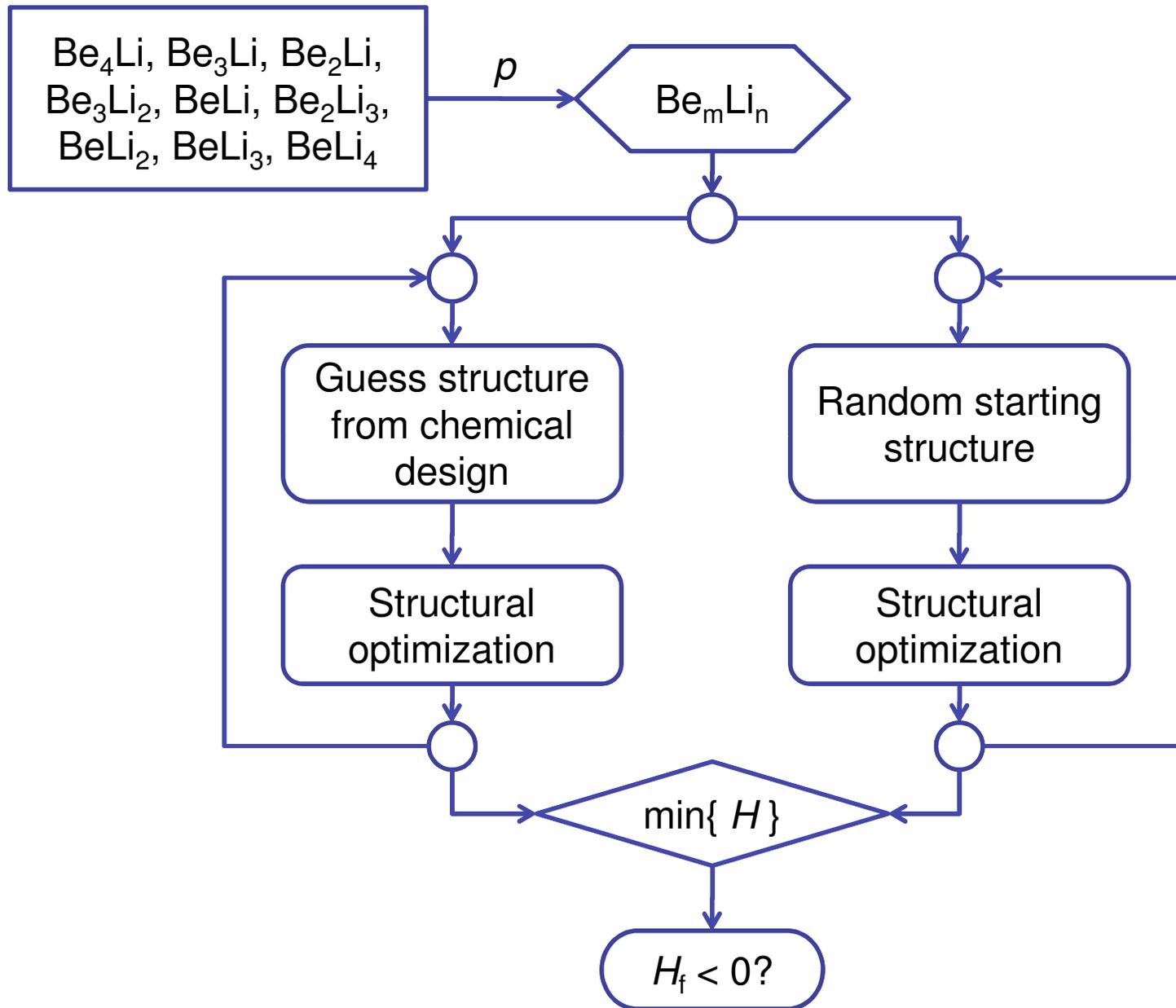
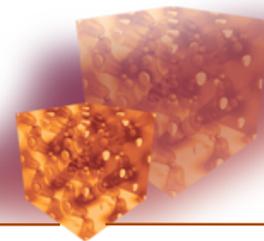
CrB



GaLi<sub>2</sub>

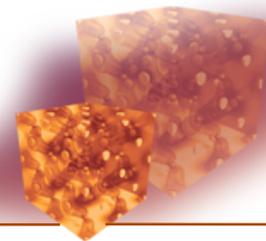
- Now, what about MgB<sub>4</sub> or Cr<sub>0.08</sub>B<sub>3.10</sub> ?
- Or prediction of high pressure phases?

# Structural Search Algorithm





# Computational Detail



## Density functional theory

- Generalized gradient approximation (PBE)
- Plane-wave basis and PAW potentials
- Optimization of all parameters

## Random structural search

- Use 20 – 50 starting structures at each selected pressure and composition
- Pressure range 0 – 200 GPa
- Symmetry identification using ISOTROPY (Stokes & Hatch, BYU)
- Check energy of higher symmetry structures
- Choice of compositions:  
 $\text{Be}_{1-x}\text{Li}_x$   $x = 0, 20, 25, 33, 40, 50, 60, 66, 75, 100 \%$

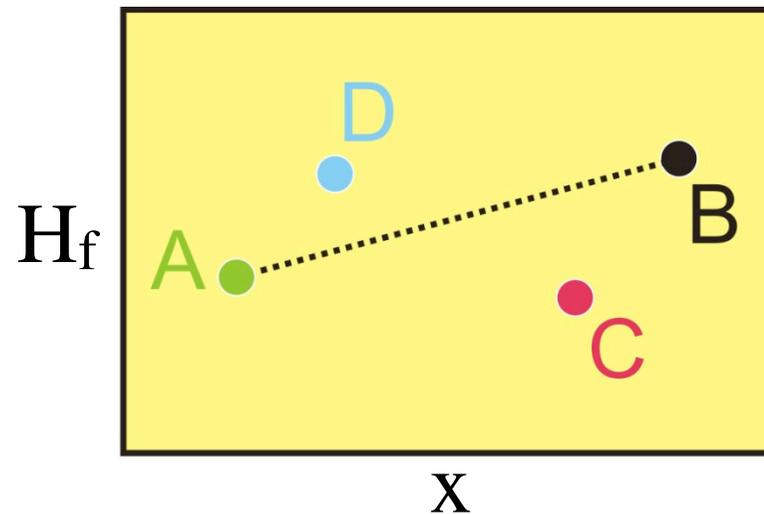
# Enthalpy of Formation & Phase Diagram



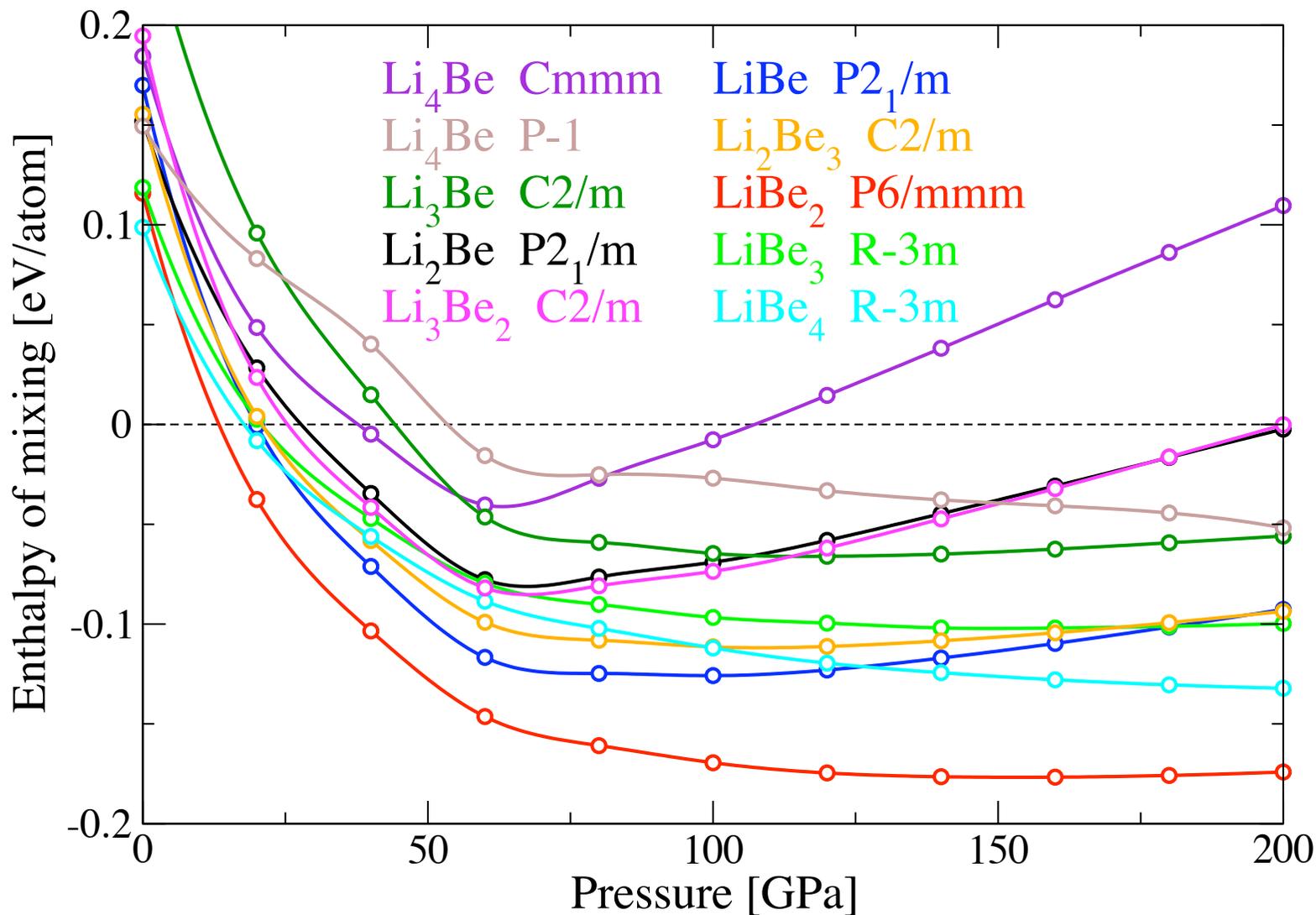
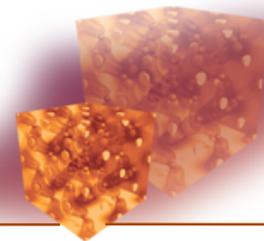
- Enthalpy of formation per atom from enthalpies at same pressure

$$H_f(\text{Be}_{1-x}\text{Li}_x) = H(\text{Be}_{1-x}\text{Li}_x) - (1 - x) \cdot H(\text{Be}) - x \cdot H(\text{Li})$$

