In search of a breakdown of quantum theory

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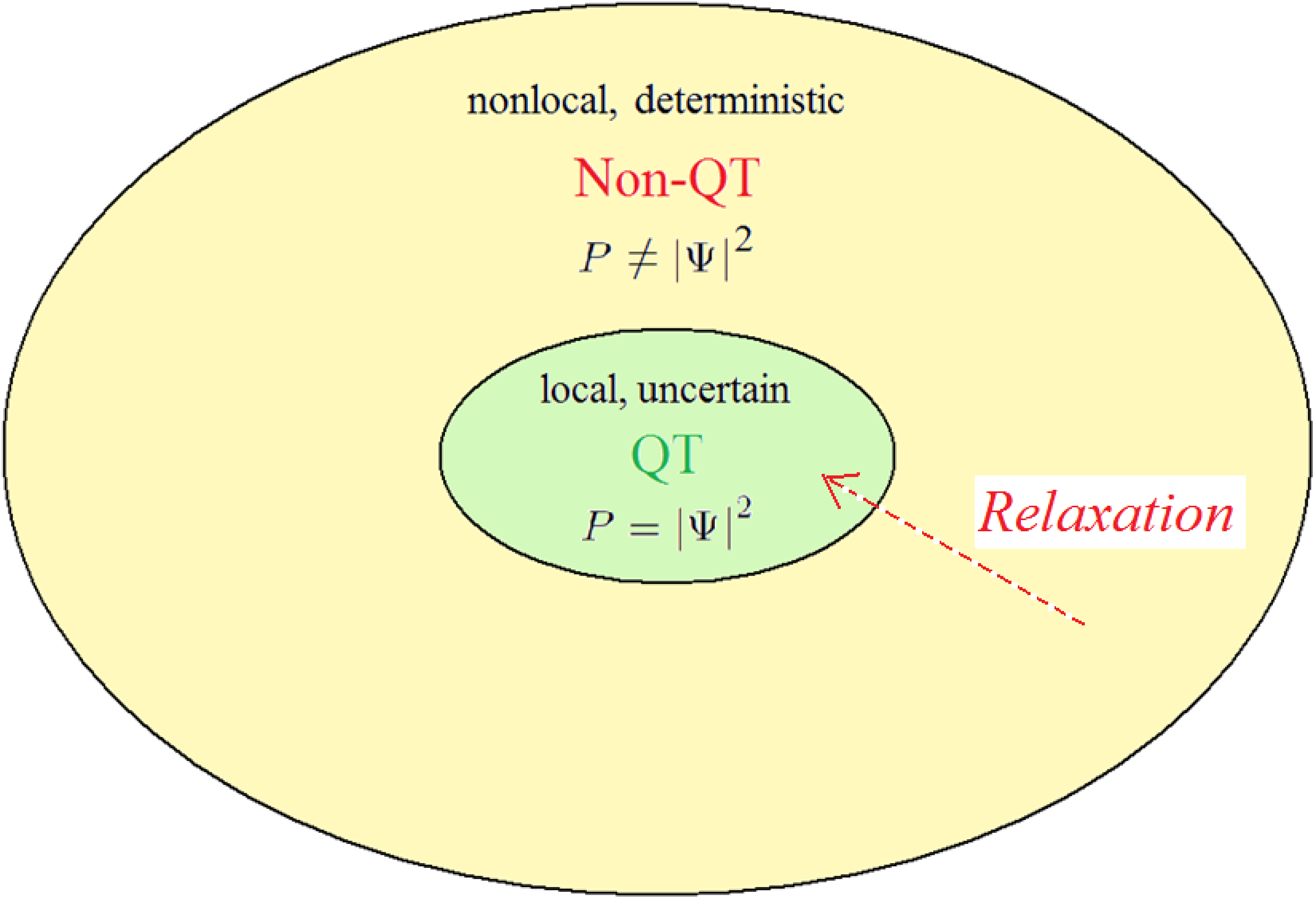
Quantum theory is a special case of a much wider physics



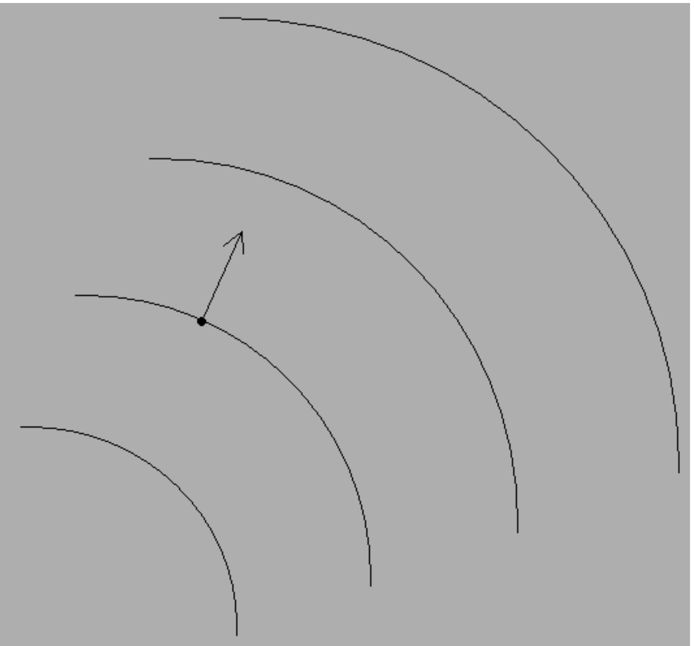
De Broglie's Pilot-Wave Dynamics (1927)

(cf. Bell 1987)

$$m_i \frac{d\mathbf{x}_i}{dt} = \nabla_i S$$

$$i \frac{\partial \Psi}{\partial t} = \sum_{i=1}^{N} -\frac{1}{2m_i} \nabla_i^2 \Psi + V\Psi$$

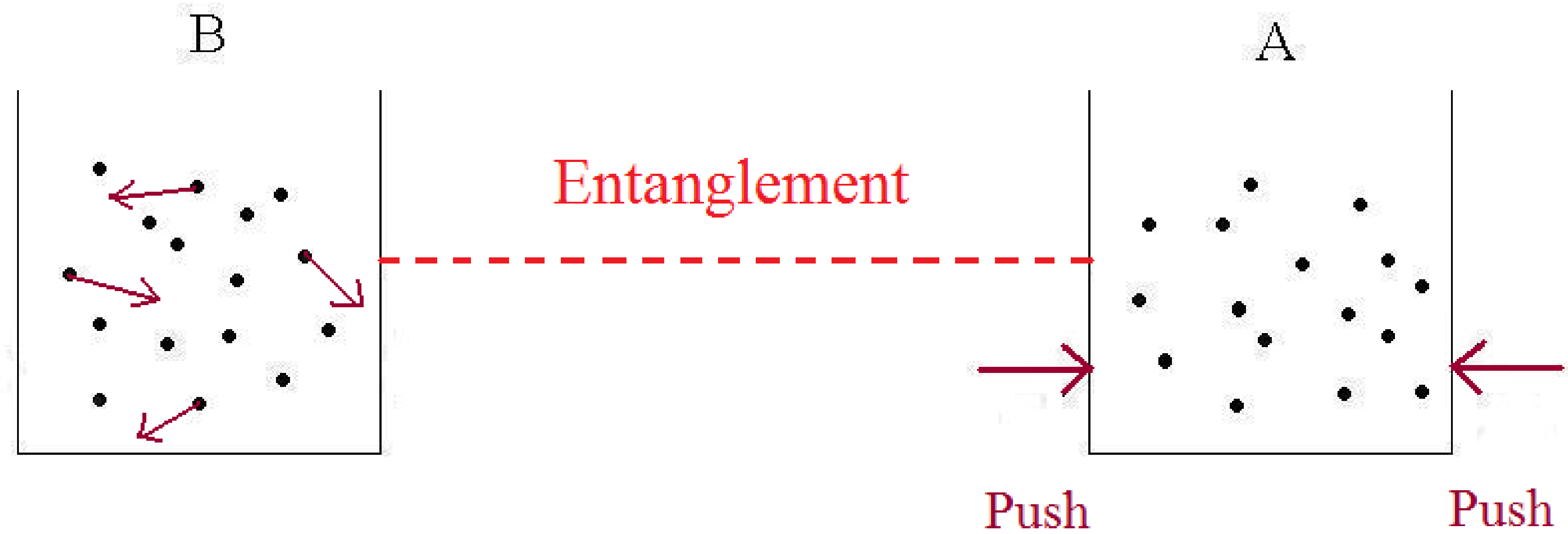
de Broglie called it "pilot-wave theory"



Get QM if assume initial $P = |\Psi|^2$

(shown fully by Bohm in 1952; apply dynamics to apparatus)

Nonequilibrium superluminal signalling

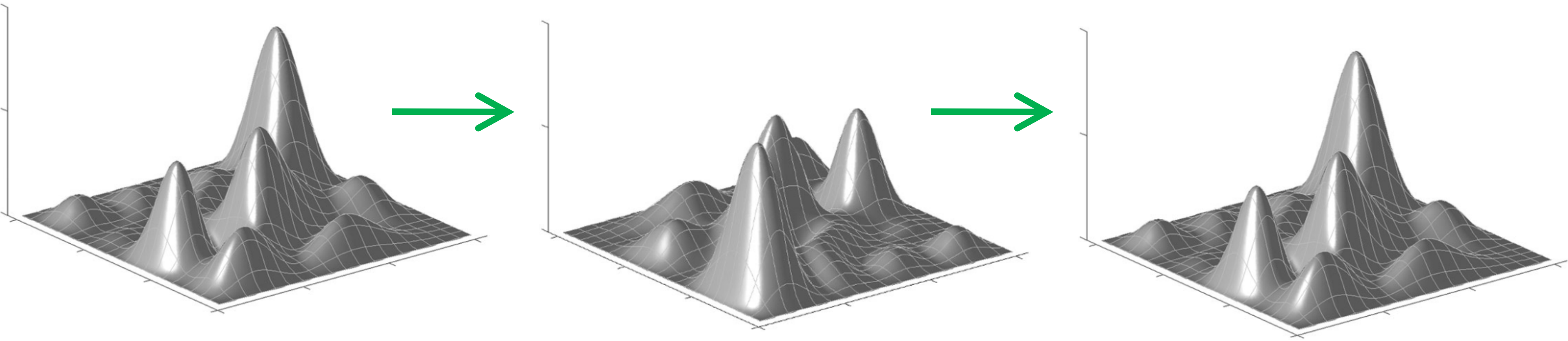


Overall effect vanishes or "cancels out" in the special state of "quantum equilibrium", but not otherwise.

