Introduction to current research in de Broglie-Bohm pilot-wave theory

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Part A: Interpretation of known pilot-wave theory

Different ways of looking at the pilot-wave physics we already have.

Different interpretations at present, possibly with different physical implications (in the long run).

Part B: Frontiers of possible new pilot-wave physics

Don't expect we will all agree on the interpretation of what we have so far.

While continuing to think and argue about these, let's also try to push back the frontiers of new physics and see what we find.

Part A: interpretation of known pilot-wave theory

- De Broglie's dynamics (1927) vs. Bohm's dynamics (1952)
- Classical limit
- Aristotelian kinematics vs. Galilean (or Einsteinian) kinematics
- Origin of the Born rule
- High-energy physics, field theory, fermions
- Lorentz invariance
- Nature of psi, configuration space vs. 3-space

De Broglie’s dynamics (1927) vs. Bohm’s dynamics (1952)

Standard historical accounts are inaccurate

Fifth Solvay conference (1927):

de Broglie presented the pilot-wave theory of a (non-relativistic) many –body system, with a pilot wave in configuration space

Quantum Theory at the Crossroads

Reconsidering the 1927 Solvay Conference

Guido Bacciagaluppi and Antony Valentini

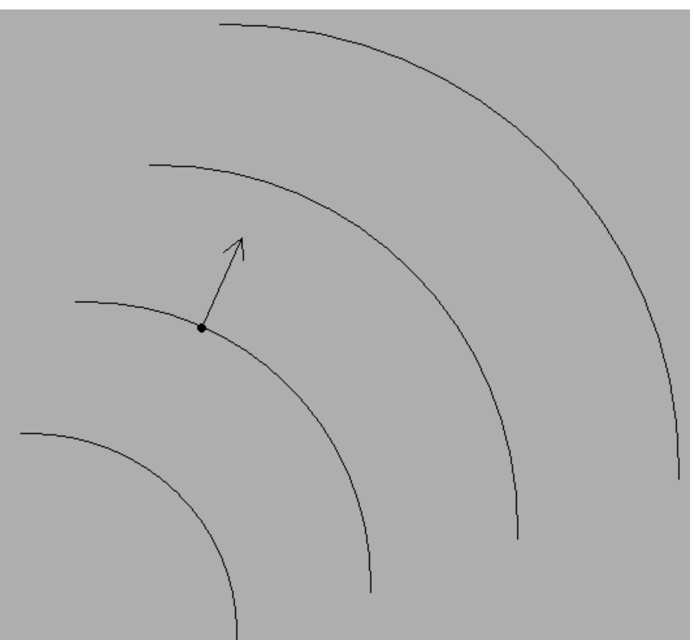
De Broglie's Pilot-Wave Dynamics (1927)

$$m_i \frac{d\mathbf{x}_i}{dt} = \nabla_i S$$

(cf. Bell 1987)

$$i\frac{\partial \Psi}{\partial t} = \sum_{i=1}^{N} -\frac{1}{2m_i}\nabla_i^2 \Psi + V\Psi$$

de Broglie called it "pilot-wave theory"



Get QM if assume initial $P = |\Psi|^2$

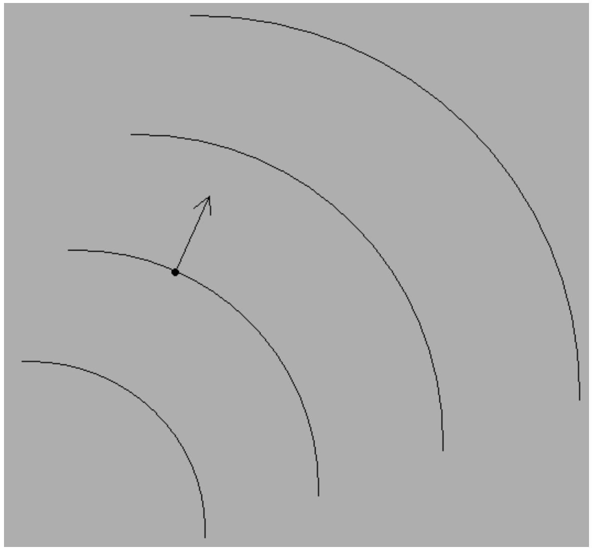
(shown fully by Bohm in 1952;
apply dynamics to apparatus)