- Also known as enthalpy of mixing
- Phase diagram from tie lines of formation enthalpies

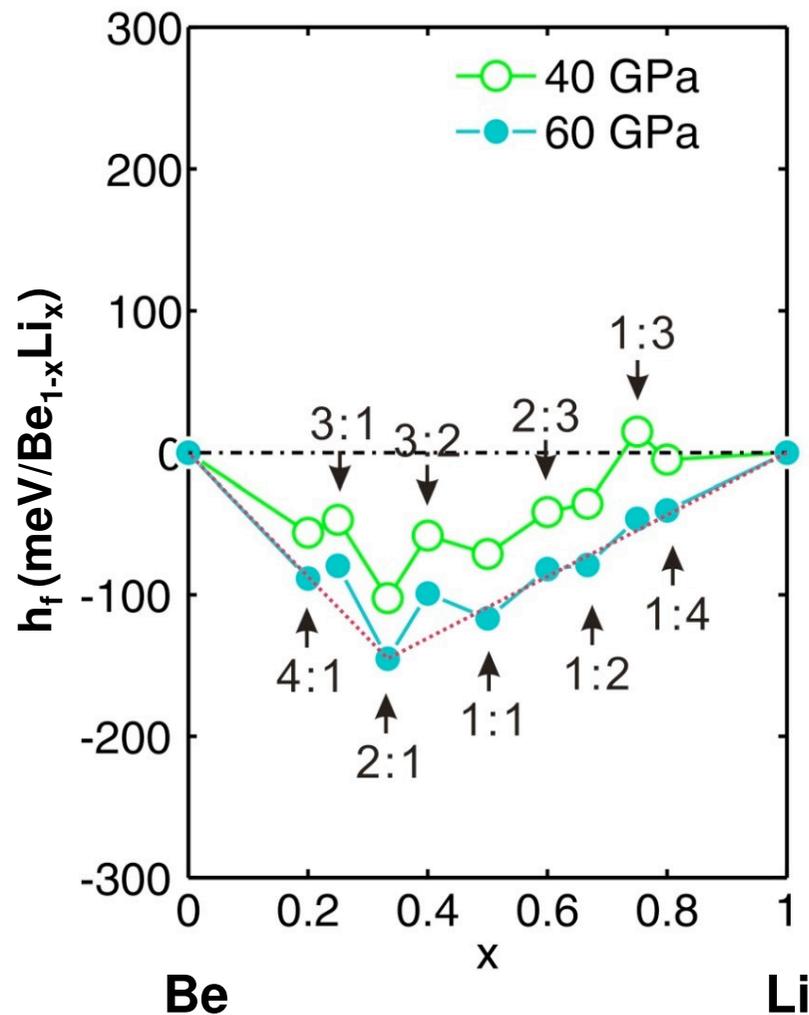
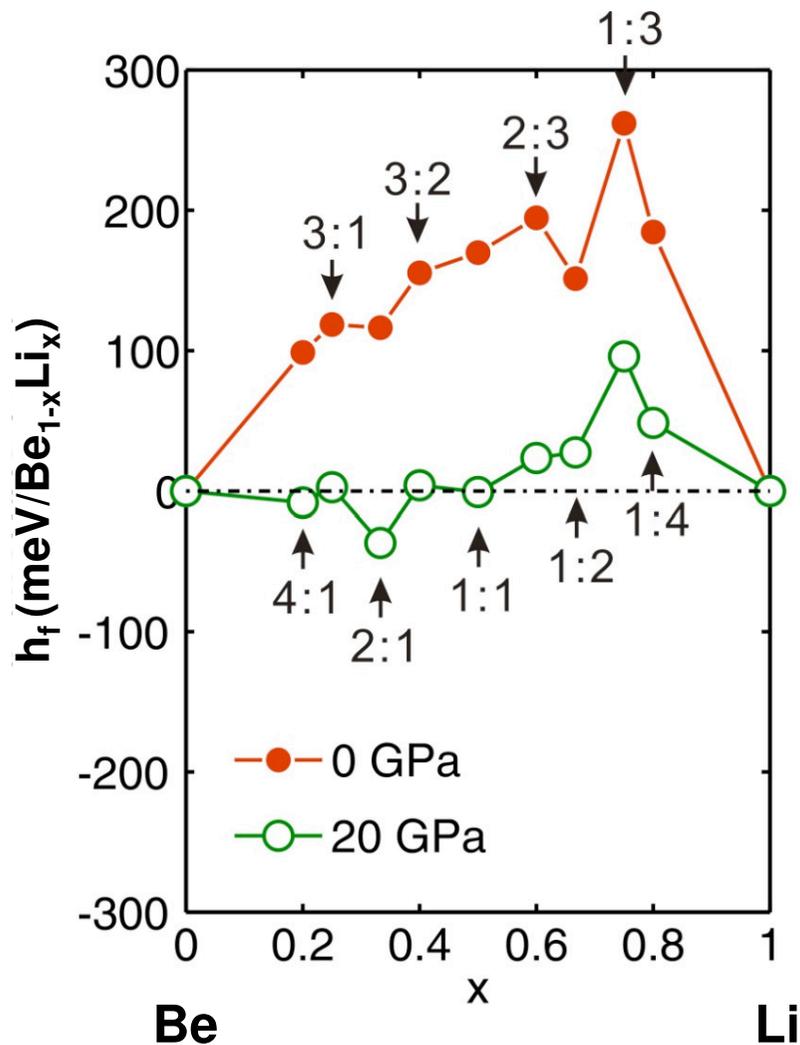
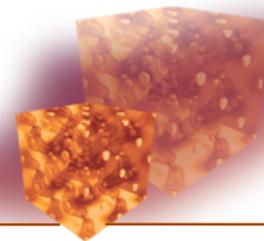


# Results for Enthalpy of Formation



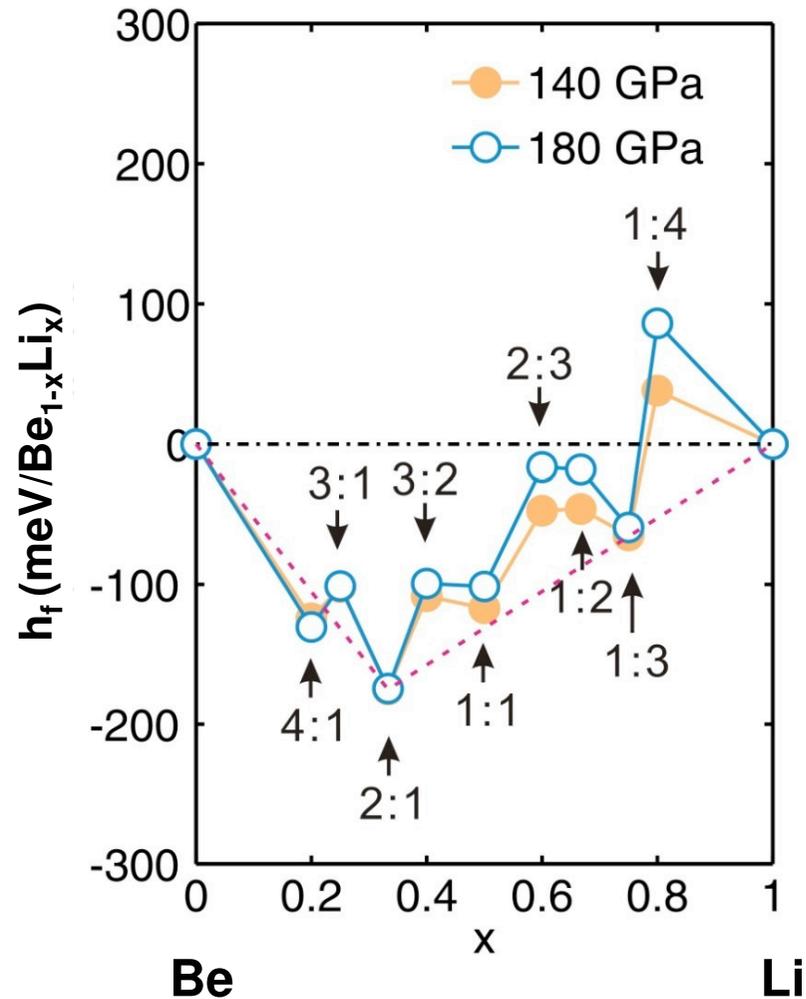
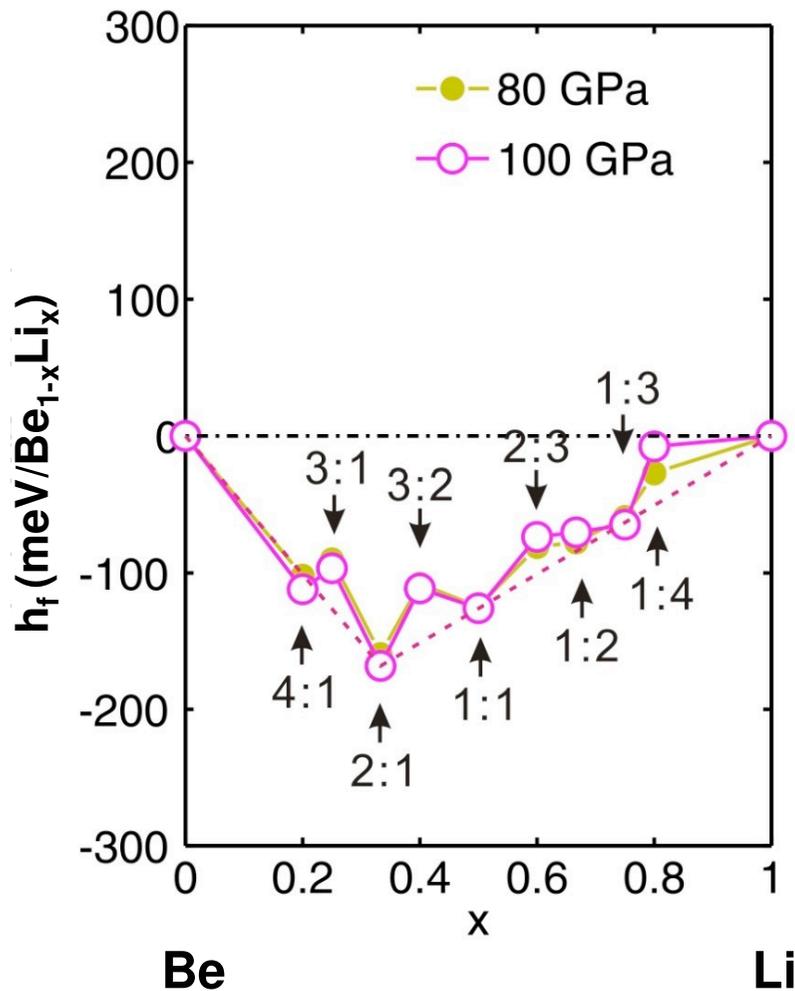
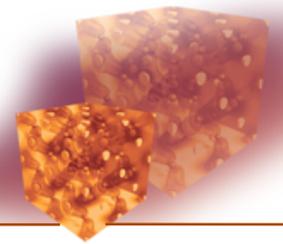
**Stability increases with pressure dramatically at low pressures**