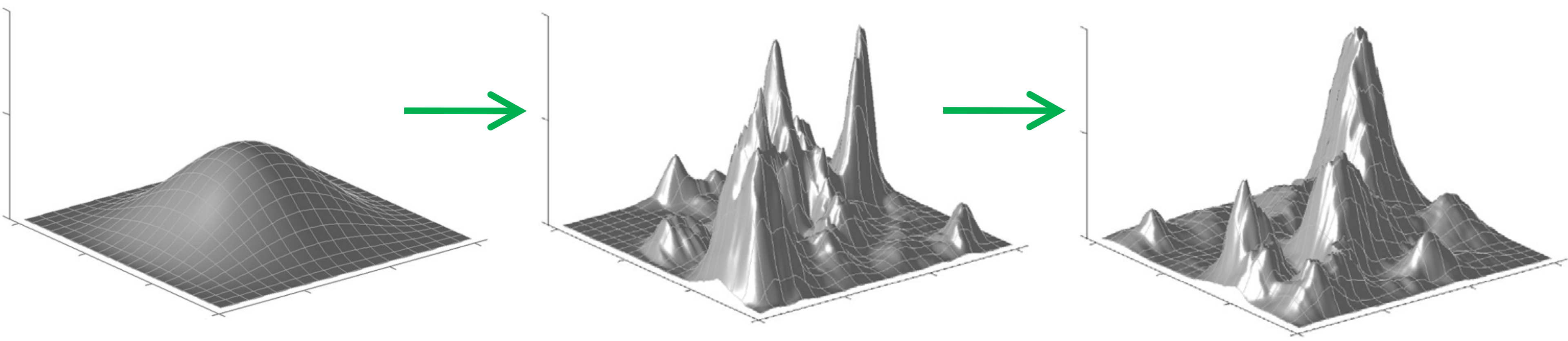
"Non-equilibrium particles" could be used to send nonlocal signals (and to do "sub-quantum" measurements on ordinary particles)

(Valentini 1991, 2002)

Equilibrium changes with time

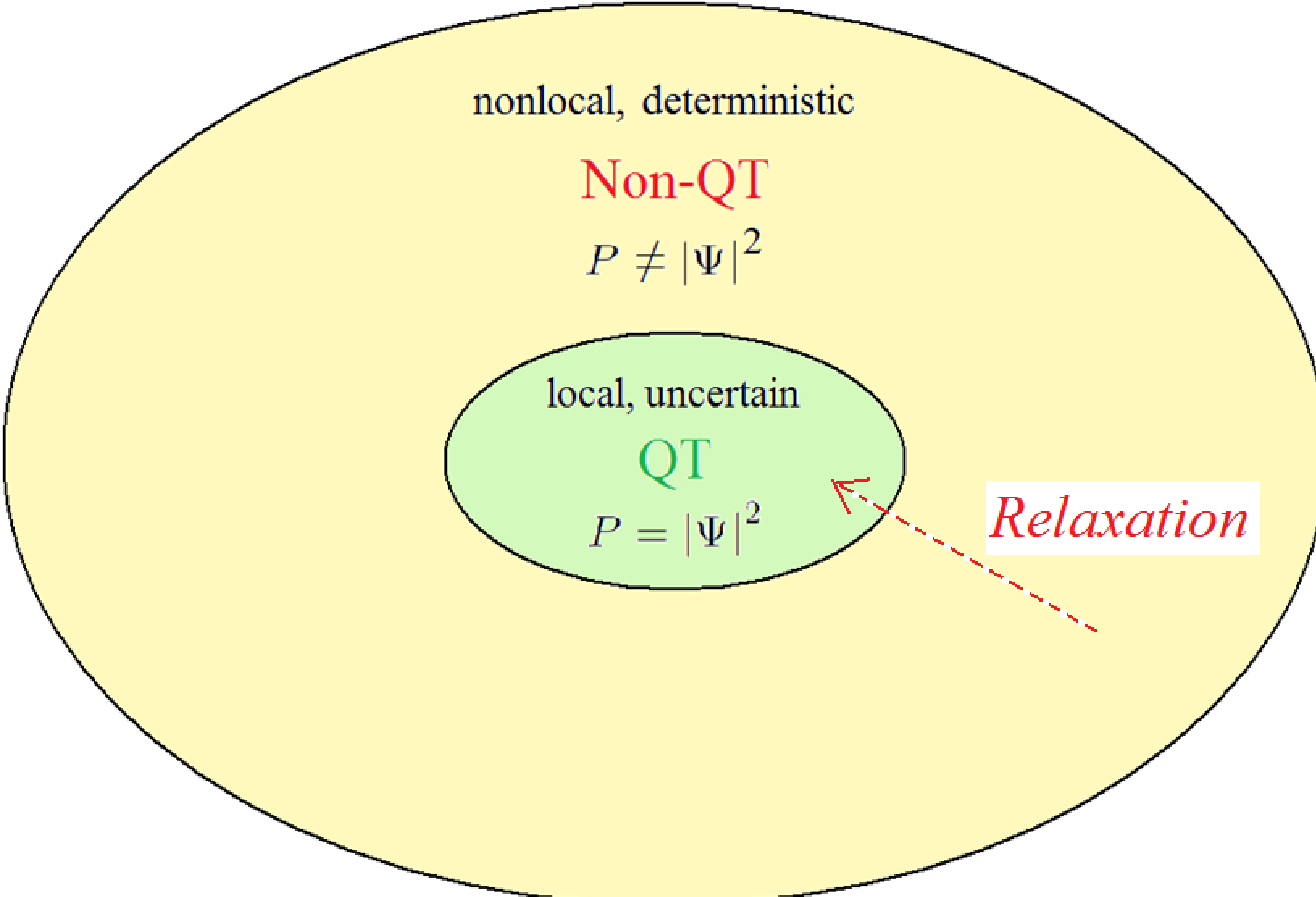


Non-equilibrium relaxes to equilibrium



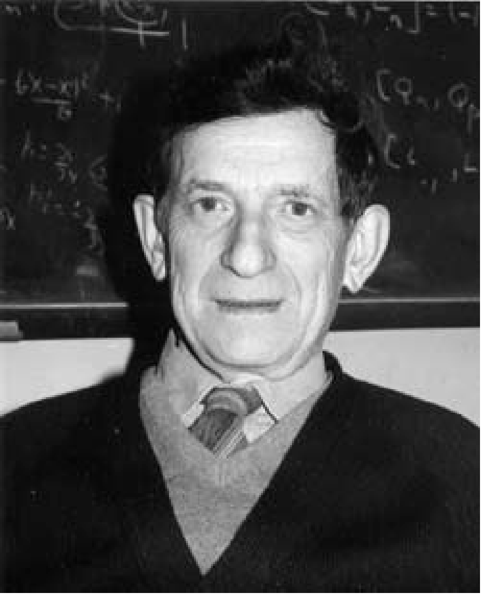
(Valentini and Westman 2005)

Quantum theory is a special case of a much wider physics



But what about doing something similar in Bohm's dynamics?

(Recent work with Samuel Colin and Ward Struyve.)



Bohm's Newtonian version (1952)

$$m_i \frac{d^2\mathbf{x}_i}{dt^2} = -\nabla_i (V + Q)$$ (law of motion)

$$Q \equiv -\sum_{i=1}^{N} \frac{1}{2m_i} \frac{\nabla_i^2 |\Psi|}{|\Psi|}$$

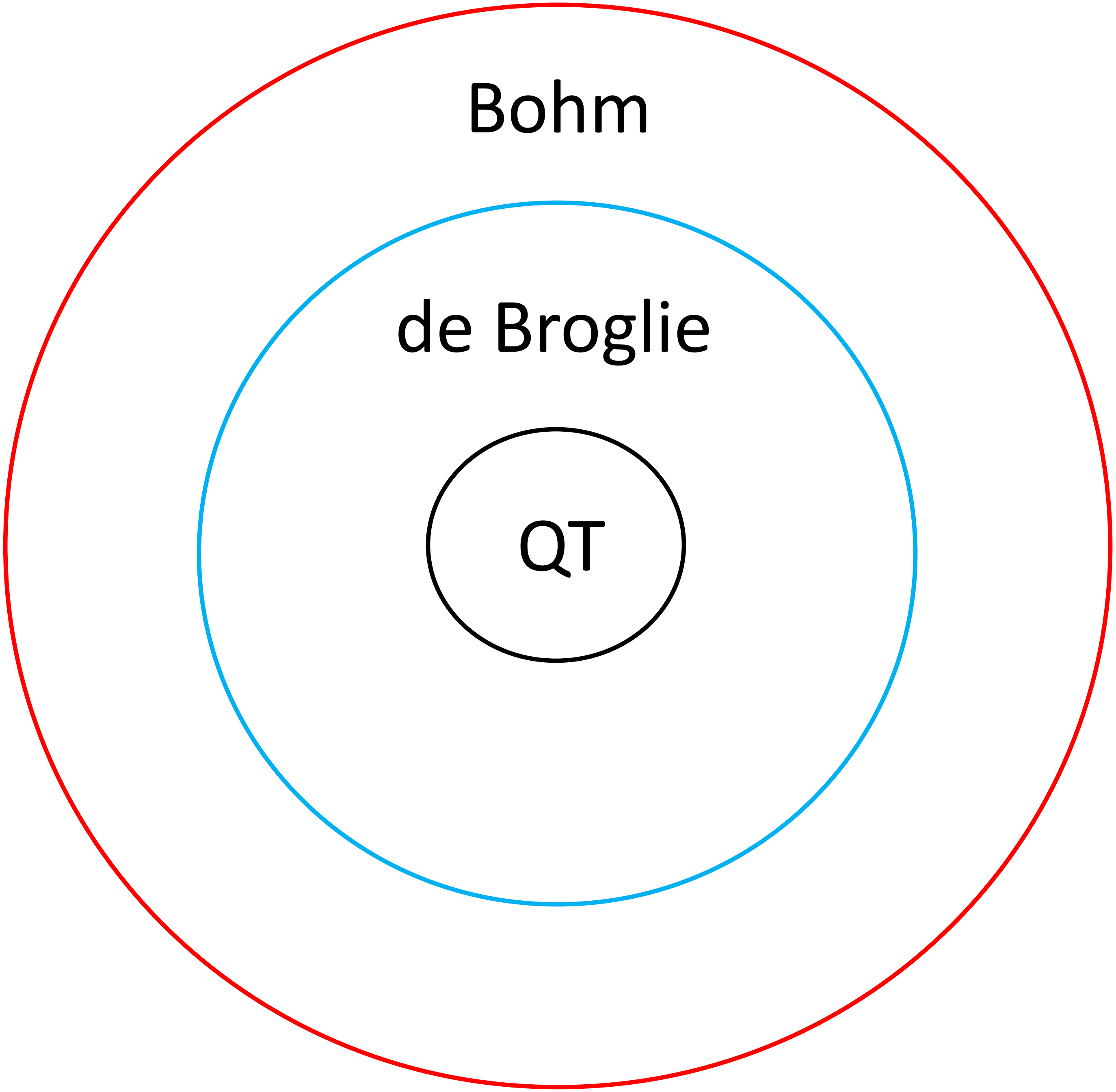
Get QM if assume initial $P = |\Psi|^2$ *and* $\mathbf{p}_i = \nabla_i S$

For Bohm, $\mathbf{p}_i = \nabla_i S$ is an initial condition; can drop it.