De Broglie's new theory of motion

* New, non-Newtonian dynamics.
Particle *velocities* are determined by the law of motion

$$m_i \frac{d\mathbf{x}_i}{dt} = \nabla_i S$$

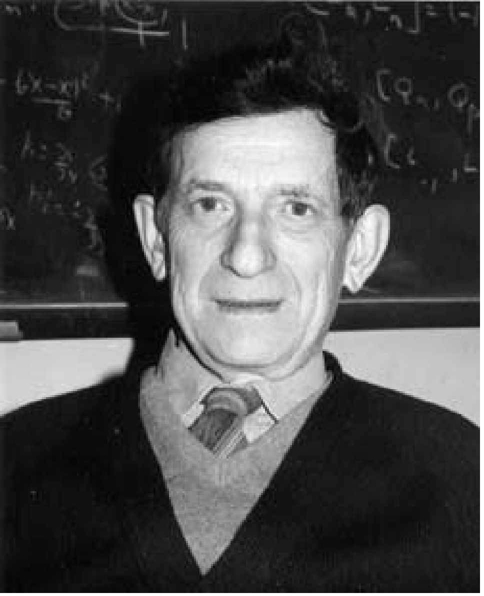
where *S* is the phase of a wave Ψ



Abandon classical dynamics
(diffraction in free space, 1923)

* Unifies principles of Maupertuis and Fermat (1924):

(one body) $\delta \int_a^b m\mathbf{v} \cdot d\mathbf{x} = 0 \quad \longleftrightarrow \quad \delta \int_a^b dS = 0$



Bohm's Newtonian version (1952)

$$m_i \frac{d^2 \mathbf{x}_i}{dt^2} = -\nabla_i (V + Q)$$ (law of motion)

$$Q \equiv -\sum_{i=1}^{N} \frac{1}{2m_i} \frac{\nabla_i^2 |\Psi|}{|\Psi|}$$

Get QM if assume initial $P = |\Psi|^2$ *and* $\mathbf{p}_i = \nabla_i S$

For Bohm, $\mathbf{p}_i = \nabla_i S$ is an initial condition; can drop it.

For de Broglie, $\mathbf{p}_i = \nabla_i S$ is the law of motion.

De Broglie's dynamics and Bohm's dynamics are different.

"Bohmian mechanics" is a misnomer for de Broglie's dynamics.

De Broglie's dynamics (1927):

First-order in time, 'Aristotelian' form of dynamics:

$\nabla_i S$ 'causes' velocity (Bell, DGZ, AV, Struyve, etc)

QT is a special case of a wider physics. with distributions in configuration space $\neq |\Psi|^2$ (tend to relax, e.g. AV and Westman 2005)

Bohm's dynamics (1952):