# Low Pressure Phase Stability



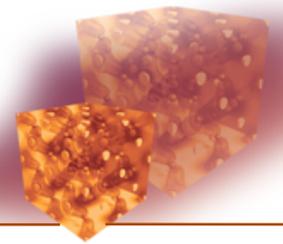
**Be<sub>2</sub>Li stable above 20 GPa and BeLi above 40 GPa**

# High Pressure Phase Stability

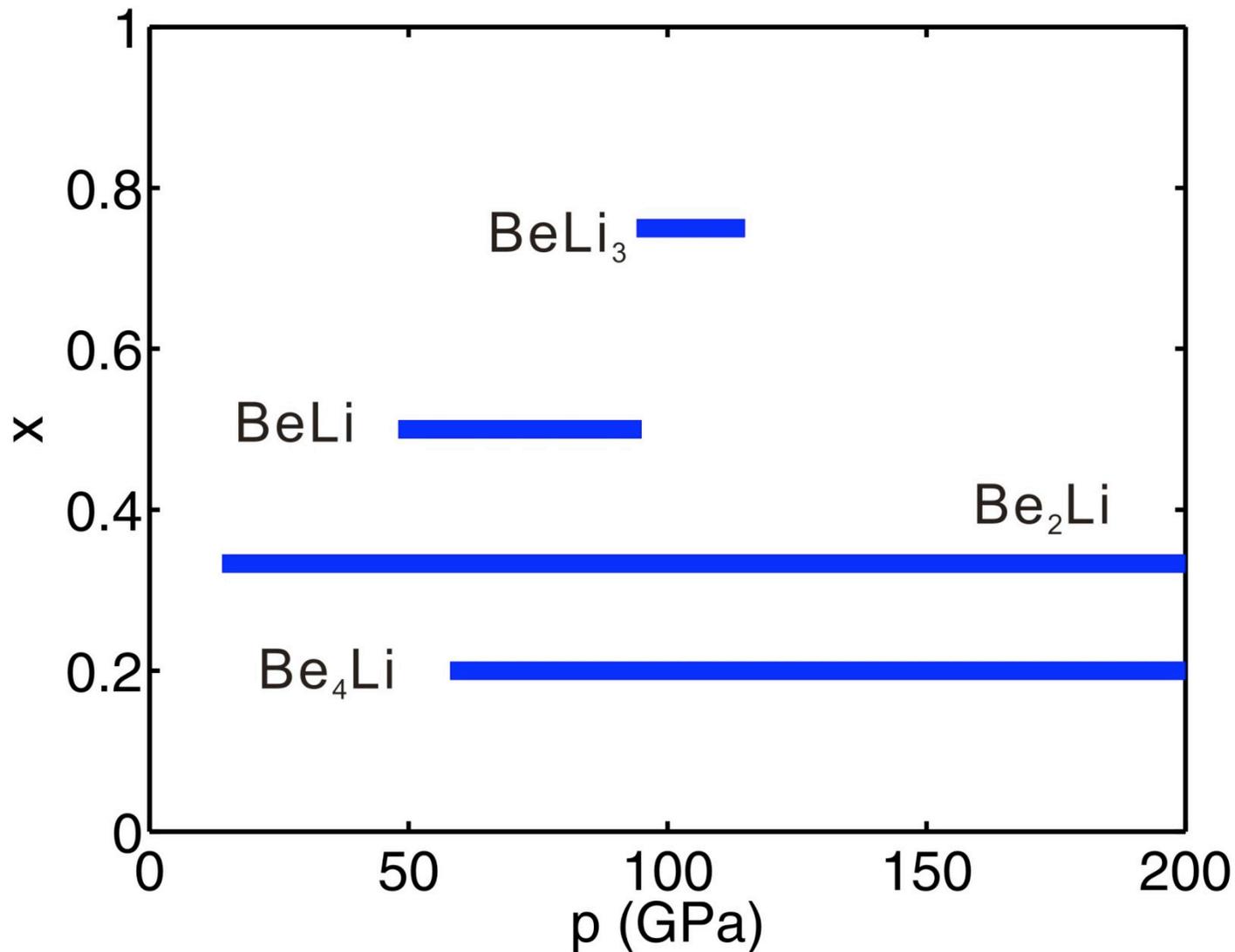


**Be<sub>4</sub>Li stable above 80 GPa and BeLi<sub>3</sub> almost stable at 140 GPa**

# Phase Diagram of LiBe under Pressure

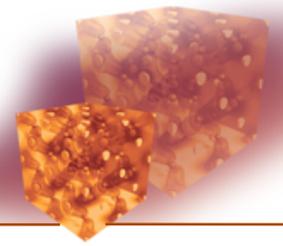


## Stability ranges

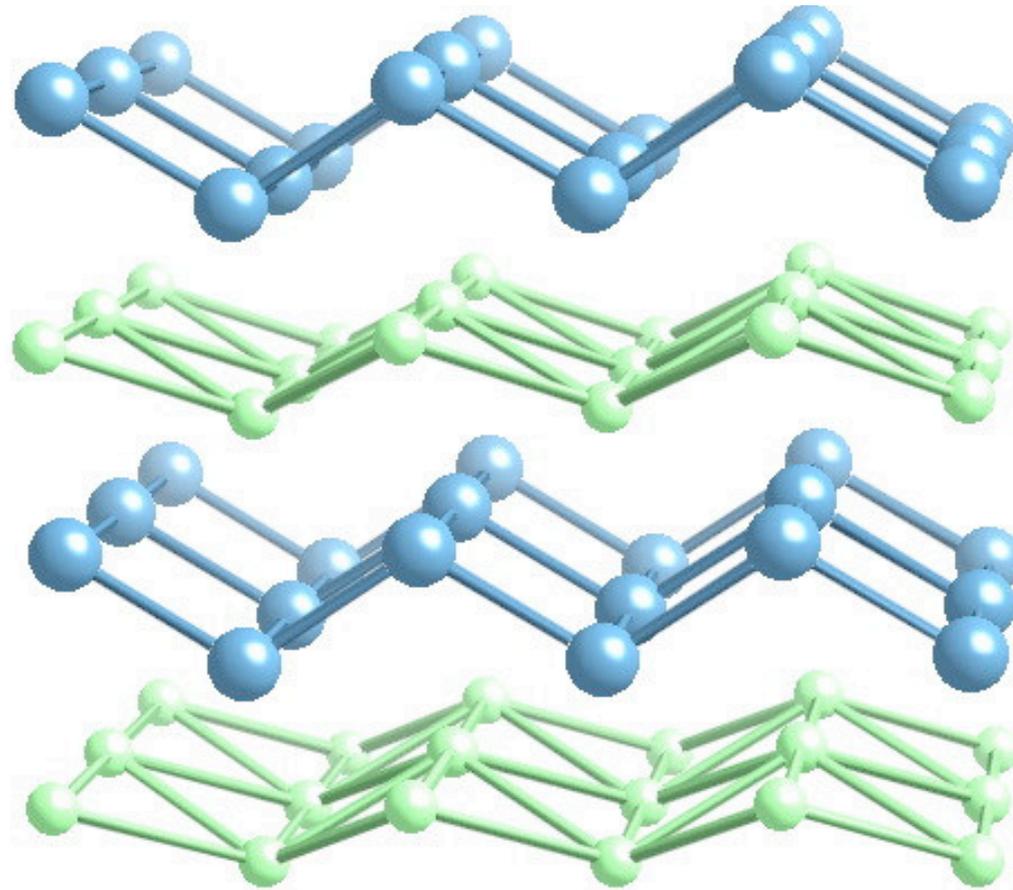


**Four novel BeLi phases become stable under pressure**

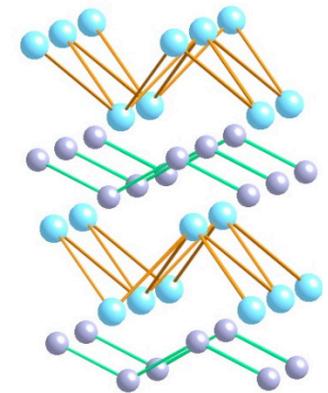
# The Stable BeLi phase ( $P2_1/m$ )