For de Broglie, $\mathbf{p}_i = \nabla_i S$ is the law of motion.

De Broglie's dynamics and Bohm's dynamics are different.

"Bohmian mechanics" is a misnomer for de Broglie's dynamics.


Bohm
de Broglie
QT

Part A: Instability of quantum equilibrium in Bohm's dynamics

(Joint work with Samuel Colin and Ward Struyve)

1. Bohm's dynamics allows "extended nonequilibrium", with momenta $p \neq \text{grad}\, S$
2. Extended nonequilibrium is unstable, does not relax in general
3. Argue that Bohm's dynamics is untenable

   (no reason to expect equilibrium today)

Instability of quantum equilibrium in Bohm's dynamics

Bohm's dynamics (1952)

$$m_i \frac{d^2 \mathbf{x}_i}{dt^2} = -\nabla_i (V + Q) \qquad Q \equiv -\sum_{i=1}^{N} \frac{1}{2m_i} \frac{\nabla_i^2 |\Psi|}{|\Psi|}$$

Get QM if assume initial $P = |\Psi|^2$ *and* $\mathbf{p}_i = \nabla_i S$

For Bohm, $\mathbf{p}_i = \nabla_i S$ is an initial condition; can be relaxed.

In phase space, the equilibrium distribution is

$$\rho_{\text{eq}}(x, p, t) = |\psi(x, t)|^2 \, \delta(p - \nabla S(x, t))$$

[(x, p) multi-dimensional]

Extended nonequilibrium in Bohm's dynamics

Bohm's dynamics allows an arbitrary $\rho(x,p,t)$, satisfying the continuity equation

$$\frac{\partial\rho}{\partial t} + \frac{\partial}{\partial x}(\rho\dot{x}) + \frac{\partial}{\partial p}(\rho\dot{p}) = 0$$

with the phase-space velocity field

$$\dot{x} = p/m, \qquad \dot{p} = -\nabla(V+Q)$$

(For simplicity, write as if one particle in 1D.)

The key question is whether 'reasonable' initial nonequilibrium conditions

$$\rho(x,p,0) \neq |\psi(x,0)|^2\,\delta(p - \nabla S(x,0))$$

tend to relax to (extended) quantum equilibrium

Our answer: they do not

Comparison with classical case

Bohm's dynamics is just Newton's dynamics
with an additional time-dependent potential $Q(x,t)$
added to the usual potential V

Because of the time dependence of Q,
the energy of the system is not conserved.

(No fixed energy surface in phase space.)

But we still have Liouville's theorem, as for
any Hamiltonian system:

$$\frac{d\rho}{dt} = \frac{\partial \rho}{\partial x}\dot{x} + \frac{\partial \rho}{\partial p}\dot{p} + \frac{\partial \rho}{\partial t}$$
$$= \frac{\partial \rho}{\partial x}\dot{x} + \frac{\partial \rho}{\partial p}\dot{p} - \frac{\partial}{\partial x}(\rho\dot{x}) - \frac{\partial}{\partial p}(\rho\dot{p})$$

and so (along a trajectory)

$$\frac{d\rho}{dt} = -\rho\left(\frac{\partial \dot{x}}{\partial x} + \frac{\partial \dot{p}}{\partial p}\right) = 0$$

Because dρ/dt = 0 , we have TWO equilibrium distributions:

(1) If $\rho(x,p,0) = c$ (for some constant c) over the available region of phase space, then

$$\rho(x,p,t) = c$$

at all times t (the classical equilibrium distribution)

(2) The quantum equilibrium distribution

$$\rho_{\text{eq}}(x,p,t) = |\psi(x,t)|^2\,\delta(p - \nabla S(x,t))$$

(also conserved by Bohm's dynamics)

This is unusual

Some comments and queries:

1. Will an arbitrary initial state relax to one of these equilibrium states? Or to neither? Might guess that the existence of two equilibrium measures will 'confuse' the system.

2. If we appeal to 'typicality' wrt to an equilibrium measure, which one should we choose?

3. Bohm's dynamics is an unusual dynamical system. Beware of standard intuitions and expectations.

Our central claim (re. Bohm’s dynamics)

While more work can and should be done, we have gathered extensive evidence that:

(1) The quantum equilibrium state is unstable in Bohm’s dynamics.

(2) There is no tendency to relax to quantum equilibrium in Bohm’s dynamics.

(3) If the universe started in a non-equilibrium state, we would (almost certainly) not see equilibrium today, and in particular there would be no bound states (atoms etc)

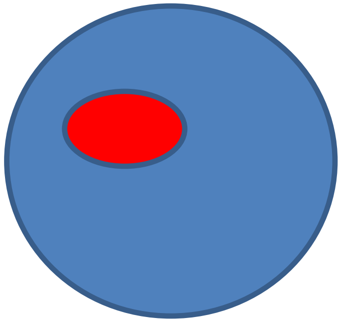
Instability of Bohm's dynamics
A simple example

Ground state of a bound system.
E.g. a hydrogen atom, or a simple harmonic oscillator (SHO)

Contrast with de Broglie's dynamics:

p = grad S = 0 , *velocities* vanish

Therefore, an initial small deviation from $P = |\Psi|^2$ stays small (and indeed static).



For superpositions, initial small deviations relax away.

Whereas, in Bohm's dynamics:

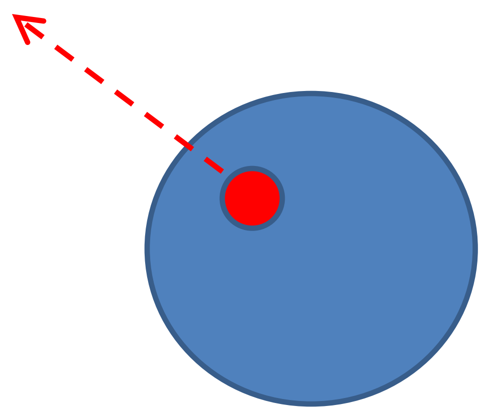
for a ground state,

$$-\,\text{grad}\, V - \text{grad}\, Q = 0 \quad , \qquad \textit{accelerations}\ \text{vanish}$$

Therefore, an initial small deviation of p from grad S (= 0) remains small (and indeed static).

But: causes a GROWTH in non-equilibrium wrt *position*.

Bound states become UNBOUND.



And for superpositions, do NOT get relaxation ... .

Non-equilibrium superpositions in Bohm's dynamics

Instability is not an artifact of simple ground state.

More complex states, with superpositions , are also unstable.

Sketch of proof, for an example:

Superposition of energy states for a harmonic oscillator.

Find lower bound on acceleration, in some x-region

$$-\operatorname{grad} V - \operatorname{grad} Q > -b/x^2 \qquad \text{(some constant } b\text{)}$$

implies *escapes to infinity* for some initial momenta
(cf. escape velocity from earth)