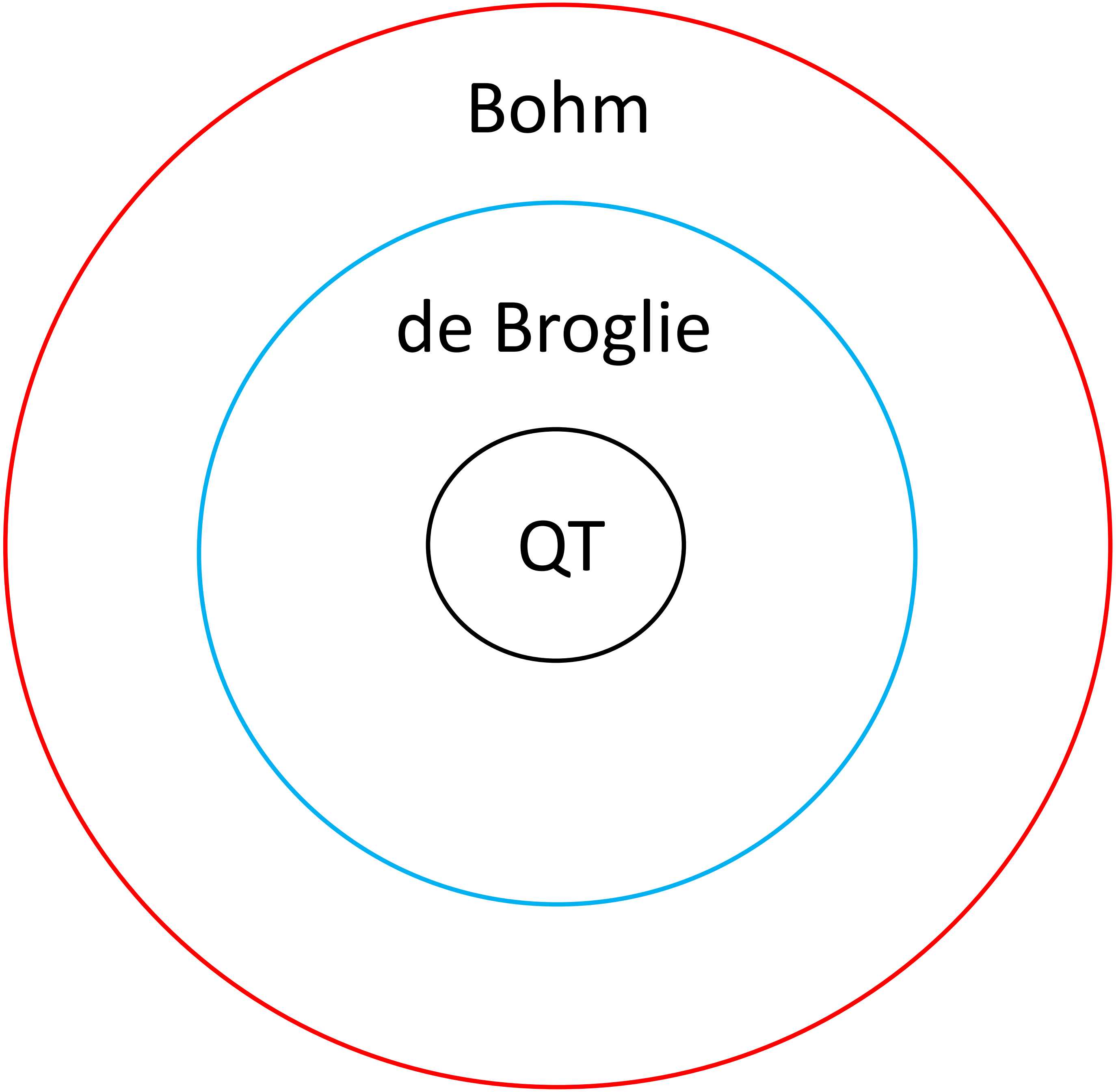
Second-order in time, 'Newtonian' form of dynamics:

$-\nabla_i(V+Q)$ 'causes' acceleration (Bohm, Holland, Hiley, etc)

QT is a special case of an even wider physics, with distributions in phase space $\neq |\psi(x,t)|^2 \delta(p - \nabla S(x,t))$

(tend *not* to relax, Colin, Struyve and Valentini 2010)

Different conceptually, *and* they allow different physics


Bohm
de Broglie
QT

Classical limit

Needs further study, beyond simple ‘textbook’ approaches (Ehrenfest theorem, WKB states, etc)

Appleby (1999) initiated study of realistic models (with environmental decoherence) in pilot-wave theory, but little done since.

Work done so far suggests can recover classical physics under reasonable conditions
(a project that is incomplete also in standard QT, e.g. for chaotic systems)