● Be  
● Li

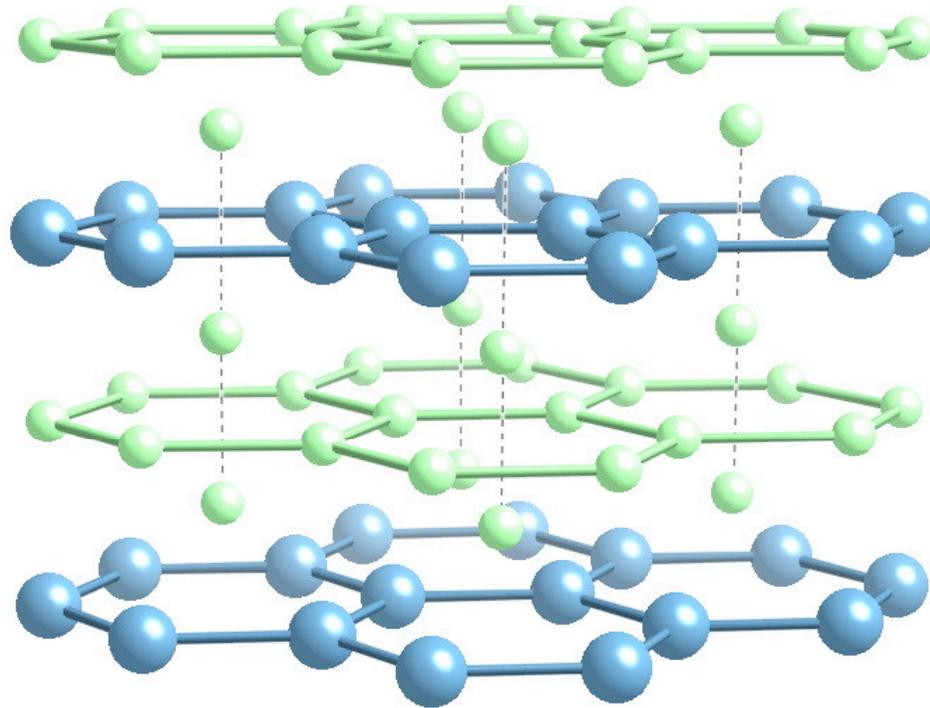
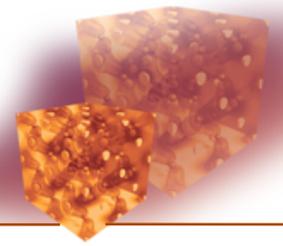


BeLi (82 GPa)  
 $P2_1/m$



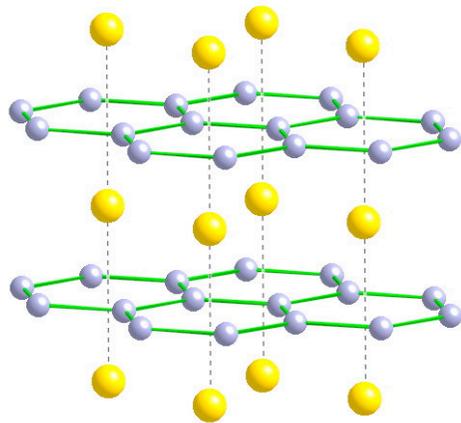
CrB

# The Stable $\text{Be}_2\text{Li}$ phase (P6/mmm)



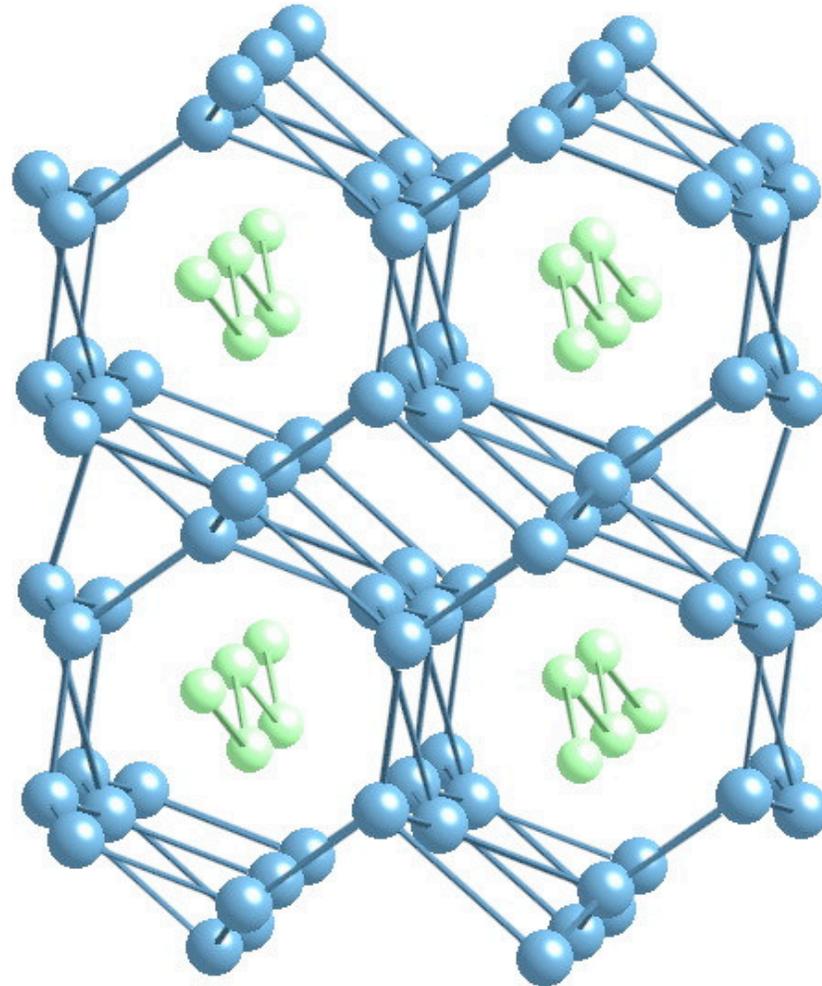
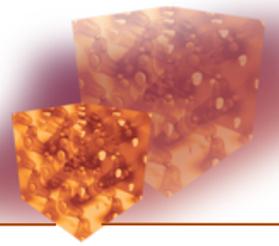
1.98 Å

$\text{Be}_2\text{Li}$  (80 GPa)  
P6/mmm

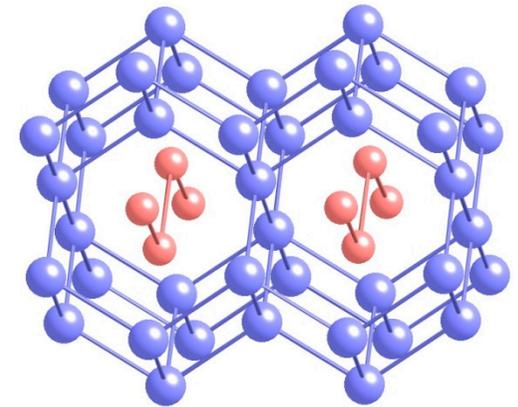


$\text{MgB}_2$

# The Stable BeLi<sub>3</sub> phase (C2/m)

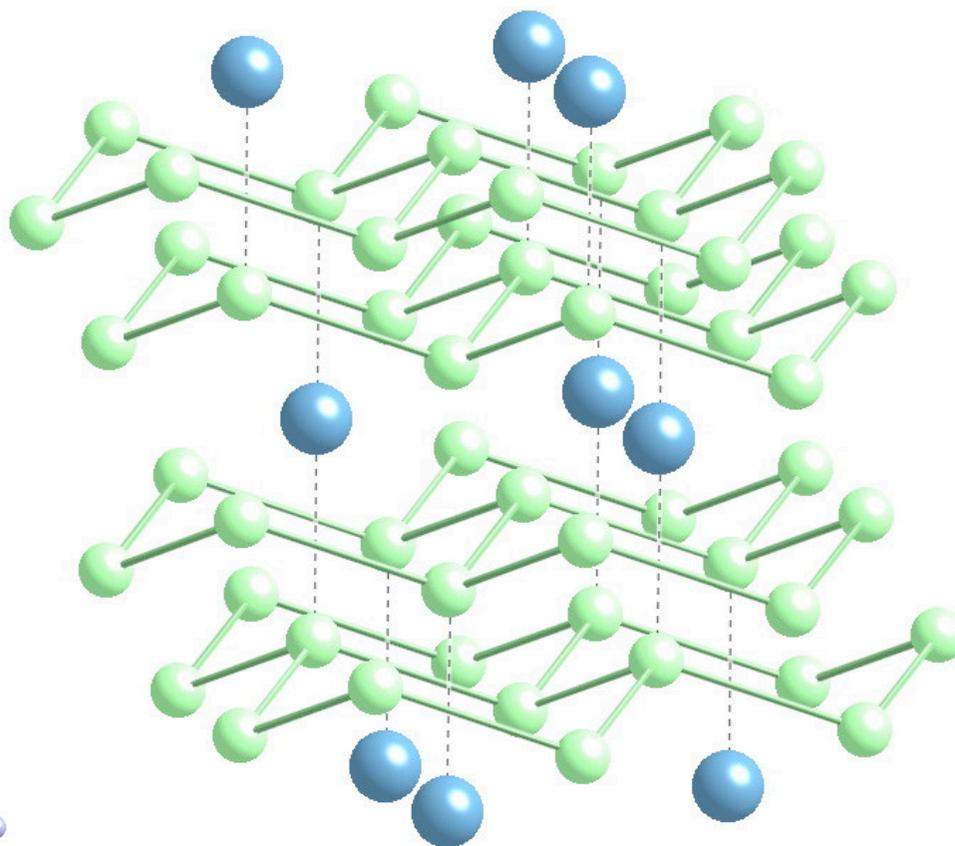
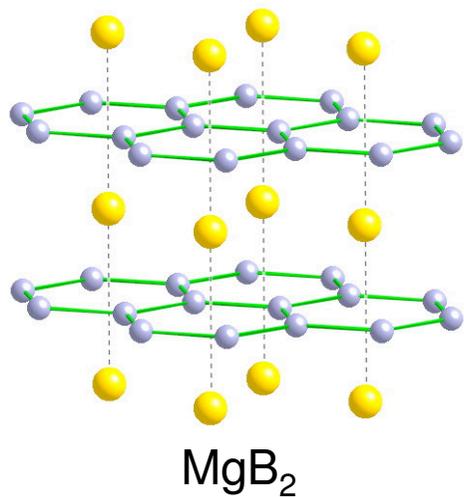
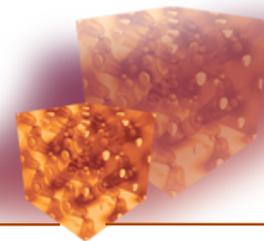