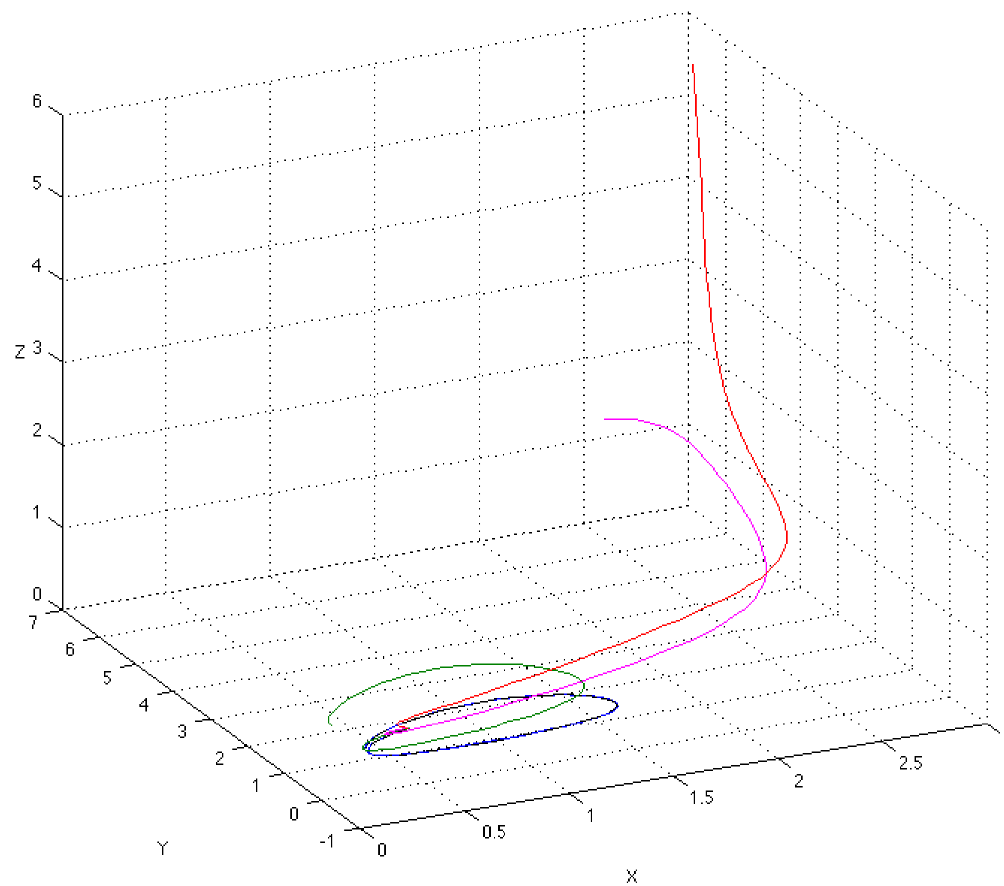
Numerical results for the hydrogen atom

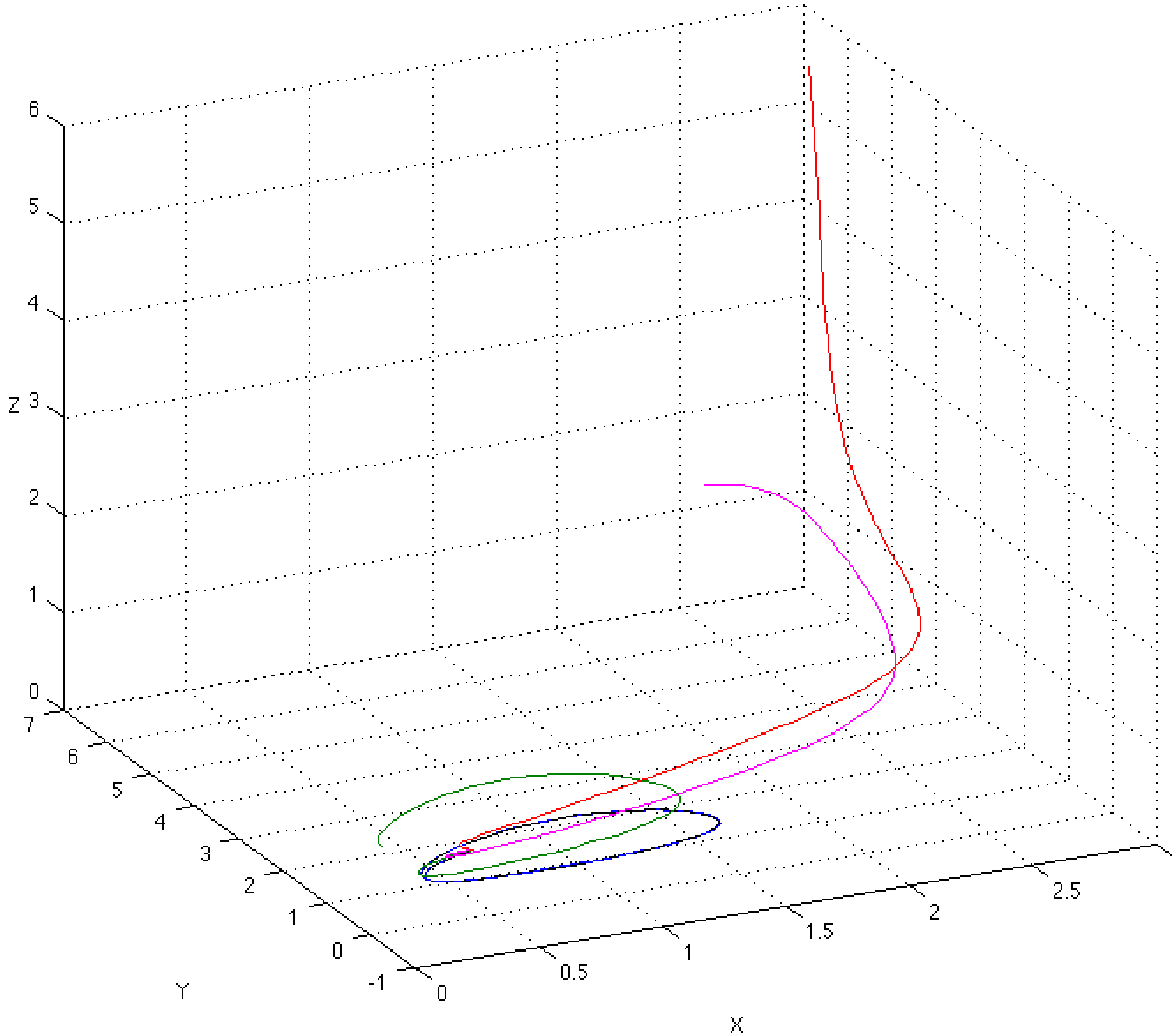
Superposition of 3 energy eigenstates, (n,l,m) = (1,0,0), (2,1,1), (3,2,-1).
Initial velocities close to those of de Broglie

Initial position: (0.5,0.5,0.5)
(Bohr radius = 1)

5 trajectories:

Black: de Broglie trajectory
Blue: Bohm trajectory with initial de Broglie velocity
(Blue and black identical)
Green: close to dB velocity
Magenta: dB velocity + (0.05,0.05,0.05)
Red: dB velocity + (0.1,0.1,0.1)





Z
0
1
2
3
4
5
6
Y
-1
0
1
2
3
4
5
6
7
X
0
0.5
1
1.5
2
2.5
3

Our central claim (again)

More work can and should be done, but we have gathered extensive evidence that:

(1) The quantum equilibrium state is unstable in Bohm’s dynamics.

(2) There is no tendency to relax to quantum equilibrium in Bohm’s dynamics.

(3) If the universe started in a non-equilibrium state, we would (almost certainly) not see equilibrium today, and in particular there would be no bound states (atoms etc)

Expect no bound states today?

Might claim that early universe will reach equilibrium long before atoms form (about 400,000 years after the big bang).

If so, would still form bound states.

Expect no bound states today?

Might claim that early universe will reach equilibrium long before atoms form (about 400,000 years after the big bang).

If so, would still form bound states.

But: *early fields will show the same instability.*

For example, a decoupled scalar field mode is mathematically the same as a 2D simple harmonic oscillator:

wave function $\psi_{\mathbf{k}}(q_{\mathbf{k}1}, q_{\mathbf{k}2}, t)$ mode amplitudes $(q_{\mathbf{k}1}, q_{\mathbf{k}2})$

$$i\frac{\partial\psi_{\mathbf{k}}}{\partial t} = -\frac{1}{2a^3}\left(\frac{\partial^2}{\partial q_{\mathbf{k}1}^2} + \frac{\partial^2}{\partial q_{\mathbf{k}2}^2}\right)\psi_{\mathbf{k}} + \frac{1}{2}ak^2\left(q_{\mathbf{k}1}^2 + q_{\mathbf{k}2}^2\right)\psi_{\mathbf{k}}$$

Same as 2D SHO with time-dependent "mass" $m = a^3$ and $\omega = k/a$

short-wavelength limit, $\lambda_{\text{phys}} << H^{-1}$,

$$a^3 \approx \text{const.}$$

Bohm's dynamics for a (short-wavelength) decoupled field mode:

$$a^3\ddot{q}_{\mathbf{k}1} = -\frac{\partial}{\partial q_{\mathbf{k}1}}(V+Q), \quad a^3\ddot{q}_{\mathbf{k}2} = -\frac{\partial}{\partial q_{\mathbf{k}2}}(V+Q)$$

$$V = \frac{1}{2}ak^2\left(q_{\mathbf{k}1}^2 + q_{\mathbf{k}2}^2\right) \qquad Q = -\frac{1}{2a^3}\frac{\nabla_{1,2}^2|\psi_{\mathbf{k}}|}{|\psi_{\mathbf{k}}|}$$

(de Broglie's equations of motion.
Initial equilibrium conditions for Bohm)

$$\dot{q}_{\mathbf{k}1} = \frac{1}{a^3}\frac{\partial s_{\mathbf{k}}}{\partial q_{\mathbf{k}1}}, \qquad \dot{q}_{\mathbf{k}2} = \frac{1}{a^3}\frac{\partial s_{\mathbf{k}}}{\partial q_{\mathbf{k}2}}$$

Field amplitudes will show same instability as found for low-energy particle case

Conclude: Bohm's dynamics is untenable

Agrees with QT only if assume very special initial conditions.

The dynamics is unstable, and small deviations from initial equilibrium do not relax.

Small departures from equilibrium (e.g. In the remote past) would in fact grow with time.

If you believe in Bohm's dynamics, it would be unreasonable to expect to see effective QT today, in contradiction with observation.

(Have found a breakdown of QT, but far too big.)

Possible responses:

1. Extended equilibrium is "absolute" (cf. DGZ in deB case). But: how justify one 'typicality' measure when there are two equilibrium distributions?
   (And in any case, the measure does too much work.)

2. Bohm's dynamics is only an approximation, corrections from a deeper theory will drive to equilibrium.
   Perhaps, but: (a) fact remains that Bohm's dynamics on its own is unstable and (we claim) untenable, and (b) the corrections will have to be large to overcome what seems a gross instability.

Our view: Bohm's pseudo-Newtonian reformulation of de Broglie's dynamics was a mistake, and we ought to regard de Broglie's original (1927) theory as the proper and bona fide theory.