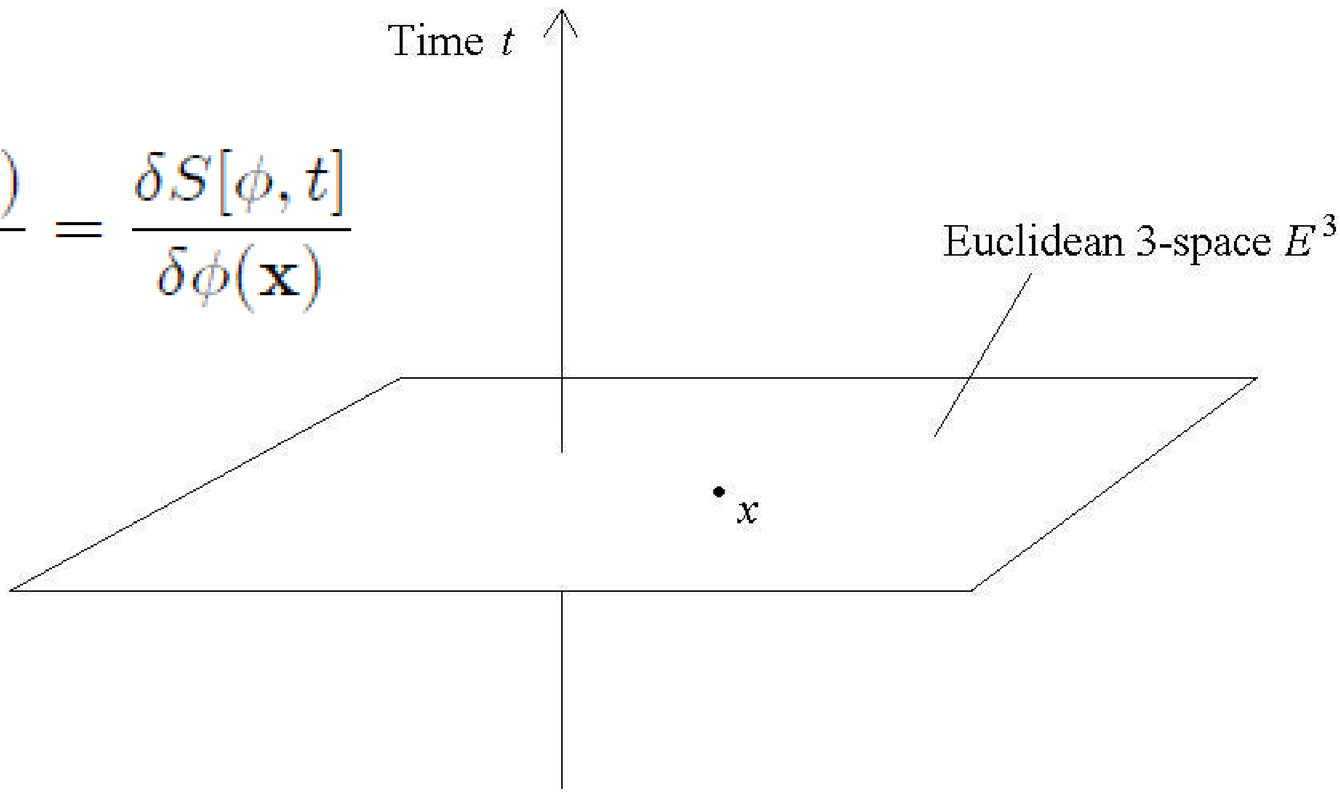
(See Rosaler talk)

Aristotelian kinematics vs. Galilean (or Einsteinian) kinematics

The high-energy theory we have contains a preferred rest frame at the fundamental level.

First discussed explicitly by Bohm and Hiley (1984) for scalar field theory. Adopted by many subsequent authors.

$$\frac{\partial \phi(\mathbf{x},t)}{\partial t} = \frac{\delta S[\phi,t]}{\delta \phi(\mathbf{x})}$$



It has been argued that hidden-variables theories generally are incompatible with fundamental Lorentz invariance (Hardy 1992, Myrvold 2002).

Seems that both the dynamics and the quantum-equilibrium distribution must be defined in a preferred rest frame (Berndl et al. 1996).

Even so, some workers continue to seek a fundamentally Lorentz-invariant theory (see Nikolic talk, and Struyve talk).

A related disagreement even in the non-relativistic theory:

$$m_i \frac{d\mathbf{x}_i}{dt} = \nabla_i S \qquad\qquad i\frac{\partial \Psi}{\partial t} = \sum_{i=1}^{N} -\frac{1}{2m_i}\nabla_i^2 \Psi + V\Psi$$

Are these equations Galilean invariant? ($\mathbf{x}_i' = \mathbf{x}_i - \mathbf{v}t$, $t' = t$)

Formally yes:

$$\Psi' = \Psi \exp i\left(\frac{1}{2}\sum_i m_i v^2 t - \sum_i m_i \mathbf{v}\cdot\mathbf{x}_i\right) \qquad \frac{d\mathbf{x}_i'}{dt} = \frac{\nabla_i' S'}{m_i} = \frac{d\mathbf{x}_i}{dt} - \mathbf{v}$$

Most regard this as a physical symmetry (e.g. Duerr et al. 1992)

AV (1997) argues that this is a fictitious symmetry involving a fictitious Aristotelian force (analogous to invariance under uniform acceleration in Newtonian mechanics).

(If the latter is true, then the natural kinematics is Aristotelian even in the non-relativistic theory.)

Origin of the Born rule

Two distinct approaches:

-- dynamical relaxation to equilibrium
(Bohm 1953, Valentini 1991)

-- 'typicality' wrt a preferred equilibrium measure
(Duerr, Goldstein and Zanghi' 1992)

General comment:

The foundations of statistical mechanics are as slippery and controversial as the foundations of quantum theory.

About 150 years of controversy in the classical case, still continues (e.g. ask Jos Uffink)

Therefore, unlikely to find a clean resolution here either.