**BeLi<sub>3</sub> (80 GPa)  
C2/m**

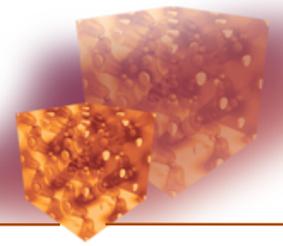


**GaLi<sub>2</sub>**

# The Stable $\text{Be}_4\text{Li}$ phase (R-3m)

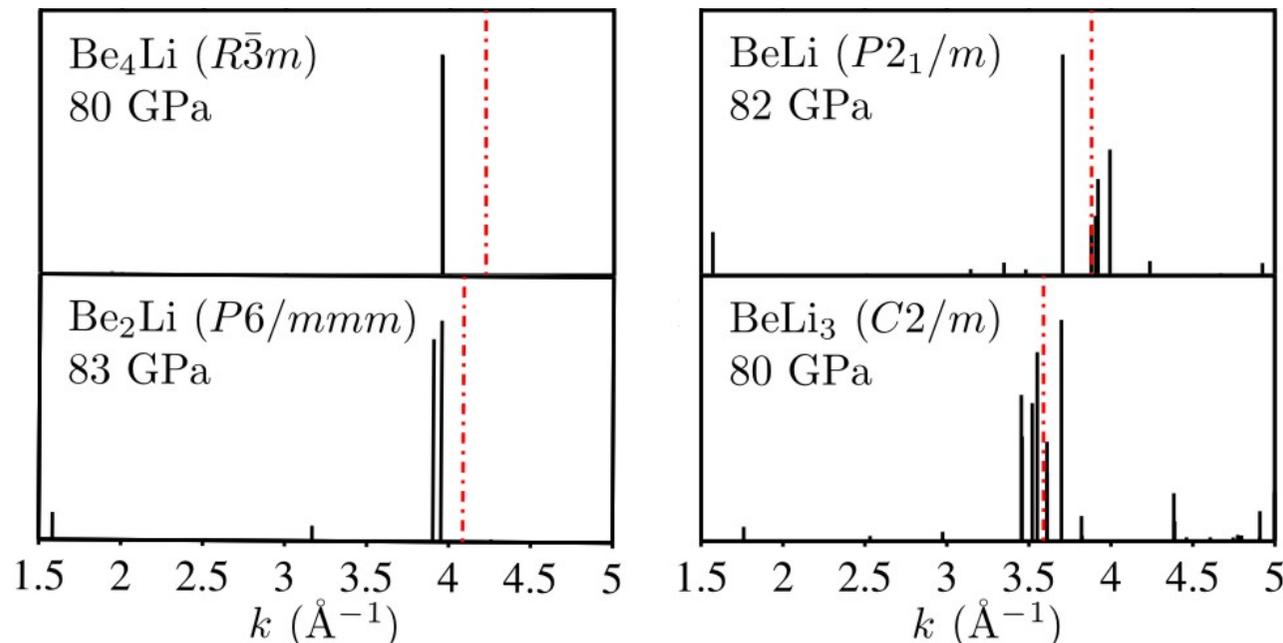


# Stabilization of Be-Li alloys



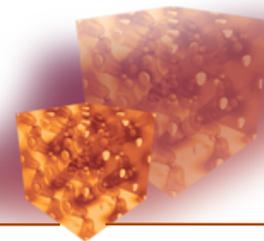
## Simulated x-ray diffraction pattern

- Cluster of diffraction peaks near  $2 k_F$
- Strong interaction between Brillouin zone and initially free-electron Fermi surface  $\Rightarrow$  Pseudogap in density of states



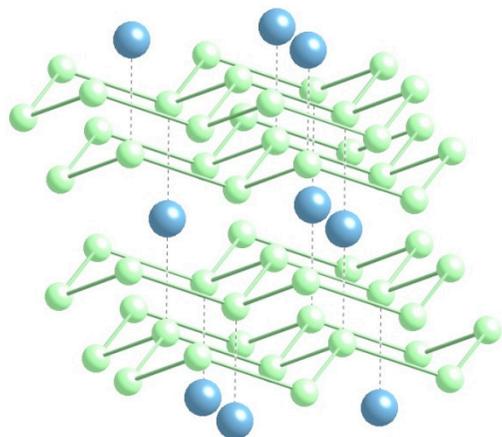
**Potential stabilization by Hume-Rothery mechanism**

# Fermi Density of States

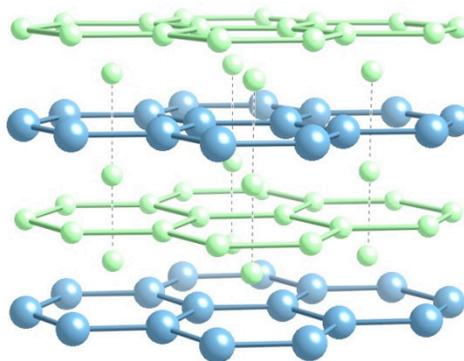


- Beryllium's DOS at the Fermi level is nearly constant over entire pressure range:  $g(\epsilon_F) = 0.04 \text{ eV}^{-1}$  per valence electron

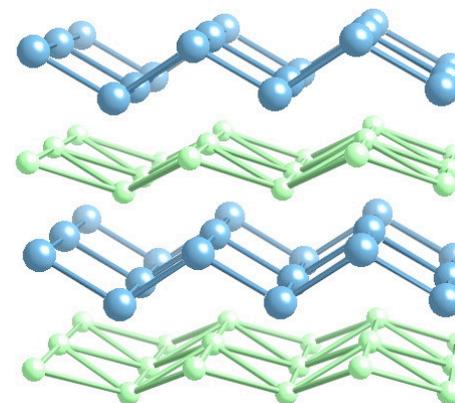
80 GPa	<b>Be<sub>4</sub>Li</b> R-3m	<b>Be<sub>2</sub>Li</b> P6/mmm	<b>BeLi</b> P2 <sub>1</sub> /m
<b><math>g(\epsilon_F)</math> in <math>\text{eV}^{-1}</math> per valence electron</b>	<b>0.06</b>	<b>0.06</b>	<b>0.12</b>



**Be<sub>4</sub>Li**



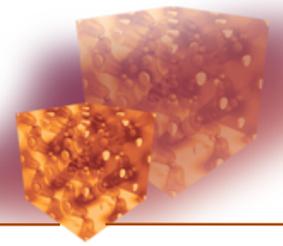
**Be<sub>2</sub>Li**



**BeLi**

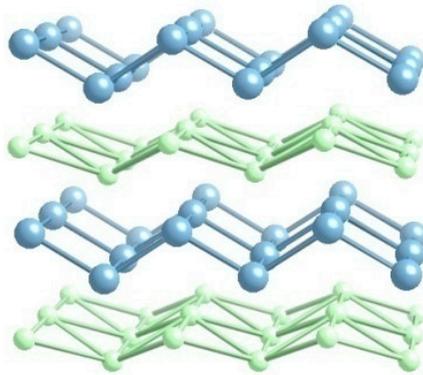
**For a comparable e-ph coupling,  $T_c$  would be about 32 K**

