Comparing solutions to the arrival time problem in de Broglie-Bohm theory and Decoherent Histories: What can we learn?

James M Yearsley

JJ Halliwell & JMY, Phys.Rev.A 79, 062101 (2009)
JJ Halliwell & JMY, Phys.Lett.A 374, 154 (2009)
JMY, Phys.Rev.A 82, 012116 (2010)

21st-century directions in de Broglie-Bohm theory and beyond.

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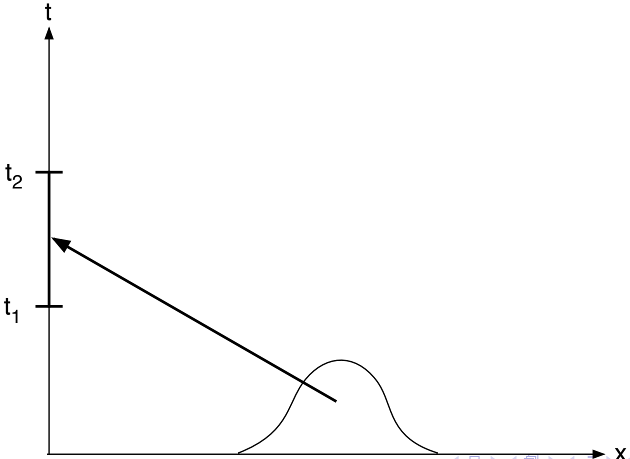
Introduction: The arrival time problem in QM

What is the probability that an incoming wave packet crosses the origin during a given time interval?



Introduction: The arrival time problem in QM

- No self adjoint operator for this in QM
- $|\psi(x,t)|^2$ is a prob dist on x but not t
- Classically, consideration of trajectories gives $\Pi_{cl}(t) = J(0,t)$, but no trajectories in "standard" QM
- Is $\Pi_{qm}(t) = J(0,t)$?
- If not, how does $\Pi_{qm}(t) \rightarrow J(0,t)$ in classical limit?
- Have additional problem $J(0,t) \ngeq 0$ even for $\tilde{\psi}(p>0) = 0$

  $\rightarrow$ "Backflow Effect"

Introduction: The arrival time problem in QM

Probabilities in QM should be of form

$$p(\alpha) = \mathsf{Tr}(P_\alpha \rho)$$

where P_α is a projector or POVM.

$\Pi(t) = J(0,t)$ does not have this form $\implies$ cannot be fundamental

The arrival time problem in dBB

dBB similar to classical case

- Can still define arrival time probabilities in terms of density of trajectories
- So naively $\Pi_{dBB}(t) = J(0,t)$
- Backflow effect explained as no "free particles" in dBB
- There are several qualitatively different proposals for $\Pi_{qm}(t)$ and they may be experimentally distinguishable...
- However not clear whether $\Pi_{dBB}(t)$ is measureable

Decoherent Histories approach to QM

What is DH?

- Formulation of QM designed for closed systems, in particular the universe!
- Aim is to assign probabilities to histories without notion of "measurement" or "observer"
- Obvious that this isn't possible in general, eg two slit experiment
- Best thought of as an extension of QM to histories, rather than as an "interpretation"...
- Most frequently used to explain emergence of classical world from QM

Decoherent Histories approach to QM

- Alternatives at a fixed moment of time represented by $\{P_a\}$

  $\sum_a P_a = 1, \quad P_a P_b = \delta_{ab} P_a$

- Histories represented by $\{C_\alpha\}$

  $C_\alpha = P_{a_n}(t_n)...P_{a_1}(t_1)$ or sums of these, $\quad \sum_\alpha C_\alpha = 1$

- Probabilities assigned to histories via $p(\alpha) = \mathrm{Tr}(C_\alpha \rho C_\alpha^\dagger)$

- Require decoherence, $D(\alpha, \beta) = \mathrm{Tr}(C_\alpha \rho C_\beta^\dagger) \approx 0, \quad \alpha \neq \beta$

- Decoherence $\implies p(\alpha) = \mathrm{Tr}(C_\alpha \rho)$

Class operators for the arrival time problem

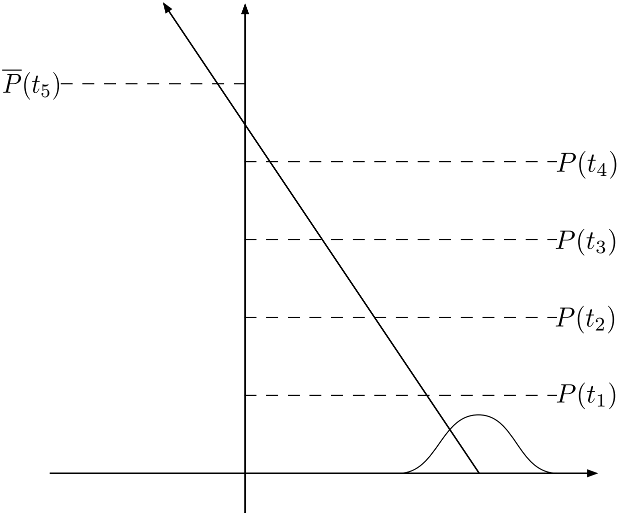
How do we formulate the arrival time problem in this general framework?