The ‘typicality’ approach:

-- assumes a $|\Psi|^2$ measure for the whole universe, from which derive the Born rule for sub-systems (DGZ 1992)

-- this is not circular, because the universal $|\Psi|^2$ is a measure of ‘typicality’, not of probability

Comments (AV 1996, 2001):

-- a different universal typicality measure would yield non-Born rule probabilities for subsystems

-- the choice of initial measure does ‘all the work’

-- does seem circular after all

-- no real difference between ‘typicality’ and ‘probability’?

The ‘dynamical relaxation’ approach:

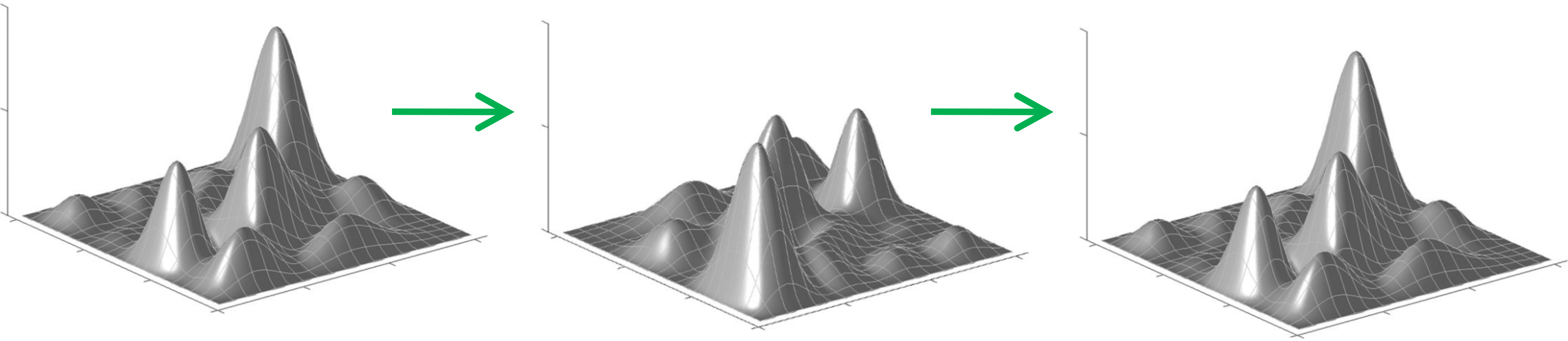
-- first considered by Bohm (1953) for a two-level atom

-- general coarse-graining *H*-theorem (Valentini 1991)

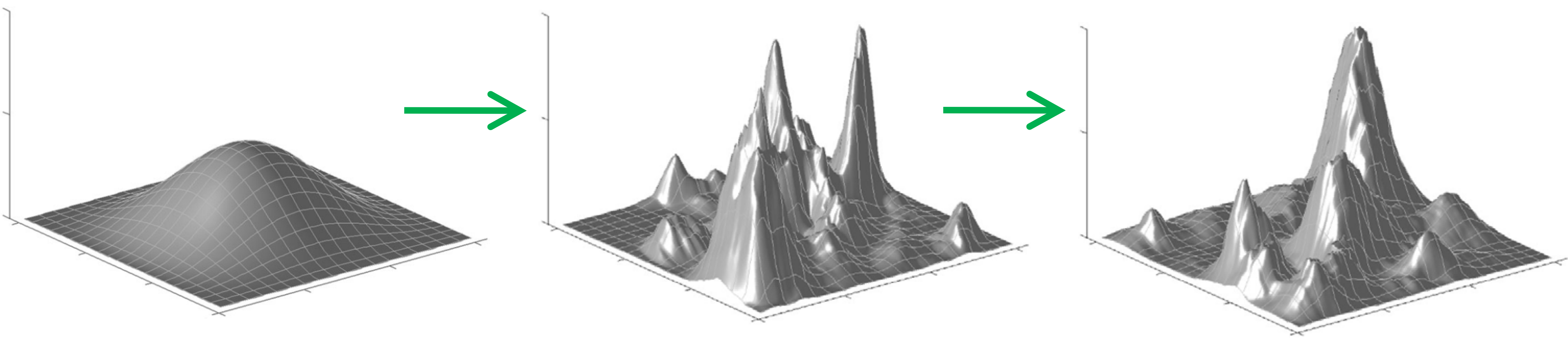
$$H = \int dq \; \rho \ln(\rho / |\psi|^2)$$

-- numerical simulations show efficient relaxation, with exponential decay of coarse-grained *H*-function

Equilibrium changes with time



Non-equilibrium relaxes to equilibrium



(Valentini and Westman 2005)

Comments on dynamical relaxation approach:

