# Pilot-wave theory and quantum fields

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## Non-relativistic de Broglie-Bohm theory

- De Broglie (1927), Bohm (1952)
- Particles moving under influence of the wave function.
- Dynamics:
  - Wave: $\psi(\mathbf{x}_1, \dots, \mathbf{x}_n, t)$ satisfies Schrödinger equation

$$\mathrm{i}\hbar\partial_t\psi = \left( - \sum_{k=1}^{n} \frac{\hbar^2}{2m_k} \nabla_k^2 + V \right) \psi$$

  - Particles: $\mathbf{x}_1(t), \dots, \mathbf{x}_n(t)$ solutions to guidance equations

$$\frac{d\mathbf{x}_k}{dt} = \mathbf{v}_k^{\psi}(\mathbf{x}_1, \dots, \mathbf{x}_n, t) = \frac{1}{m_k} \boldsymbol{\nabla}_k S(\mathbf{x}_1, \dots, \mathbf{x}_n, t), \qquad \psi = |\psi| e^{\mathrm{i}S/\hbar}$$

- Quantum equilibrium:

  for an ensemble of systems with wave function $\psi$, the distribution $\rho(x)$ of particle positions is given by $|\psi(x)|^2$.

- Quantum equilibrium is preserved by the particle motion because it satisfies the continuity equation:

$$\partial_t |\psi|^2 + \sum_{k=1}^{n} \boldsymbol{\nabla}_k \cdot (\mathbf{v}_k^{\psi} |\psi|^2) = 0$$

$\rightarrow$ For other Schrödinger equations, the continuity equation of $|\psi|^2$ may be used to find a suitable guidance law.

That is

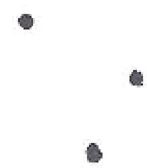
$$\partial_t |\psi|^2 + \mathrm{div} j^{\psi} = 0$$

suggest the guidance law

$$\dot{X} = \frac{j^{\psi}}{|\psi|^2}$$

- **Treatment of spin:**
  - No need to introduce extra beables representing spin.

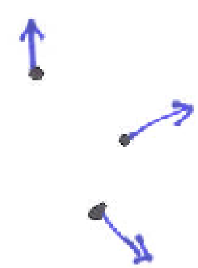
    Just positions:



  - One could introduce extra beables. E.g. the spin vector:

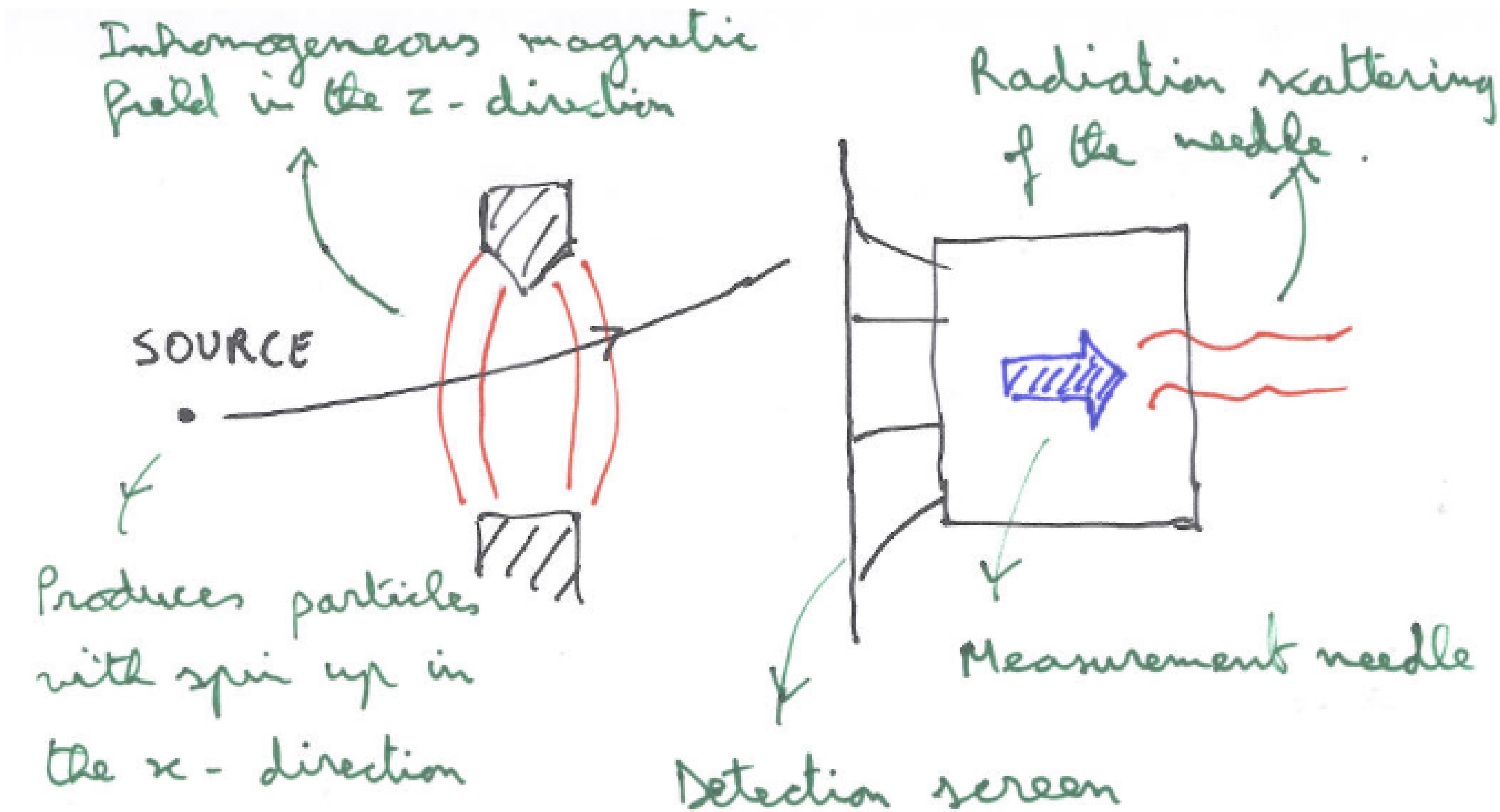
$$\mathbf{s}(t) = \frac{\psi^\dagger(\mathbf{x},t)\boldsymbol{\sigma}\psi(\mathbf{x},t)}{\psi^\dagger(\mathbf{x},t)\psi(\mathbf{x},t)}\Bigg|_{\mathbf{x}=\mathbf{x}(t)}$$

    which is constructed from the wavefunction and the actual position of the particle.