Part B: Relic non-equilibrium in de Broglie's dynamics

Towards a prediction for a breakdown of QT

(Valentini: arXiv 2008, Phys. Rev. D 2010)

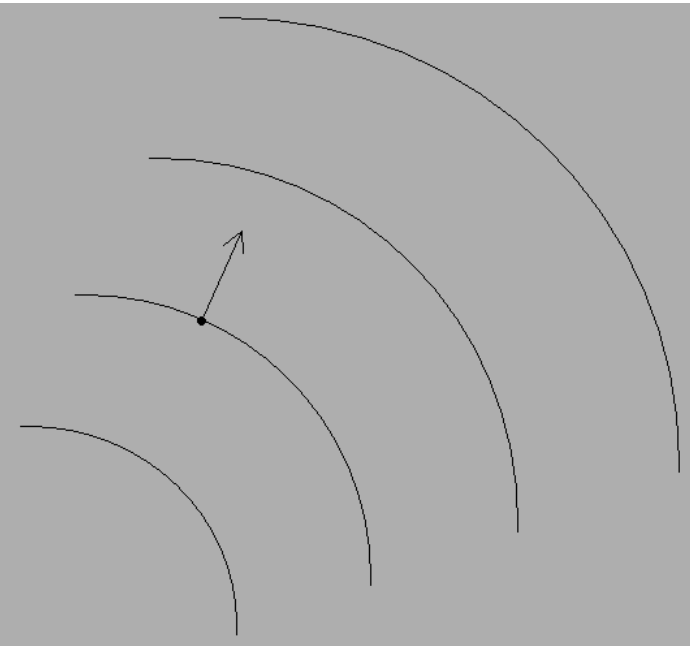
1. De Broglie's dynamics allows non-equilibrium with configuration-distributions $P \neq |\Psi|^2$, which tend to relax to equilibrium $P = |\Psi|^2$
2. But on expanding space: relaxation can be suppressed at long wavelengths
3. Update on work towards a precise prediction for relic non-equilibrium today (or in the CMB)


Scanned at the American Institute of Physics

De Broglie's Pilot-Wave Dynamics (1927)

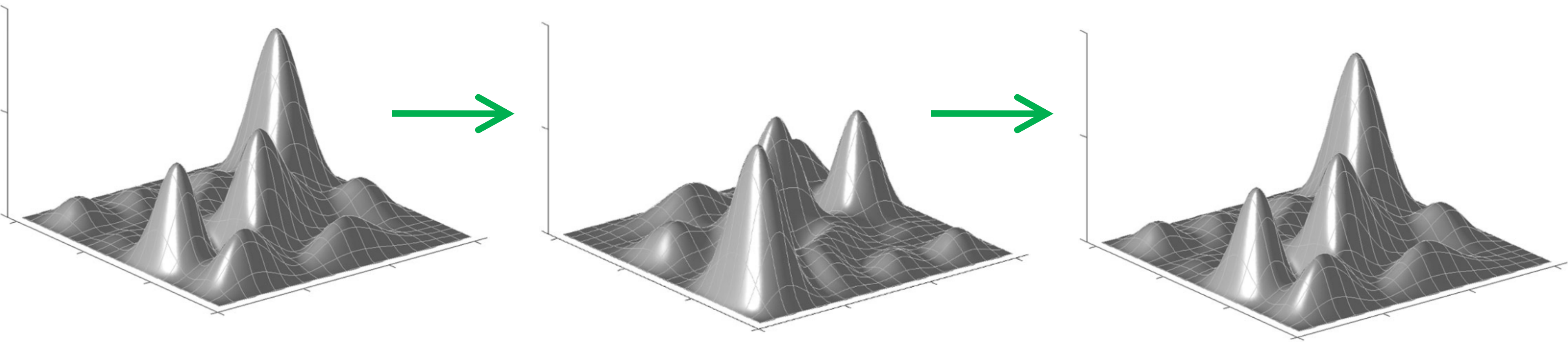
$$m_i \frac{d\mathbf{x}_i}{dt} = \nabla_i S$$

$$i\frac{\partial \Psi}{\partial t} = \sum_{i=1}^{N} -\frac{1}{2m_i}\nabla_i^2 \Psi + V\Psi$$

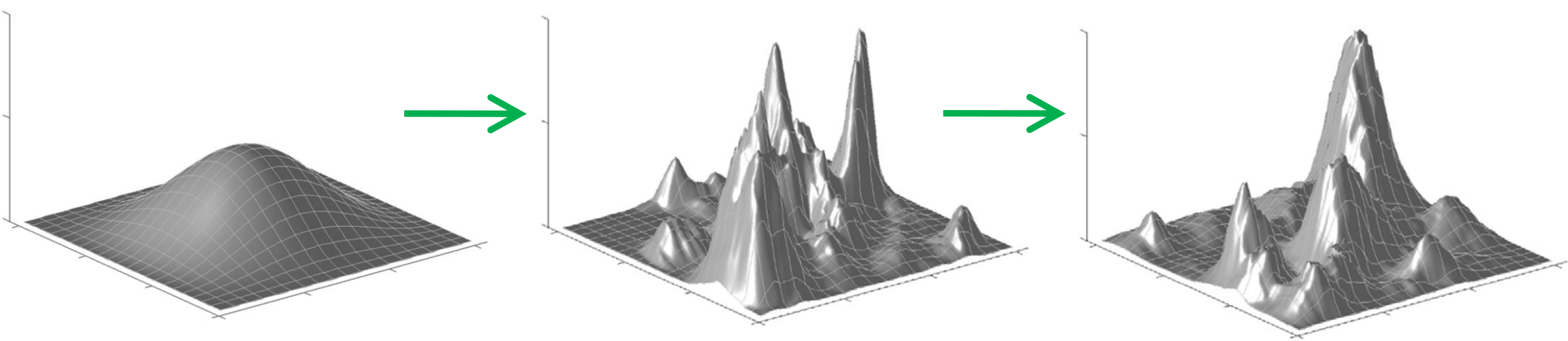


Get QM if assume initial $P = |\Psi|^2$

Equilibrium changes with time



Non-equilibrium relaxes to equilibrium



A long time ago (long and violent astrophysical history)

Key idea

Relaxation on expanding space

Can be suppressed at long wavelengths

(cf. freezing of classical perturbations)

For a given cosmology, if we assume initial nonequilibrium, we can deduce where "residual nonequilibrium" can be found.

Pilot-wave field theory on expanding space

Flat metric $$d\tau^2 = dt^2 - a^2 d\mathbf{x}^2$$

$H \equiv \dot{a}/a$ is the Hubble parameter

physical wavelengths $\lambda_{\text{phys}} = a(t)\lambda$, where $\lambda = 2\pi/k$ is a comoving wavelength

free (minimally-coupled) massless scalar field ϕ

Hamiltonian density $$\mathcal{H} = \tfrac{1}{2}\frac{\pi^2}{a^3} + \tfrac{1}{2}a(\nabla\phi)^2$$

Fourier components $$\phi_{\mathbf{k}} = \frac{\sqrt{V}}{(2\pi)^{3/2}}\left(q_{\mathbf{k}1} + iq_{\mathbf{k}2}\right)$$

Hamiltonian $H = \int d^3\mathbf{x}\,\mathcal{H}$ becomes $H = \sum_{\mathbf{k}r} H_{\mathbf{k}r}$

with $$H_{\mathbf{k}r} = \frac{1}{2a^3}\pi_{\mathbf{k}r}^2 + \frac{1}{2}ak^2q_{\mathbf{k}r}^2$$

Schrödinger equation for $\Psi = \Psi[q_{\mathbf{k}r}, t]$ is

$$i\frac{\partial\Psi}{\partial t} = \sum_{\mathbf{k}r}\left(-\frac{1}{2a^3}\frac{\partial^2}{\partial q_{\mathbf{k}r}^2} + \frac{1}{2}ak^2q_{\mathbf{k}r}^2\right)\Psi$$

implies the continuity equation

$$\frac{\partial\left|\Psi\right|^2}{\partial t} + \sum_{\mathbf{k}r}\frac{\partial}{\partial q_{\mathbf{k}r}}\left(\left|\Psi\right|^2\frac{1}{a^3}\frac{\partial S}{\partial q_{\mathbf{k}r}}\right) = 0$$

and the de Broglie velocities

$$\frac{dq_{\mathbf{k}r}}{dt} = \frac{1}{a^3}\frac{\partial S}{\partial q_{\mathbf{k}r}}$$

initial distribution $P[q_{\mathbf{k}r}, t_i]$,

time evolution $P[q_{\mathbf{k}r}, t]$ will be determined by

$$\frac{\partial P}{\partial t} + \sum_{\mathbf{k}r}\frac{\partial}{\partial q_{\mathbf{k}r}}\left(P\frac{1}{a^3}\frac{\partial S}{\partial q_{\mathbf{k}r}}\right) = 0$$

Relaxation in the short-wavelength limit

Roughly: short-wavelength limit, $\lambda_{\mathrm{phys}} << H^{-1}$,

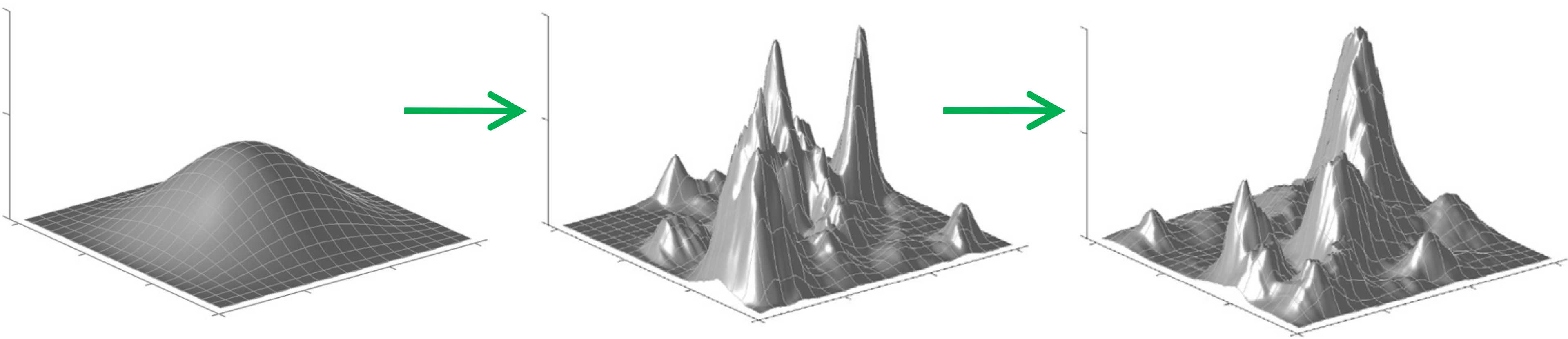
$$\downarrow$$

(strictly, $\lambda_{\mathrm{phys}} << \Delta n_{\mathbf{k}} \cdot H^{-1}$)

Minkowski

the timescale $\Delta t \propto \lambda_{\mathrm{phys}}$ over which $\psi_{\mathbf{k}} = \psi_{\mathbf{k}}(q_{\mathbf{k}1}, q_{\mathbf{k}2}, t)$ evolves will be much smaller than the expansion timescale $H^{-1} \equiv a/\dot{a}$

Obtain usual efficient relaxation (decoupled mode)



But: relaxation *can* be suppressed at *long* wavelengths

Will now derive a rigorous condition for non-equilibrium freezing:

- arbitrary time interval $[t_i, t_f]$
- any quantum state (entangled, mixed)
- interacting fields