-- no time-reversal invariant theory can generate relaxation for *all* initial conditions

-- need some assumption about initial state (e.g. no fine-grained structure)

--- in the end, an empirical question? (AV 2001)

Current work on dynamical relaxation:

Russell, Towler and AV, further simulations, relaxation rates (see Towler talk)

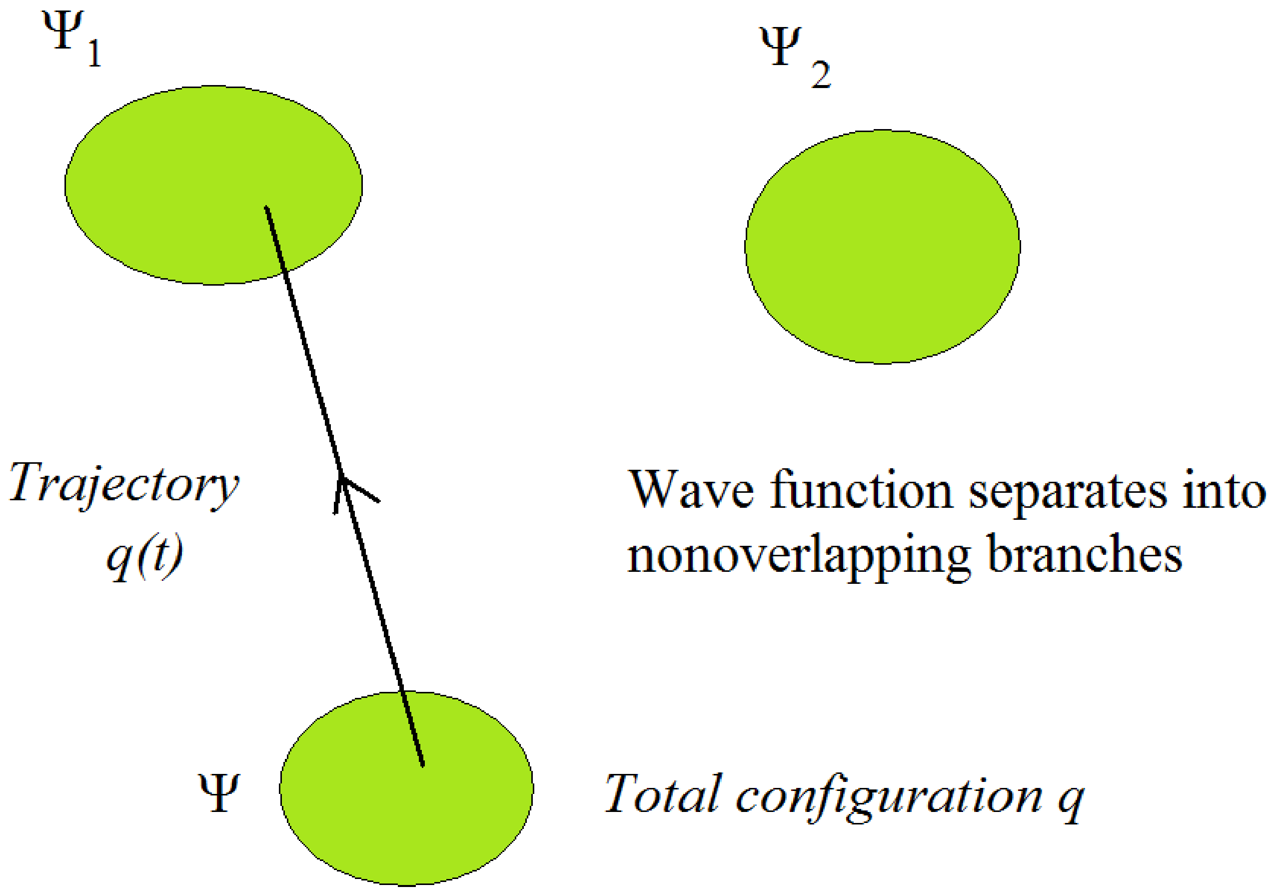
Cf. Chaos, role of nodes (Efthimiopoulos talk)

Bennett: better understanding of what is going on during relaxation, from a fluid-dynamics perspective

High-energy physics, field theory, fermions

We have the essentials of a pilot-wave theory of high-energy physics (with an underlying preferred time). But needs further study.

Quantum measurements in field theory (Struyve, Schmelzer). Separation of packets in field-configuration space.