- Key step is deriving the class operators $\{C_\alpha\}$

Consider initially discrete moments of time $\{t_1, t_2 ... T\}$

- Arrival time between t_k and t_{k+1} means particle was in $x > 0$ at $t_1 ... t_k$ and in $x < 0$ at t_{k+1}
- So $C(t_{k+1}, t_k) = \overline{P}(t_{k+1}) P(t_k) ... P(t_1)$
- Class operator for not crossing $C_{nc} = P(T) ... P(t_1)$

Histories



$$C(t_4, t_5) = \overline{P}(t_5)P(t_4)P(t_3)P(t_2)P(t_1)$$

Class operators for the arrival time problem

Seems natural to take the "continuum limit"

- Let $t_k = k\epsilon$, $T = N\epsilon$
- Then take $N \to \infty$, $\epsilon \to 0$ leaving $T = N\epsilon$ finite

$$C_{nc} = \lim_{\epsilon \longrightarrow 0} P(N\epsilon)...P(\epsilon) = P\exp(-iPHPT)$$

$\Longrightarrow$ restricted propagation!

Quantum Zeno Effect. Monitoring the state too closely stops it from leaving the subspace.

Class operators for the arrival time problem

Have to leave ϵ finite $\implies$ cannot specify arrival time with arbitrary precision

- Need to find a way of working with

$$C(t_{k+1}, t_k) = \overline{P}((k+1)\epsilon)P(k\epsilon)...P(\epsilon)$$

Two options:

- Semi-classical approximation
- Projections $\Leftrightarrow$ Complex potentials

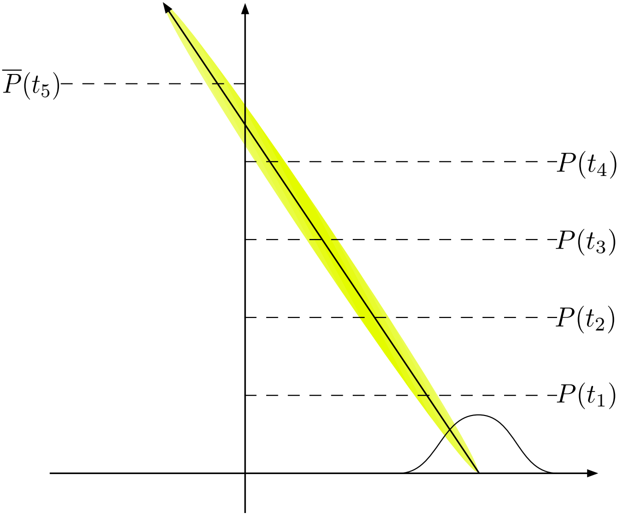
Class operators for the arrival time problem

Semi-classical approximation

- Path integral representation of propagator is dominated by straight line path
- $P(k\epsilon)...P(\epsilon) \approx P(k\epsilon)$
- State doesn't "see" projections earlier than crossing time
- It follows that, for $t_k = k\epsilon$,

$$C(t_{k+1}, t_k) \approx P(t_k) - P(t_{k+1})$$

Class operators for the arrival time problem



$$C(t_4, t_5) \approx \overline{P}(t_5)P(t_4)$$

Arrival time probabilities in DH

Now we have our class operators, what is $\Pi(t)$?

- Suppose decoherence,

$$\begin{aligned} p(t_k, t_{k+1}) &= \mathsf{Tr}(C(t_k, t_{k+1})\rho) = \mathsf{Tr}(P(t_k)\rho) - \mathsf{Tr}(P(t_{k+1})\rho) \\ &= \int_{t_k}^{t_{k+1}} dt J(0,t) \end{aligned}$$

  Standard result!

Arrival time probabilities in DH

When do we have decoherence?

Free particle:

- Gaussian wavepacket or orthogonal superpositions $\implies$ no interference effects
- $p < 0$, left moving
- Decoherence condition

$$E\epsilon >> 1$$

Note $p = \mathrm{Tr}(C\rho C^\dagger)$ so decoherence implies $J(0, t) > 0$

Arrival time probabilities in DH

Free particle with environment: Specifically quantum brownian motion

- Arbitrary wavefunctions
- $p < 0$ left moving
- Decoherence condition

$E\epsilon >> 1$, and also initial evolution for $t >> t_l$

Discussion

- DH and dBB both give $\Pi_{qm}(t) = J(0,t)$
- Are DH and dBB equivalent? No
  - In addition DH imposes conditions of form $E\Delta t >> 1$
  - Related to fact that if decoherence then predicted probabilities are the ones you would actually measure
- More general line of research: Can we translate decoherence condition into dBB language, and use it to analyze when dBB probabilities can be measured?

Conclusions

- Copenhagen QM does not supply $\Pi_{qm}(t)$
- dBB and DH both give solution, and can be extended eg dwell times, tunneling times, quantum cosmology...
- However status of solution different in two theories:
  - Always $J(0,t)$ in dBB
  - Sometimes not defined in DH
- Does this teach us something interesting about dBB, or DH, or both?