# Electronic Structure of BeLi ( $P2_1/m$ )

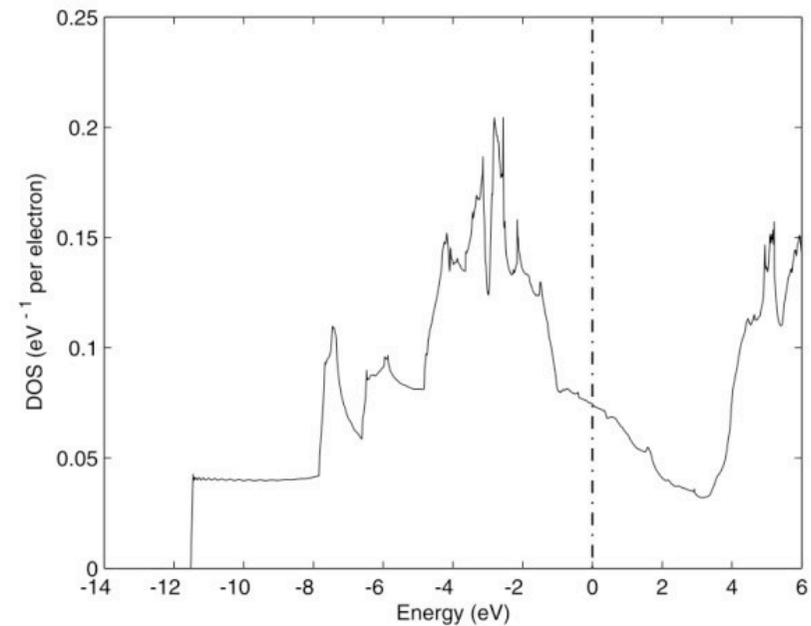
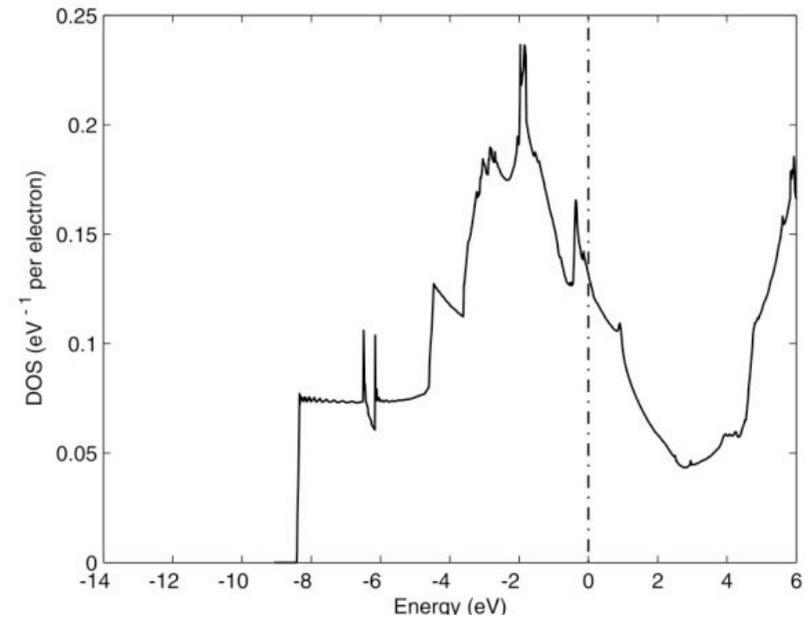
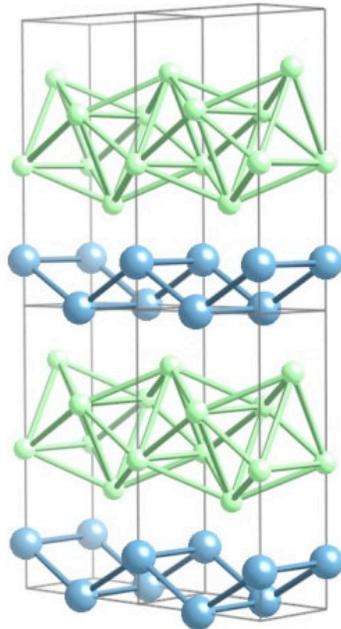


BeLi  
 $P2_1/m$

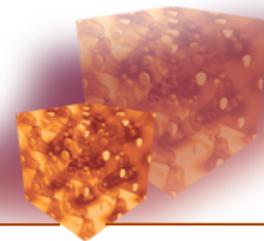
● Be  
● Li



Be<sub>2</sub>Li  
Pmcm

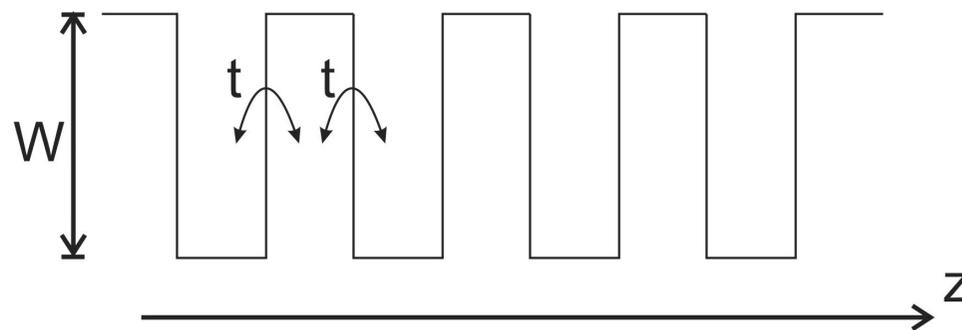
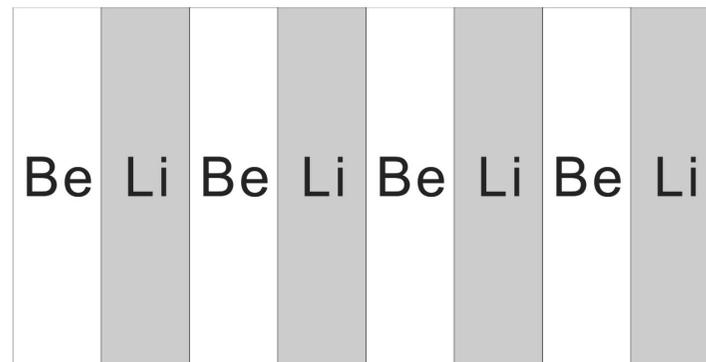
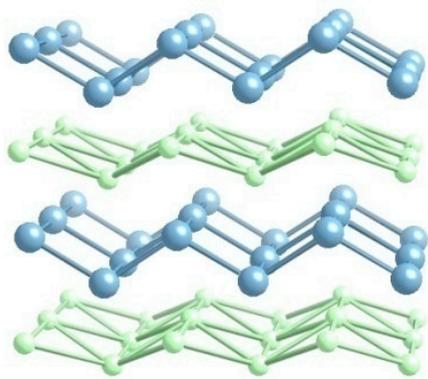


# Electronic Structure of BeLi (P2<sub>1</sub>/m)



BeLi  
P2<sub>1</sub>/m

● Be  
● Li

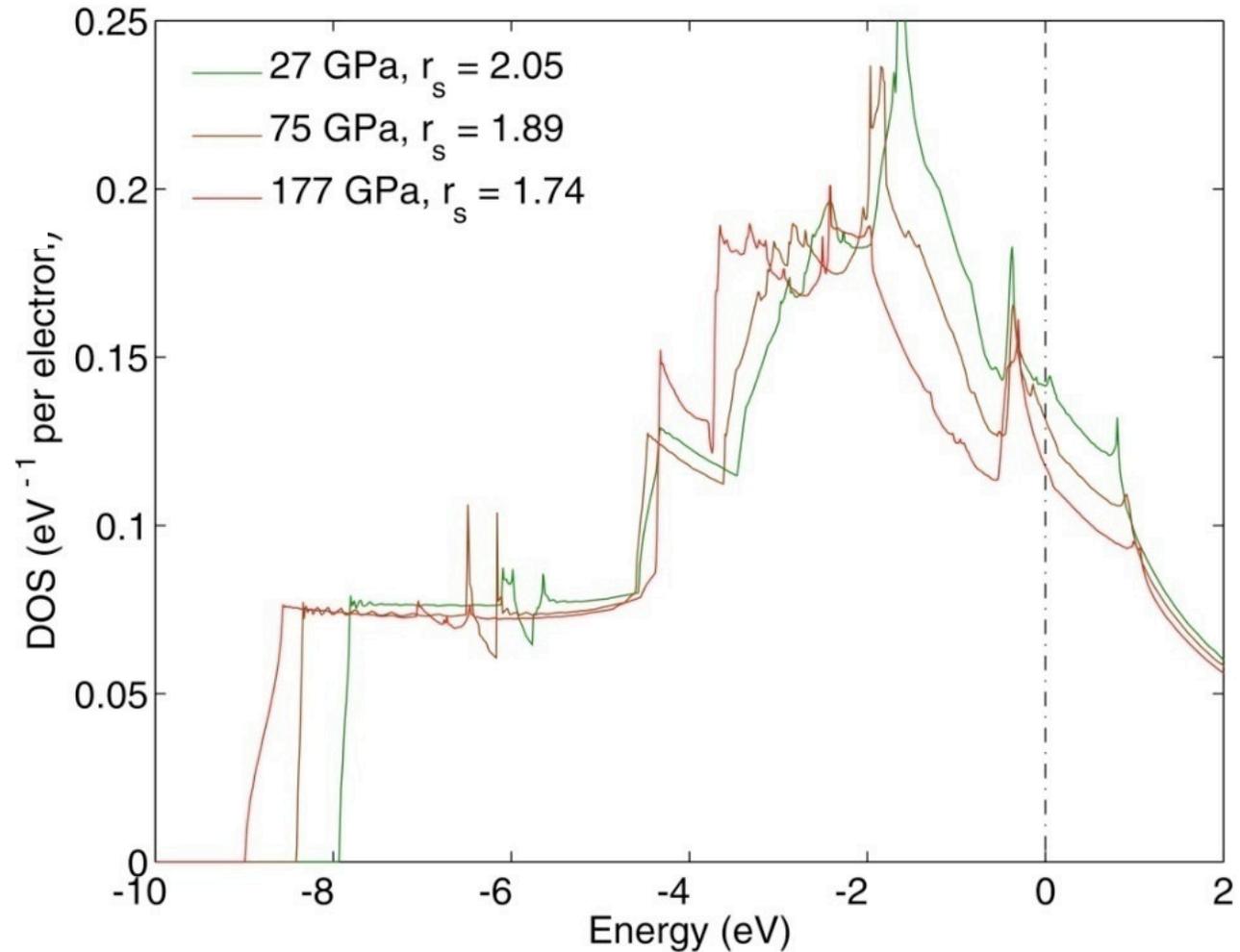
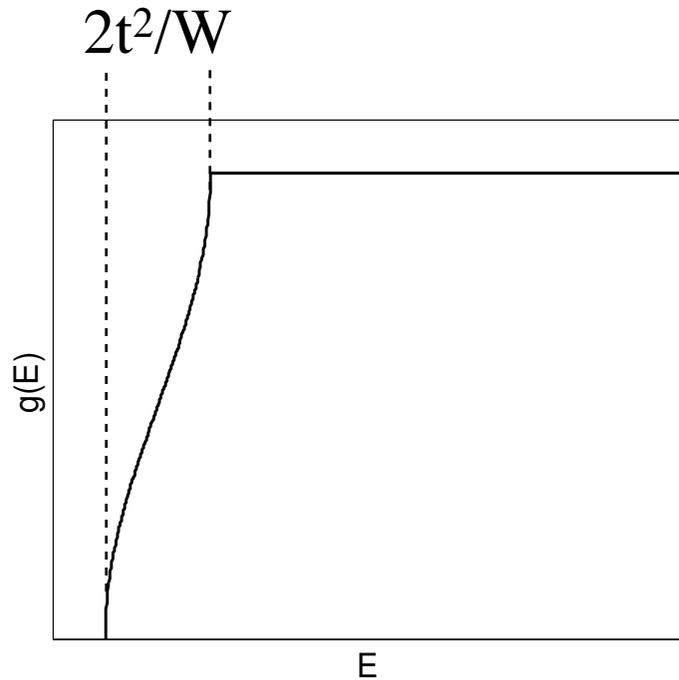
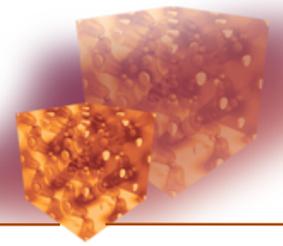