Fermions:

Simplest model seems to be the 'Dirac sea model' of Bohm-Hiley-Kaloyerou (1987, 1993), with a many-body Dirac equation and guidance equations. Many-body generalisations of:

$$i\frac{\partial\psi}{\partial t} = -i\alpha\cdot\nabla\psi + m\beta\psi \qquad \frac{d\mathbf{x}}{dt} = \frac{\psi^{\dagger}\alpha\psi}{\psi^{\dagger}\psi}$$

Derived by Colin (2003) as the continuum limit of Bell's stochastic model. (Limit is deterministic.)

Relation to quantum field theory clarified by Colin and Struyve (2007).

--- Grassmann *field* model (AV 1992) seems to be a formal construction only (Struyve 2010)

--- New *field* theory for fermions by Schmelzer (talk)

Alternative continuum limit of Bell model (Duerr et al. 2004):

Alternative approach to continuum limit of Bell's model. Result not deterministic (during pair creation).

(Fermion number not conserved, unlike for Colin.)

Remarkable if pair creation forces indeterminism upon us

(AV's view: Duerr et al. took the wrong definition of fermion number in their interpretation of Bell's model.)

Duerr et al.:

fermion number = no. of particles plus no. of anti-particles.

Standard QFT:

fermion no. = no. of particles minus no. of anti-particles = F

Colin:

fermion no. = no. of positive-energy particles plus no. of negative-energy particles

Lorentz invariance

Example:

$$i\frac{\partial\Psi}{\partial t} = \int d^3\mathbf{x}\,\frac{1}{2}\left(-\frac{\delta^2}{\delta\phi^2} + (\nabla\phi)^2 + m^2\phi^2\right)\Psi$$

$$i\frac{\partial\Psi'}{\partial t'} = \int d^3\mathbf{x}'\,\frac{1}{2}\left(-\frac{\delta^2}{\delta\phi'^2} + (\nabla'\phi')^2 + m^2\phi'^2\right)\Psi'$$

$$\Psi'[\phi(\mathbf{x}),t] = \hat{U}\Psi[\phi(\mathbf{x}),t]$$

Schroedinger equation is Lorentz covariant, but not manifestly so (for the Hamiltonians we encounter in nature).

Lorentz symmetry *broken* by the guidance equation (e.g. vacuum)

$$\frac{\partial\phi(\mathbf{x},t)}{\partial t} = \frac{\delta S[\phi,t]}{\delta\phi(\mathbf{x})}$$

In equilibrium, cannot see trajectories, drop guidance equation.
Equilibrium statistics 'inherit' the Lorentz group from the mathematical structure of the Schroedinger equation.

Is this satisfactory?

Nature of psi, configuration space vs. 3-space, etc

Ontological (Bell, Bohm, Holland, AV, et al.)

vs.

law-like (Duerr et al.)

Or: something in between? (Hardy talk)

If ontological:

seem to have an extraordinary new kind of 'thing' grounded in configuration space

If law-like:

hard to understand the complex contingency in psi

3-space only?

Some workers try to reduce psi down to 3-space fields only (Norsen talk), or to interpret psi in 3-space terms (Riggs (V))

Real fields *R* and *S*?

Finally, some think of psi in terms of two real fields *R* and *S*, with

psi = *R* exp (*iS*) (e.g. Bohm and Hiley 1993)

This is problematic:

the two equations

$$\frac{\partial S}{\partial t} + \frac{(\nabla S)^2}{2m} + V + Q = 0 \qquad \frac{\partial R^2}{\partial t} + \nabla \cdot \left(R^2 \frac{\nabla S}{m} \right) = 0$$

are *not* equivalent to the Schroedinger equation.

For psi single-valued and continuous, require

$$\oint d\mathbf{x} \cdot \nabla S = 2\pi n$$

(cf. Derakhshani talk)

Part B: frontiers of possible new pilot-wave physics

- Beyond conventional quantum theory? (in equilibrium)
- New physics of nonequilibrium, in de Broglie's dynamics and in Bohm's dynamics
- Quantum gravity, problem of time
- Relation to more general hidden-variables theories
- Numerical uses of pilot-wave theory
- Analogue models of pilot-wave theory
- Derivation from a deeper theory?
- The later Bohm

Beyond conventional quantum theory?
(in equilibrium)

Are there situations where, even in equilibrium, pilot-wave theory gives different answers and/or is able to do something quantum theory cannot?