Inequality for the freezing of quantum nonequilibrium

(Valentini 2008)

examine the behaviour of the trajectories themselves

First: general (entangled) pure quantum state of a free field
(Generalise later to mixed, interacting.)

collection of non-interacting one-dimensional harmonic oscillators

$$\hat{H} = \sum_{\mathbf{k}r} \hat{H}_{\mathbf{k}r}, \text{ with } \hat{H}_{\mathbf{k}r} = \frac{\hat{\pi}_{\mathbf{k}r}^2}{2a^3} + \frac{1}{2}a^3\omega^2\hat{q}_{\mathbf{k}r}^2$$

arbitrary wave functional $\Psi[q_{\mathbf{k}r}, t]$,

de Broglie velocity field is given by $\dfrac{dq_{\mathbf{k}r}}{dt} = \dfrac{1}{a^3}\dfrac{\partial S}{\partial q_{\mathbf{k}r}}$

evolution of an arbitrary ensemble distribution $P[q_{\mathbf{k}r}, t]$

$$\frac{\partial P}{\partial t} + \sum_{\mathbf{k}r} \frac{\partial}{\partial q_{\mathbf{k}r}} \left(P \frac{1}{a^3} \frac{\partial S}{\partial q_{\mathbf{k}r}} \right) = 0$$

Initial nonequilibrium distribution $P[q_{\mathbf{k}r}, t_i] \neq |\Psi[q_{\mathbf{k}r}, t_i]|^2$

Can relax (in general, on a coarse-grained level)

only if the trajectories move far enough

(Cf. gas molecules in a box)

Final displacement $\delta q_{\mathbf{k}r}(t_f) = \int_{t_i}^{t_f} dt\ \dot{q}_{\mathbf{k}r}(t)$

Simple condition for "freezing": $|\delta q_{\mathbf{k}r}(t_f)| << \Delta q_{\mathbf{k}r}(t_f)$

(magnitude of final displacement smaller than width of wave packet)

Too strong. Take weaker condition $\langle|\delta q_{\mathbf{k}r}(t_f)|\rangle_{\mathrm{eq}} < \Delta q_{\mathbf{k}r}(t_f)$

(equilibrium mean smaller than width of wave packet)

Implies that most of the ensemble cannot move

by much more than $\Delta q_{\mathbf{k}r}(t_f)$

If $\langle|\delta q_{\mathbf{k}r}(t_f)|\rangle_{\rm eq} < \Delta q_{\mathbf{k}r}(t_f)$

relaxation will in general be suppressed (for the mode **k***r*)

Now: $$\langle|\delta q_{\mathbf{k}r}(t_f)|\rangle_{\rm eq} \leq \left\langle \int_{t_i}^{t_f} dt \ \ |\dot{q}_{\mathbf{k}r}(t)| \right\rangle_{\rm eq} = \int_{t_i}^{t_f} dt \ \ \langle|\dot{q}_{\mathbf{k}r}(t)|\rangle_{\rm eq} \ ,$$

(where $\langle|\dot{q}_{\mathbf{k}r}(t)|\rangle_{\rm eq} = \int dq \ |\Psi[q,t]|^2 |\dot{q}_{\mathbf{k}r}(q,t)|$)

Or

$$\langle|\delta q_{\mathbf{k}r}(t_f)|\rangle_{\rm eq} \leq \int_{t_i}^{t_f} dt \ \sqrt{\left\langle |\dot{q}_{\mathbf{k}r}(t)|^2 \right\rangle_{\rm eq}} \ .$$

We have

$$a^6 \left\langle |\dot{q}_{\mathbf{k}r}|^2 \right\rangle_{\rm eq} \leq \left\langle \hat{\pi}_{\mathbf{k}r}^2 \right\rangle \ , \quad (*)$$

and so

$$\langle|\delta q_{\mathbf{k}r}(t_f)|\rangle_{\rm eq} \leq \int_{t_i}^{t_f} dt \ \frac{1}{a^3} \sqrt{\langle \hat{\pi}_{\mathbf{k}r}^2 \rangle}$$

and so

$$\langle|\delta q_{\mathbf{k}r}(t_f)|\rangle_{\mathrm{eq}} \leq \int_{t_i}^{t_f} dt\ \frac{1}{a^3}\sqrt{\langle\hat{\pi}_{\mathbf{k}r}^2\rangle}$$

Since $\langle\hat{q}_{\mathbf{k}r}^2\rangle > 0$, we also have

$$\langle\hat{\pi}_{\mathbf{k}r}^2\rangle < 2a^3\left\langle\hat{H}_{\mathbf{k}r}\right\rangle\ ,$$

and so

$$\langle|\delta q_{\mathbf{k}r}(t_f)|\rangle_{\mathrm{eq}} < \int_{t_i}^{t_f} dt\ \frac{1}{a^3}\sqrt{2a^3\left\langle\hat{H}_{\mathbf{k}r}\right\rangle}\ .$$

(Can use to estimate relaxation time, agrees with LOUIS for *a* = 1.)

Introducing the number operator $\hat{n}_{\mathbf{k}r}$,

$$\langle|\delta q_{\mathbf{k}r}(t_f)|\rangle_{\mathrm{eq}} < \int_{t_i}^{t_f} dt\ \frac{1}{a^2}\sqrt{2k(\langle\hat{n}_{\mathbf{k}r}\rangle+1/2)}\ .$$

The mean $\langle|\delta q_{\mathbf{k}r}(t_f)|\rangle_{\mathrm{eq}}$ at time t_f is to be compared with the width $\Delta q_{\mathbf{k}r}(t_f)$

Using uncertainty relation $\Delta q_{\mathbf{k}r} \Delta \pi_{\mathbf{k}r} \geq \frac{1}{2}$ and $\Delta \pi_{\mathbf{k}r} \leq \sqrt{\langle \hat{\pi}^2_{\mathbf{k}r} \rangle}$,

we have

$$1/\Delta q_{\mathbf{k}r} < 2\sqrt{2a^3 \left\langle \hat{H}_{\mathbf{k}r} \right\rangle} = 2a\sqrt{2k(\langle \hat{n}_{\mathbf{k}r} \rangle + 1/2)} \; .$$

Combine with previous

$$\langle |\delta q_{\mathbf{k}r}(t_f)| \rangle_{\text{eq}} < \int_{t_i}^{t_f} dt \; \frac{1}{a^2} \sqrt{2k(\langle \hat{n}_{\mathbf{k}r} \rangle + 1/2)} \; .$$

Have upper bound for the ratio

$$\boxed{\frac{\langle |\delta q_{\mathbf{k}r}(t_f)| \rangle_{\text{eq}}}{\Delta q_{\mathbf{k}r}(t_f)} < 4k a_f \sqrt{\langle \hat{n}_{\mathbf{k}r} \rangle_f + 1/2} \int_{t_i}^{t_f} dt \; \frac{1}{a^2} \sqrt{\langle \hat{n}_{\mathbf{k}r} \rangle + 1/2}}$$

(where $a_f \equiv a(t_f)$, and so on). Note that $\langle \hat{n}_{\mathbf{k}r} \rangle$ is in general a function of time

True for arbitrary entangled state Ψ.

“Freezing inequality”: right-hand side is less than one ... .

Freezing inequality

$$\frac{1}{k} > 4a_f \sqrt{\langle \hat{n}_{\mathbf{k}r} \rangle_f + 1/2} \int_{t_i}^{t_f} dt \; \frac{1}{a^2} \sqrt{\langle \hat{n}_{\mathbf{k}r} \rangle + 1/2} \, .$$

"Frozen" nonequilibrium will exist at later times $t_f > t_i$

for modes satisfying the inequality

Mixed states:

- Statistical mixture of physically-real pilot waves
- Consider freezing inequality for each pure subensemble separately
- Might hold for some subensembles and not for others (or for all of them, or none)

Interacting fields:

- Finite models with a cutoff (ignore divergences)
- Scalar ϕ interacts with other fields
- Only difference is in time evolution of $\langle \hat{n}_{\mathbf{k}r} \rangle$

General implication of the freezing inequality

$$\frac{1}{k} > 4a_f \sqrt{\langle \hat{n}_{\mathbf{k}r} \rangle_f + 1/2} \int_{t_i}^{t_f} dt \; \frac{1}{a^2} \sqrt{\langle \hat{n}_{\mathbf{k}r} \rangle + 1/2} \; .$$

Satisfied? Depends on history of expansion,
and on time evolution of quantum state

For a radiation-dominated expansion on $[t_i, t_f]$, with $a(t) = a_f (t/t_f)^{1/2}$,

the physical wavelength $\lambda_{\text{phys}}(t_f) = a_f (2\pi/k)$ at time t_f

must be larger than

the Hubble radius H_f^{-1} at time t_f

Since $\langle \hat{n}_{\mathbf{k}r} \rangle \geq 0$,

$$\frac{1}{k} > 2a_f \int_{t_i}^{t_f} dt \; \frac{1}{a^2} = \frac{2t_f}{a_f} \ln(t_f/t_i) \; , \quad \text{or} \quad \lambda_{\text{phys}}(t_f) > 2\pi H_f^{-1} \ln(t_f/t_i)$$

(where $H_f^{-1} = 2t_f$)

Possible consequences of early nonequilibrium freezing

Write $\lambda_{\rm phys}(t_f) > 2\pi H_f^{-1}\ln(t_f/t_i)$ as $\lambda_{\rm phys}(t_f) > 4\pi H_f^{-1}\ln(T_i/T_f)$.

(A necessary but not sufficient condition.)

Points to where nonequilibrium *could* be found.