$$\mathcal{H}_{xy} = \sum_{\mathbf{k}} \frac{\hbar^2 k^2}{2m} c_{\mathbf{k}}^\dagger c_{\mathbf{k}}$$

$$\mathcal{H}_z = \epsilon_0 \sum_i n_i + W \sum_i n_{2i+1} - t \sum_i \left( c_i^\dagger c_{i+1} + h.c. \right)$$

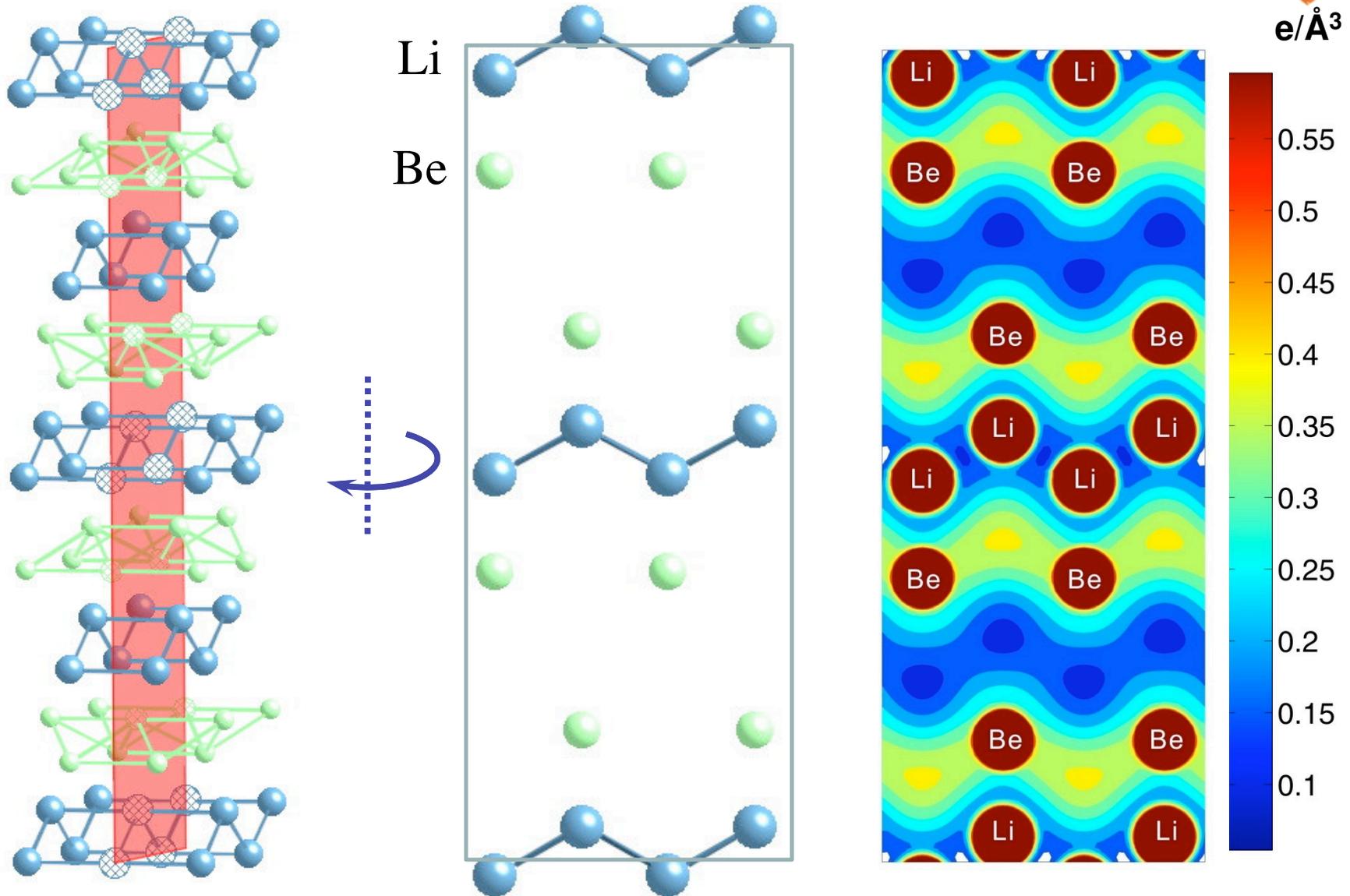
Simple model Hamiltonian can be solved exactly

# Electronic Structure of BeLi ( $P2_1/m$ )



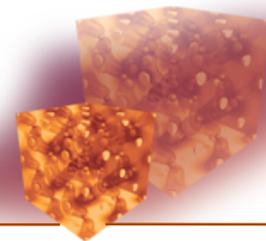
**The electronic density of states from model Hamiltonian matches the calculated one.**

# Electron Density of BeLi (P2<sub>1</sub>/m)

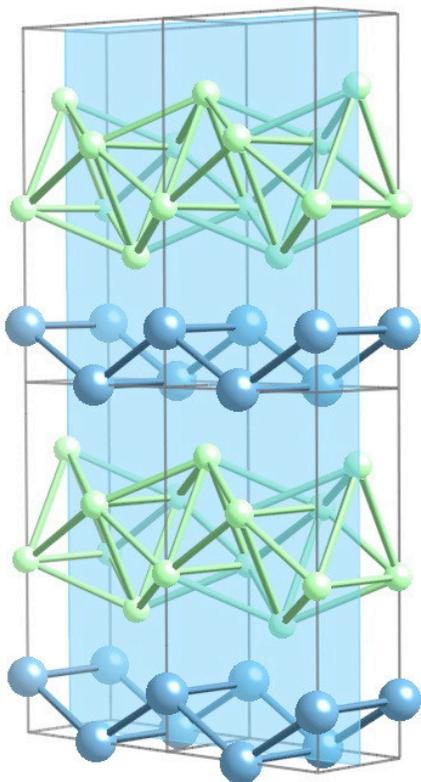


**Electron density shows layered structure**

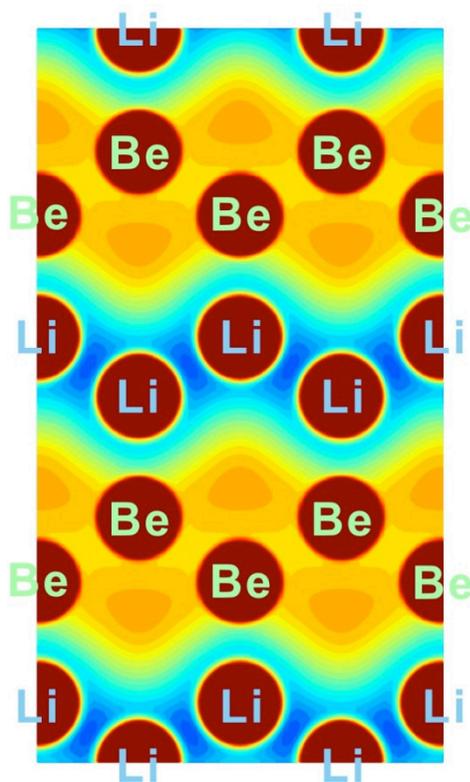
# Electron Density of metastable $\text{Be}_2\text{Li}$ (Pmcm)