- Arrival times, tunnelling times, etc (Yearsley)
- Cosmological perturbations (Peter)
- Riggs

Even if not true, thinking about these exotic possibilities will probably teach us something about pilot-wave theory (at least)

New physics of nonequilibrium, in de Broglie's dynamics and in Bohm's dynamics

De Broglie's dynamics in non-equilibrium, new physics developed extensively (AV 1991 ff):

superluminal signalling, sub-quantum measurements, distinguishing non-orthogonal states, breaking quantum cryptography, etc

Theoretically conceivable with relic cosmological particles (AV talk).

Potentially observable in cosmic microwave background (AV 2008, 2010)

Bohm's dynamics in non-equilibrium, new physics not developed at all:

non-standard momenta (different from grad *S*) would imply numerous new phenomena

However, no relaxation, unstable, suggests untenable (AV talk).

Quantum gravity, problem of time

Wheeler-DeWitt equation is 'timeless' ($\Psi = \Psi[g_{ij}]$)

$$\left(-G_{ijkl}\frac{\delta^2}{\delta g_{ij}\delta g_{kl}} - g^{1/2}R\right)\Psi = 0$$

From classical expression for canonical momenta, can write down a 'de Broglie guidance equation'

$$\frac{\partial g_{ij}}{\partial t} = 2NG_{ijkl}\frac{\delta S}{\delta g_{kl}} + N_{i|j} + N_{j|i}$$

Is this theory consistent?

Yes, according to Pinto-Neto et al. (see talk):

consistency of the dynamics requires that each solution psi of the W-D equation determines a preferred foliation of spacetime.

If true:

pilot-wave theory would solve the problem of time

Relation to more general hidden-variables theories

What features of pilot-wave theory are necessarily features of any 'reasonable' hidden-variables theory?

We have already learned that:

--- non-locality is a general feature (Bell 1964)
(assuming no backwards causation)

--- contextuality is a general feature
(Bell-Kochen-Specker, 1966, 1967)

-- non-equilibrium nonlocal signalling is a general feature
(AV 1991, 2002)

Important open question:

Is the ontological wave function a general feature?

Can one construct a hidden-variables theory without it?

Question has been studied by Montina (2007):

For a general class of hidden-variables theories, the number of continuous degrees of freedom must be at least as many as those contained in psi.

Suggests:

psi must be ontological in any hidden-variables theory?

However:

-- assumes continuity (and?)
-- assumes Markovian (no 'memory')

Can have a smaller number of variables for non-Markovian theories (Montina). Are such theories 'reasonable'?

Question seems still open

Related subjects:

--- More general frameworks (Hardy talk)?

--- Nelson's stochastic mechanics:

Claims to derive quantum theory from a stochastic theory of particle motion, with psi = R exp (iS) a derived (and non-ontological) quantity.

If correct, would provide a hidden-variables theory with no ontological psi.

However, suffers from phase problem (Wallstrom 1995), need to impose $\oint d\mathbf{x} \cdot \nabla S = 2\pi n$

(See Derakhshani talk)

Numerical uses of pilot-wave theory

Can the pilot-wave dynamics of particle trajectories provide a numerically more efficient means of calculating in some situations?

See Oriols talk

Analogue models of pilot-wave theory

Cf. analogue models of gravity: now a booming subject

See Batelaan, Bush talks

Derivation from a deeper theory?

Hiley, theory based on Clifford algebras.

--- what is the ontology?

--- technical issue: derives equations for *R* and *S*, these are not equivalent to the Schroedinger equation (the phase problem, again)

The later Bohm

de Broglie: a realist and a scientist, concerned with atomic physics, arguing from experimental puzzles to objective theories to explain them

Three Bohm's:

-- the 'early' Bohm fascinated by complementarity and Bohr

-- the Bohm of 1952 and 1950s, strong 'materialism'

-- the 'later' Bohm of the 1960s onwards, influenced by ancient Indian thought, the guru Krishnamurti, etc.

Important to distinguish between these three 'Bohm's'.

Only the Bohm of 1952 is closely related to the de Broglie of the 1920s.

"The substratum is that in which the properties -- latent, active or unmanifest – inhere". (*Yoga—Sūtras*, Patañjali)

Commentary by Taimni (1961): This means that

"... all natural phenomena are due to the continual appearance and disappearance of all kinds of properties in a substratum which is their repository".

The later Bohm's suggestion that

"... the things that appear to our senses are derivative forms and their true meaning can be seen only when we consider the plenum, in which they are generated and sustained, and into which they must ultimately vanish"

(Bohm 1980, *Wholeness and the Implicate Order*, p. 192)

is an elaboration of this old idea in a more modern context.

(Cf. Pylkkanen talk)