We should search for nonequilibrium above a specific critical wavelength

Two main areas of research (in progress)

Corrections to Inflationary Predictions for the CMB

Relic Nonequilibrium Particles

Corrections to Inflationary Predictions for the CMB

inflaton perturbation ϕ $\longrightarrow$ curvature perturbation $\mathcal{R}_{\mathbf{k}}$

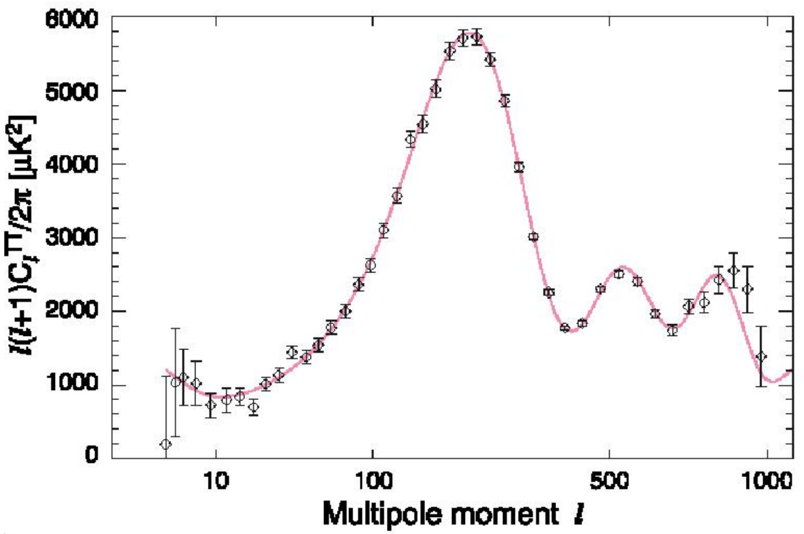
curvature perturbation $\mathcal{R}_{\mathbf{k}}$ $\longrightarrow$ temperature anisotropy a_{lm}

$$\mathcal{R}_{\mathbf{k}} = -\left[\frac{H}{\dot{\phi}_0}\phi_{\mathbf{k}}\right]_{t=t_*(k)}$$

$$a_{lm} = \frac{i^l}{2\pi^2}\int d^3\mathbf{k}\; \mathcal{T}(k,l)\mathcal{R}_{\mathbf{k}} Y_{lm}(\hat{\mathbf{k}})$$

$$\frac{\Delta T(\theta,\phi)}{\bar{T}} = \sum_{l=2}^{\infty}\sum_{m=-l}^{+l} a_{lm} Y_{lm}(\theta,\phi)$$

Angular power spectrum $C_l \equiv \left\langle |a_{lm}|^2 \right\rangle$



$$C_l = \frac{1}{2\pi^2}\int_0^\infty \frac{dk}{k}\,\mathcal{T}^2(k,l)\mathcal{P}_{\mathcal{R}}(k)$$

$$\mathcal{P}_{\mathcal{R}}(k) \equiv \frac{4\pi k^3}{V}\left\langle |\mathcal{R}_{\mathbf{k}}|^2 \right\rangle$$

Quantum equilibrium

$$\langle |\phi_{\mathbf{k}}|^2 \rangle_{\mathrm{QT}} = \frac{V}{2(2\pi)^3}\frac{H^2}{k^3} \qquad \mathcal{P}_\phi^{\mathrm{QT}}(k) \equiv \frac{4\pi k^3}{V}\left\langle |\phi_{\mathbf{k}}|^2 \right\rangle_{\mathrm{QT}} = \frac{H^2}{4\pi^2}$$

Quantum nonequilibrium (no relaxation during inflation; product state)

$$\langle |\phi_{\mathbf{k}}|^2 \rangle = \langle |\phi_{\mathbf{k}}|^2 \rangle_{\mathrm{QT}}\,\xi(k) \qquad \mathcal{P}_{\mathcal{R}}(k) = \mathcal{P}_{\mathcal{R}}^{\mathrm{QT}}(k)\xi(k)$$

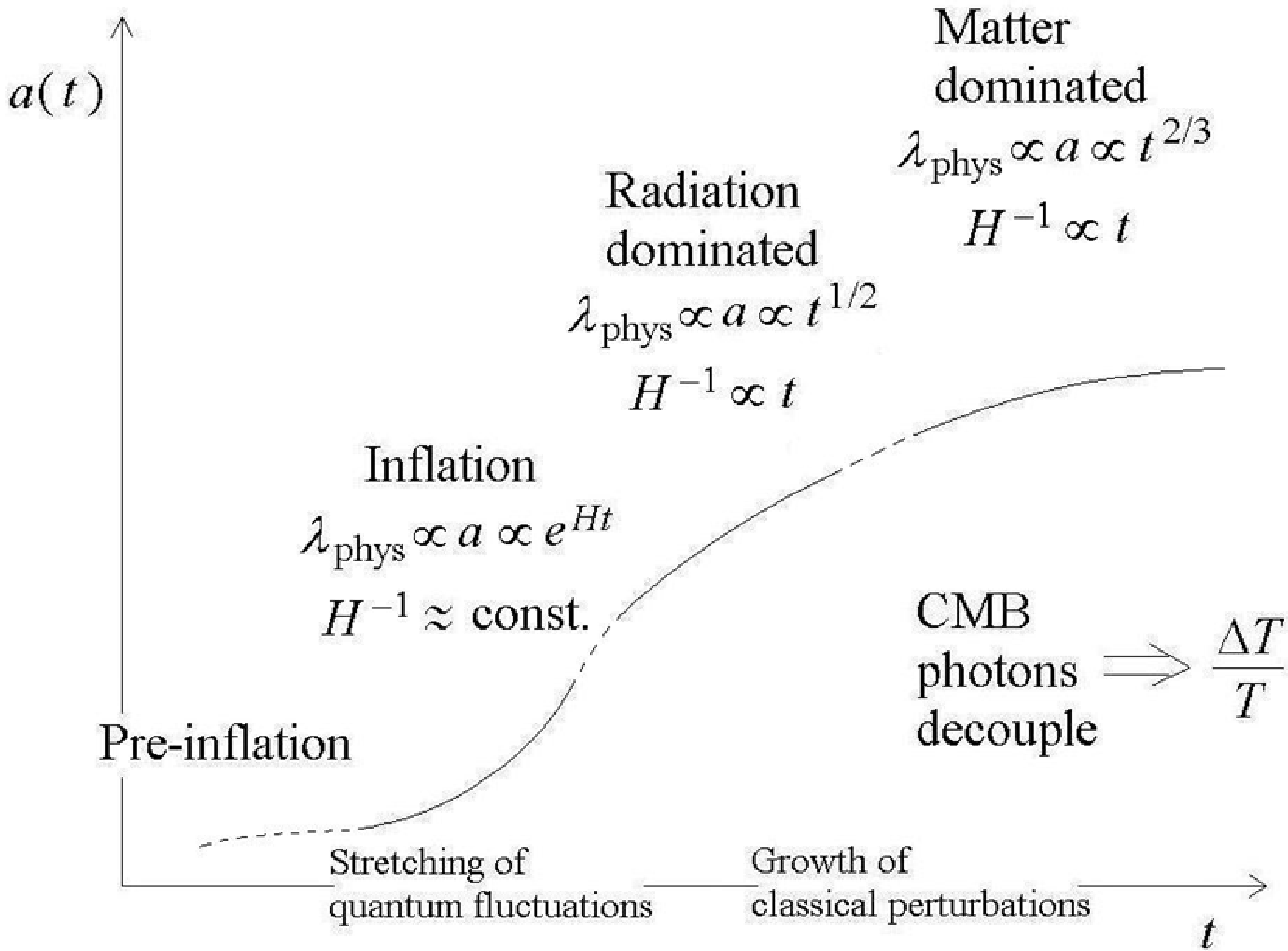
Can set empirical limits on $\xi(k)$ (Valentini 2007, 2008, PRD 2010)

But can we *predict* something about $\xi(k)$?

Can we *predict* something about $\xi(k)$?
(Work in progress)

One possible strategy:

- Consider a pre-inflationary (radiation-dominated) era
- Derive constraints on relic nonequilibrium from that era



Plausible scenario:

- Pre-inflation: nonequilibrium at super-Hubble wavelengths (all $\lambda_{\rm phys} > H^{-1}$ at sufficiently early times)
- Some nonequilibrium modes enter the Hubble radius, and do not completely relax by the time inflation begins
- Larger wavelengths enter later, less likely to relax before inflation begins
- Nonequilibrium possible only for λ larger than infra-red cutoff $\lambda_{\rm c}$
- Try to predict $\lambda_{\rm c}$ (depends on cosmology)

There is some (weak) evidence for an infra-red cutoff in the primordial power spectrum

Relic Nonequilibrium Particles

(to be detected today)

(Work in progress: rough scenario only)

Lower bound

$\lambda_{\rm phys}(t_f) > 2\pi H_f^{-1} \ln(t_f/t_i)$ or $\lambda_{\rm phys}(t_f) > 4\pi H_f^{-1} \ln(T_i/T_f)$.

To maximise our chances, minimise the right-hand-side

Want t_f as small as possible (subject to the constraint that further relaxation can be neglected)

Take t_f to be time $t_{\rm dec}$ of decoupling

Nonequilibrium present at $t_{\rm dec}$ *might* persist until much later

Then have lower bound $\lambda_{\rm phys}(t_{\rm dec}) > 4\pi H_{\rm dec}^{-1} \ln(k_{\rm B}T_i/k_{\rm B}T_{\rm dec})$

Rough estimates using the lower bound

$$\lambda_{\text{phys}}(t_{\text{dec}}) > 4\pi H_{\text{dec}}^{-1} \ln(k_{\text{B}} T_i / k_{\text{B}} T_{\text{dec}})$$

$$\lambda_{\text{phys}}(t_{\text{dec}}) = a_{\text{dec}} \lambda, \text{ where } a_{\text{dec}} = T_0 / T_{\text{dec}} \text{ (with } T_0 \simeq 2.7 \text{ K}$$

$H_{\text{dec}}^{-1} = 2t_{\text{dec}}$, where t_{dec} may be expressed in terms of T_{dec} using

$$t \sim (1\ \text{s}) \left(\frac{1\ \text{MeV}}{k_{\text{B}} T} \right)^2$$

$$\lambda \gtrsim 8\pi c (1\ \text{s}) \left(\frac{1\ \text{MeV}}{k_{\text{B}} T_{\text{dec}}} \right) \left(\frac{1\ \text{MeV}}{k_{\text{B}} T_0} \right) \ln \left(\frac{k_{\text{B}} T_i}{k_{\text{B}} T_{\text{dec}}} \right)$$

or

$$\lambda \gtrsim (3.3 \times 10^{21}\ \text{cm}) \left(\frac{1\ \text{MeV}}{k_{\text{B}} T_{\text{dec}}} \right) \ln \left(\frac{k_{\text{B}} T_i}{k_{\text{B}} T_{\text{dec}}} \right)$$

Lower bound on wavelength *today*,
at which nonequilibrium *could* be found.

standard Friedmann cosmology with no inflationary period

initial conditions at the Planck era $k_B T_i \sim k_B T_P \sim 10^{19}$ GeV

(Or: nonequilibrium relics could be produced by inflaton decay.)