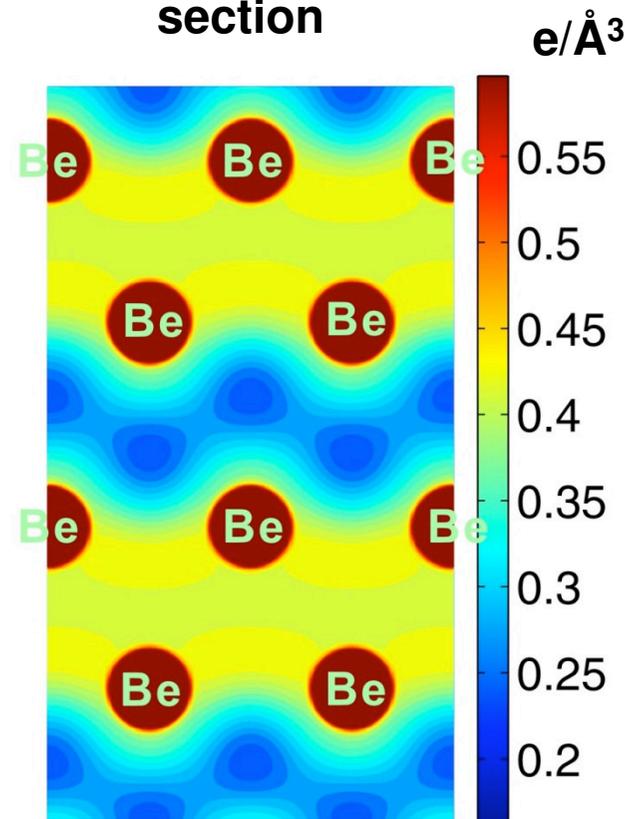
$\text{Be}_2\text{Li}$   
Pmcm



Front face of  
the cells

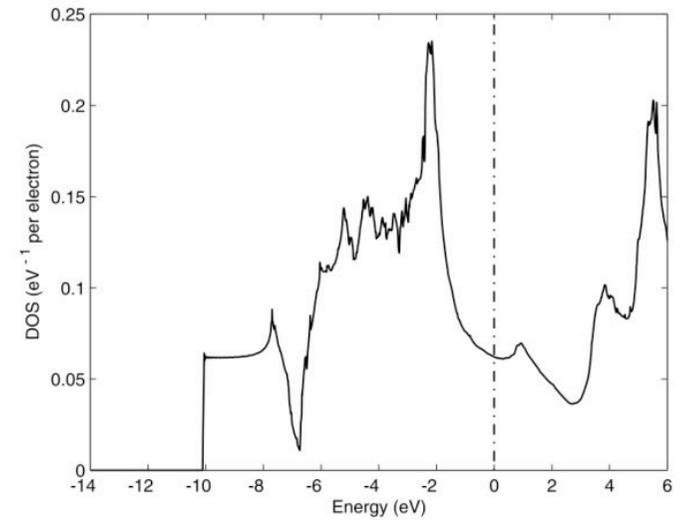
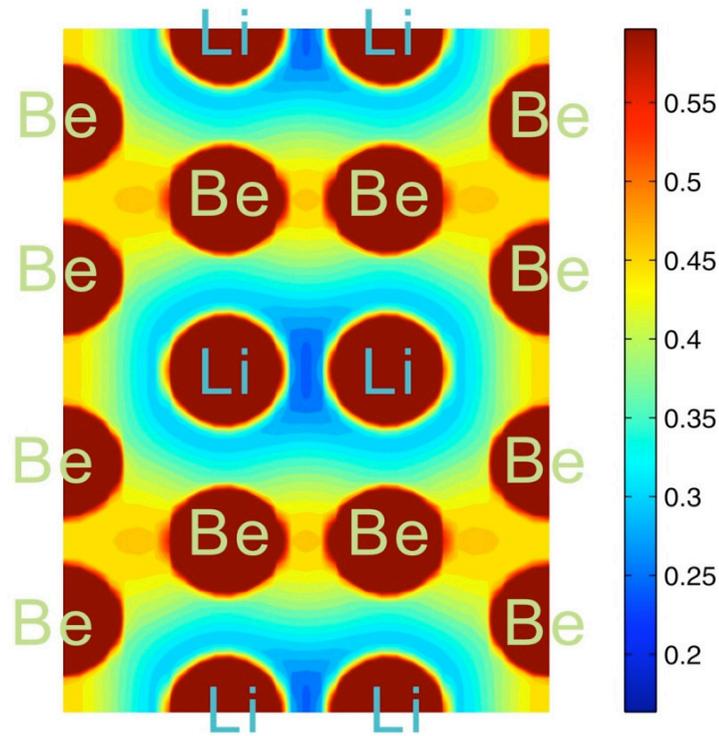
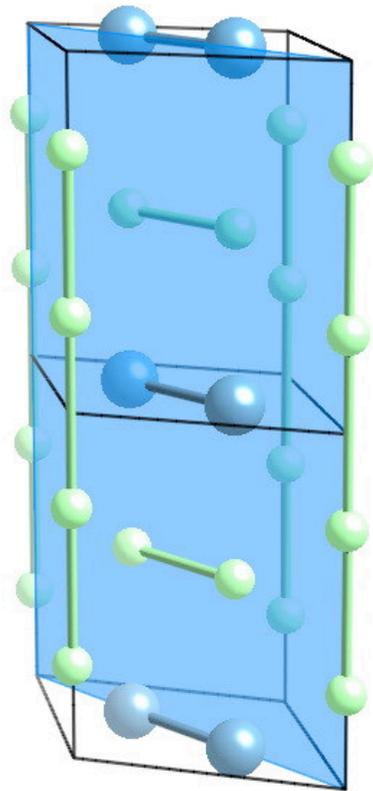
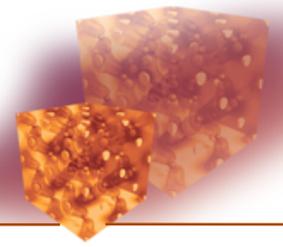


“Blue” cross-  
section

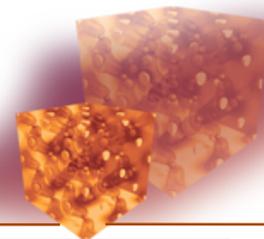


**Electron density shows layered structure**

# Electron Density of stable $\text{Be}_2\text{Li}$ (P6/mmm)



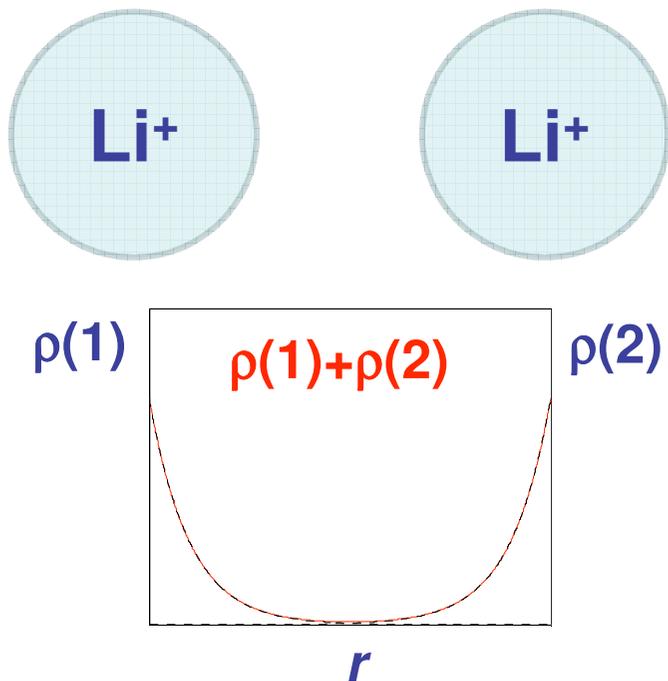
# Origin of Charge Disproportionation



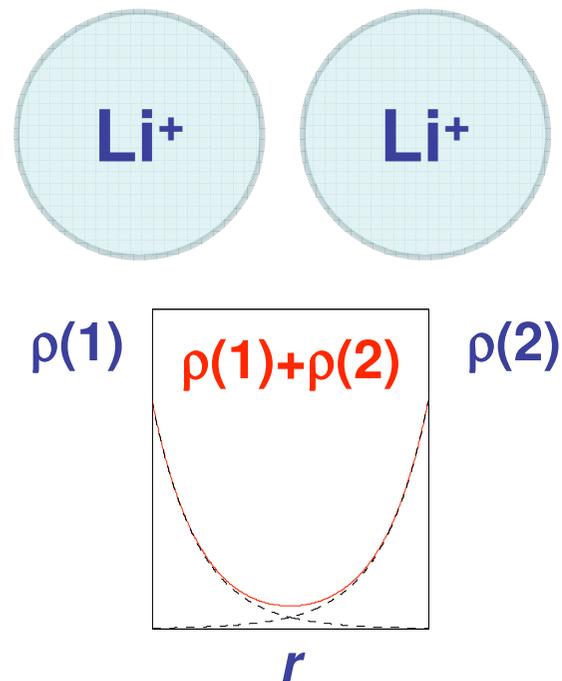
- Large difference in core size of Be and Li

	Li	Be
Z	3	4
Ionic Radius (Å)	0.76	0.27

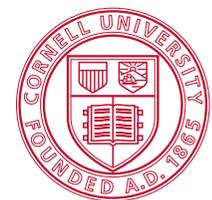
Low density



High density



Overlapping core makes internuclear region less accessible to valence electrons



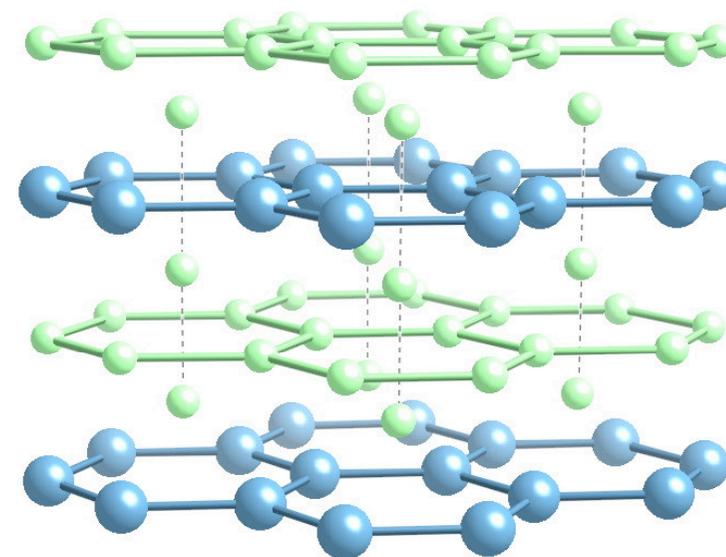
# Emergent quasi-2d behaviour in a



Richard G. Hennig, Ji Feng, Neil Ashcroft and Roald Hoffmann

**Do Be and Li form alloys? What is their electronic structure?  
Can they have higher superconducting temperatures than pure Be and Li?**

- Be and Li form intermetallic compounds under pressure
- Possible enhancement of  $T_c$  through increased DOS
- Larger core of Li and smaller core of Be push valence electron density into 2D electron gas
- Fascinating high-pressure chemistry of alloys from simple elements



**They used to be called the simple elements**