Photons decouple from matter at $k_B (T_{dec})_\gamma \sim 0.3$ eV

$\lambda_\gamma \gtrsim 0.7 \times 10^{30}$ cm (Ridiculous)

neutrinos, which decouple at $k_B (T_{dec})_\nu \sim 1$ MeV

$\lambda_\nu \gtrsim 1.7 \times 10^{23}$ cm $\simeq 5.5 \times 10^4$ pc (or $\sim 10^5$ light years) (Hopeless)

Gravitons decouple at a temperature $(T_{dec})_g \lesssim T_P$

$k_B (T_{dec})_g \equiv x_g (k_B T_P) \simeq x_g (10^{19}$ GeV$) \quad x_g \lesssim 1$

$\lambda_g \gtrsim (0.3 \text{ cm})(1/x_g) \ln(1/x_g) \qquad \lambda_{max}(1 \text{ K}) \simeq 0.3$ cm

(But can't detect relic gravitons. Still hopeless)

Ray of hope:

unstable gravitino $\tilde{G}$,

Decouples very early, decay products (e.g. photons) might be observable today

$$k_{\mathrm{B}}(T_{\mathrm{dec}})_{\tilde{G}} \equiv x_{\tilde{G}}(k_{\mathrm{B}}T_{\mathrm{P}}) \approx (1\ \mathrm{TeV})\left(\frac{g_*}{230}\right)^{1/2}\left(\frac{m_{\tilde{G}}}{10\ \mathrm{keV}}\right)^2\left(\frac{1\ \mathrm{TeV}}{m_{gl}}\right)^2$$

g_* is the number of spin degrees of freedom at the temperature $(T_{\mathrm{dec}})_{\tilde{G}}$

m_{gl} is the gluino mass,

$m_{\tilde{G}}$ is the gravitino mass

$$\lambda_{\tilde{G}} \gtrsim (0.3\ \mathrm{cm})(1/x_{\tilde{G}})\ln(1/x_{\tilde{G}})$$

Illustration: take $(g_*/230)^{1/2} \sim 1$ and $(1\ \mathrm{TeV}/m_{gl})^2 \sim 1$.

$$x_{\tilde{G}} \approx \left(\frac{m_{\tilde{G}}}{10^3\ \mathrm{GeV}}\right)^2$$

If, for example, $m_{\tilde{G}} \approx 100$ GeV, then $x_{\tilde{G}} \approx 10^{-2}$ and $\lambda_{\tilde{G}} \gtrsim 140$ cm.

Low energies, perhaps accessible (dark matter decay?)

Inflaton decay

'reheating', perturbative decay of the inflaton

Require:

particles are created at a temperature below their decoupling temperature.

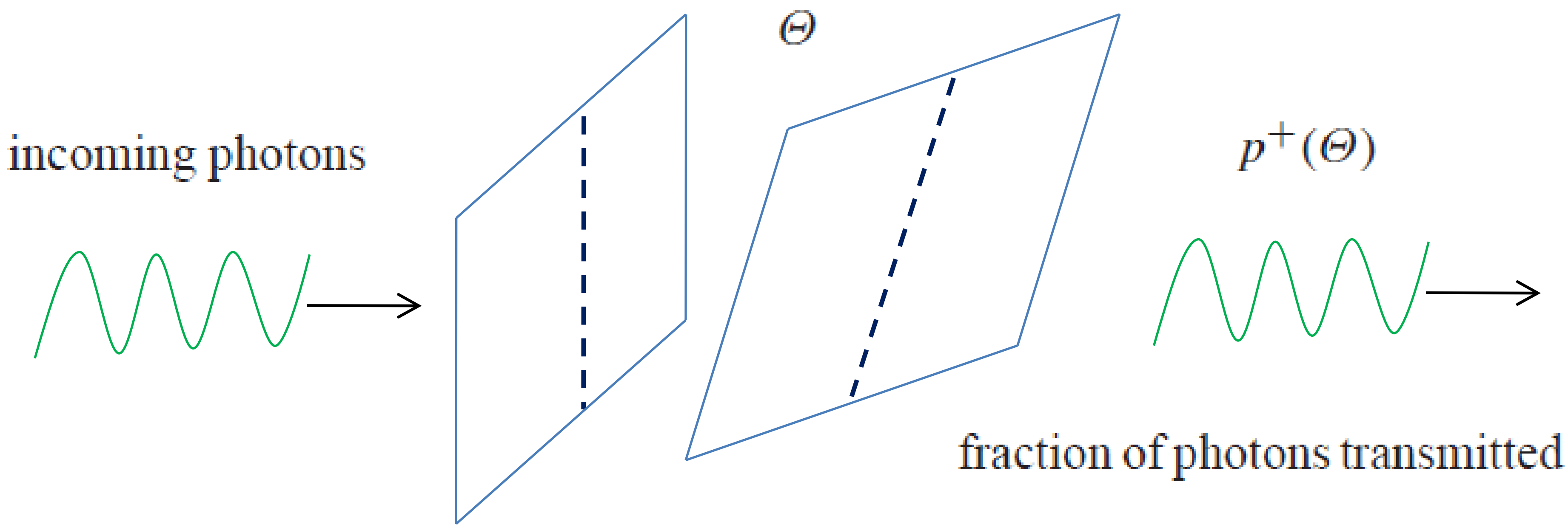
Gravitinos

copiously produced by inflaton decay

significant component of dark matter ?

Decay photons? Violations of Malus' law?

Decay photons? Violations of Malus' law?



$p^+(\Theta)$ deviates from $p^+_{\text{eq}}(\Theta) = \cos^2\Theta$???

(above a certain wavelength)

More precise implications of the freezing inequality

$$\frac{1}{k} > 4a_f \sqrt{\langle \hat{n}_{\mathbf{k}r} \rangle_f + 1/2} \int_{t_i}^{t_f} dt \; \frac{1}{a^2} \sqrt{\langle \hat{n}_{\mathbf{k}r} \rangle + 1/2} \; .$$

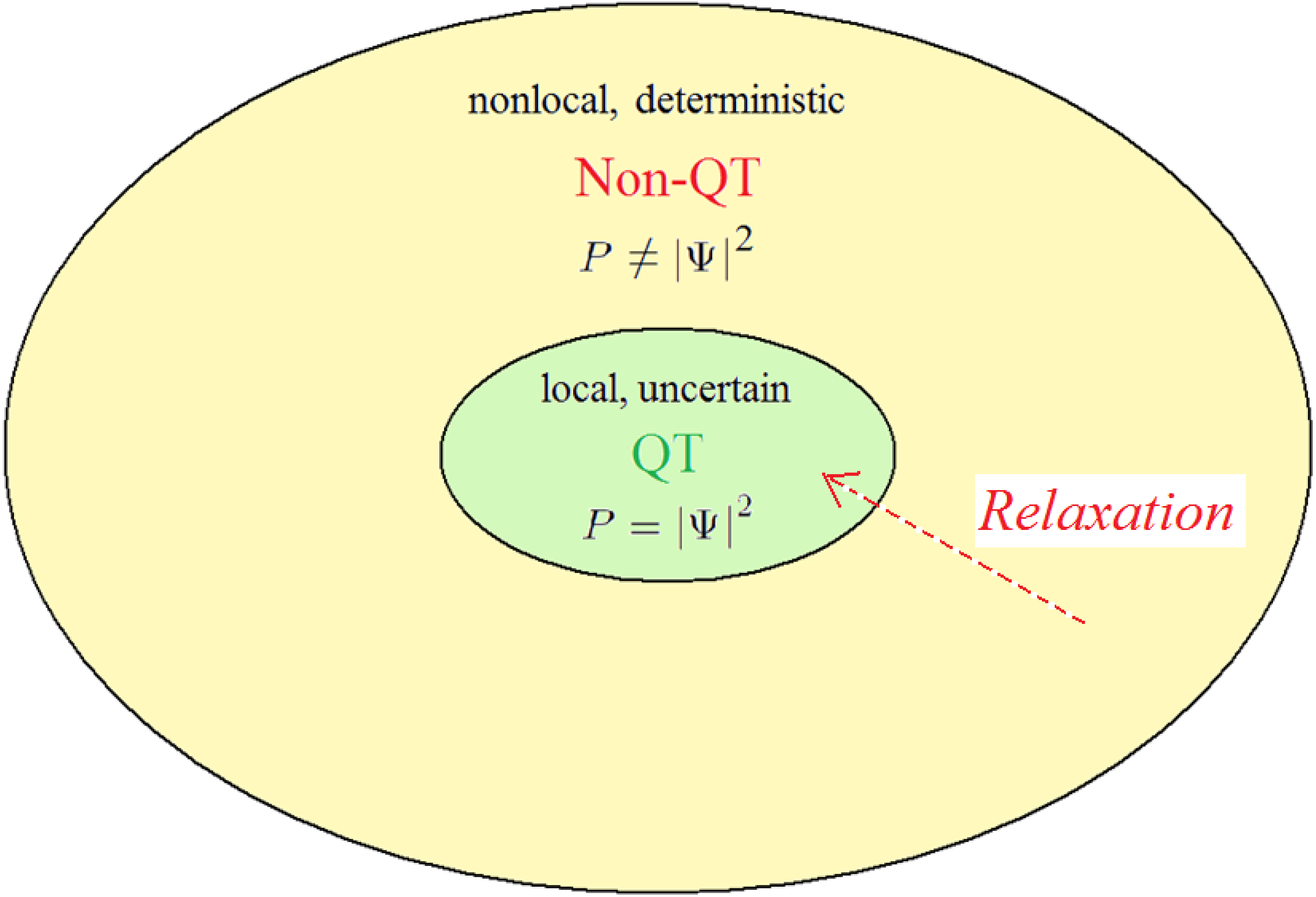
Satisfied? Depends on history of expansion,
and on time evolution of quantum state.

Super-Hubble wavelengths are necessary
but not sufficient.

Calculate time evolution of $\langle \hat{n}_{\mathbf{k}r} \rangle$ (for a given $a(t)$)
and *find out* which sub-ensembles satisfy the inequality.

On arXiv soon ....

Quantum theory is a special case of a much wider physics



Now know roughly where to look. More work to be done!