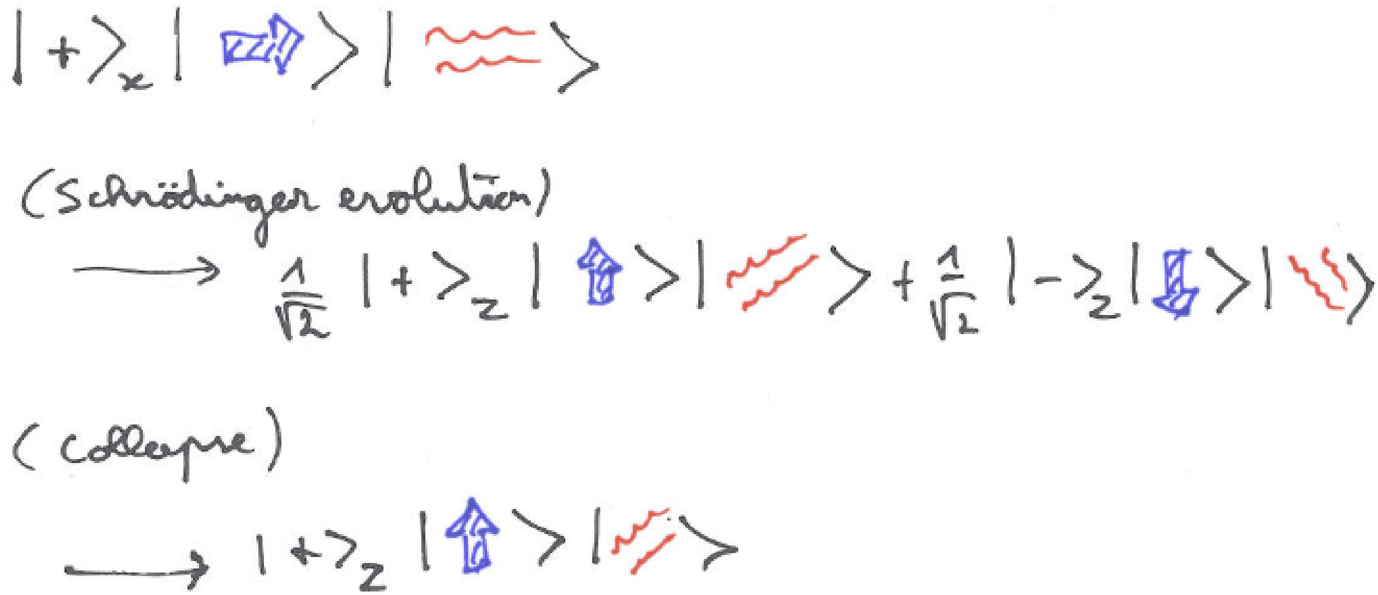


- **Stern-Gerlach experiment**

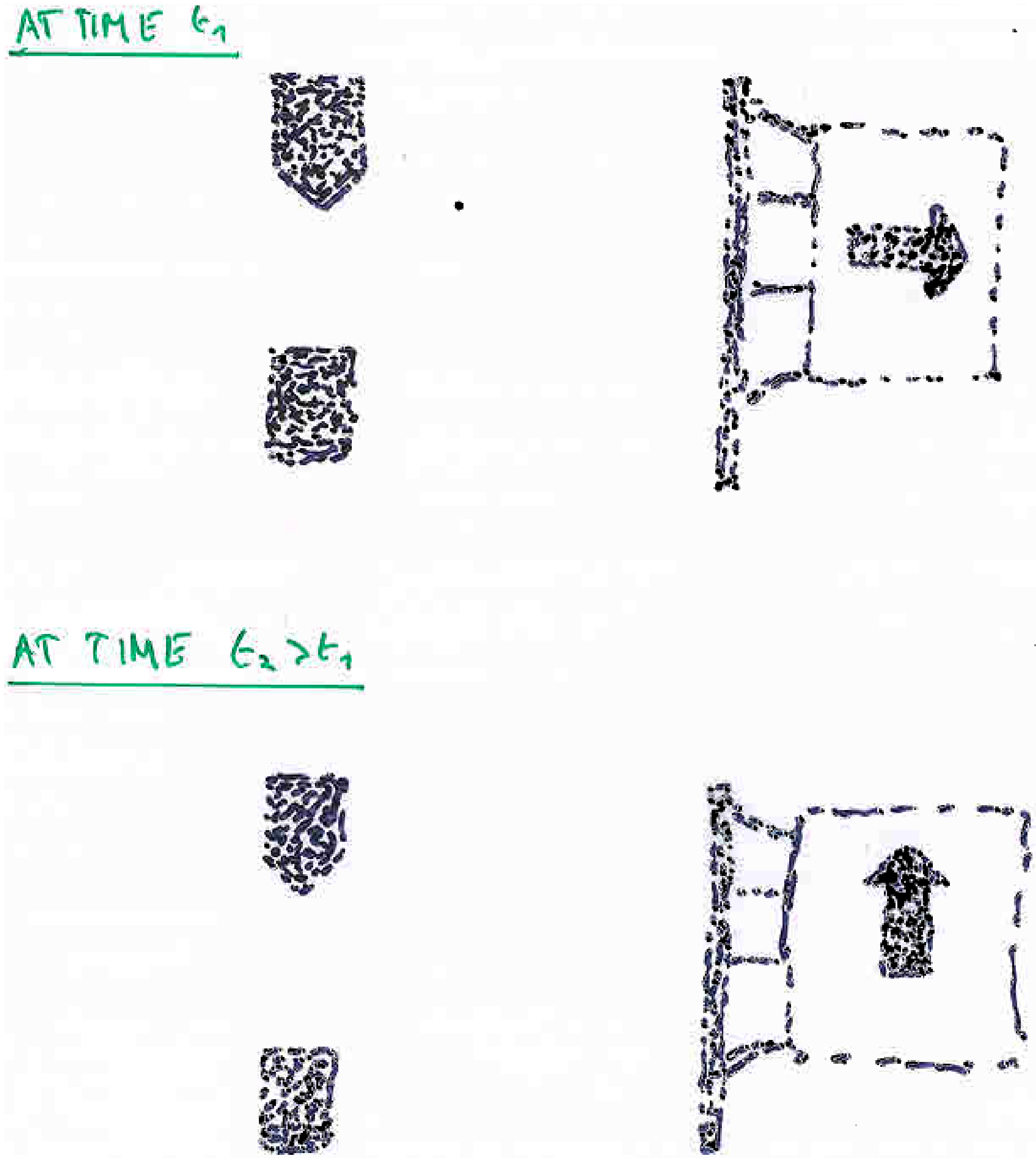
  Sketch of the setup:



Quantum mechanical description:



De Broglie-Bohm picture:



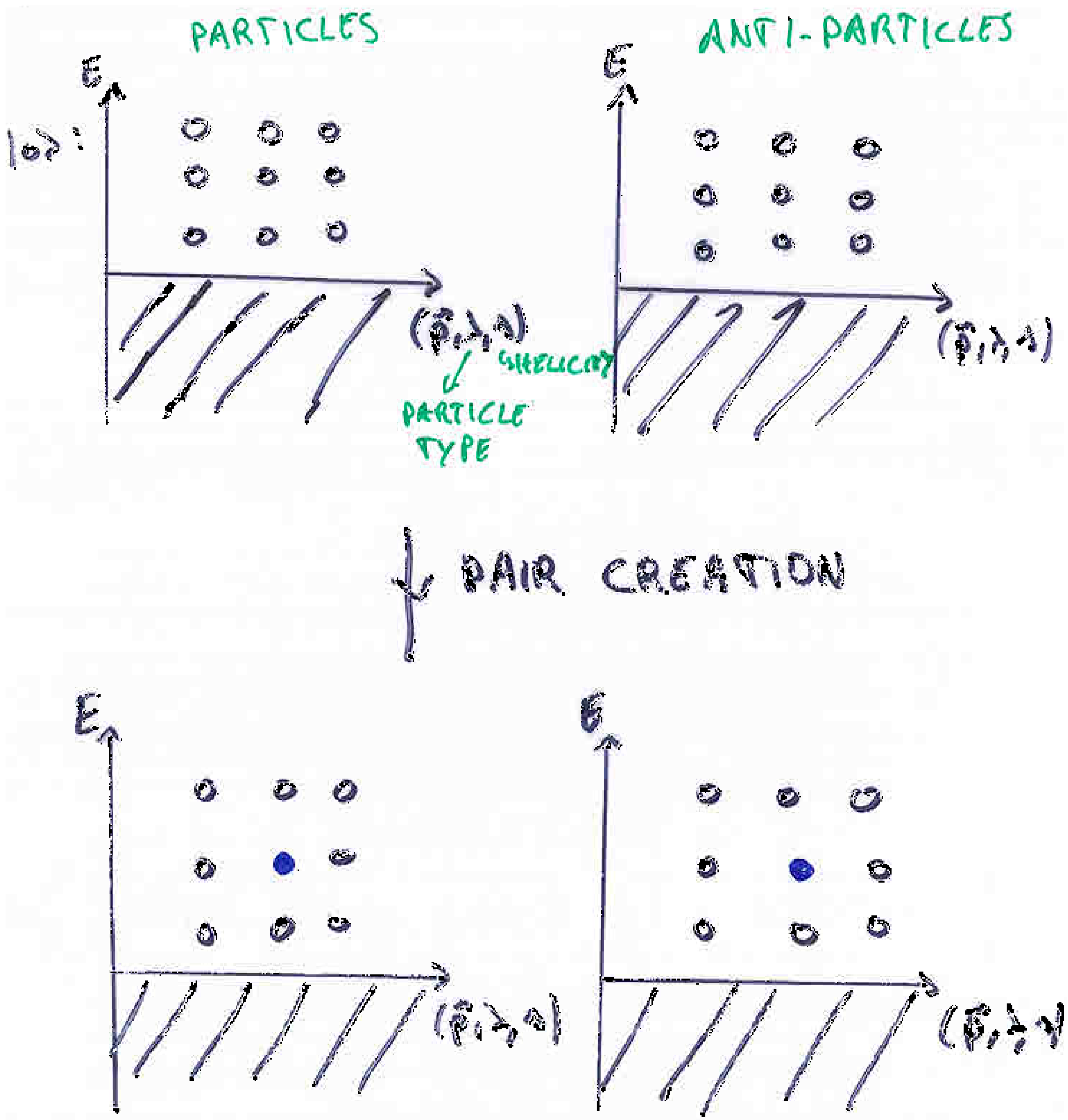
## Quantum field theory

- Need suitable local beables. Particles, fields?
- Need wave function and Schrödinger evolution.
- We assume suitable regulators which make equations well-defined.
- We assume preferred reference frame. Quantum equilibrium holds in this frame.

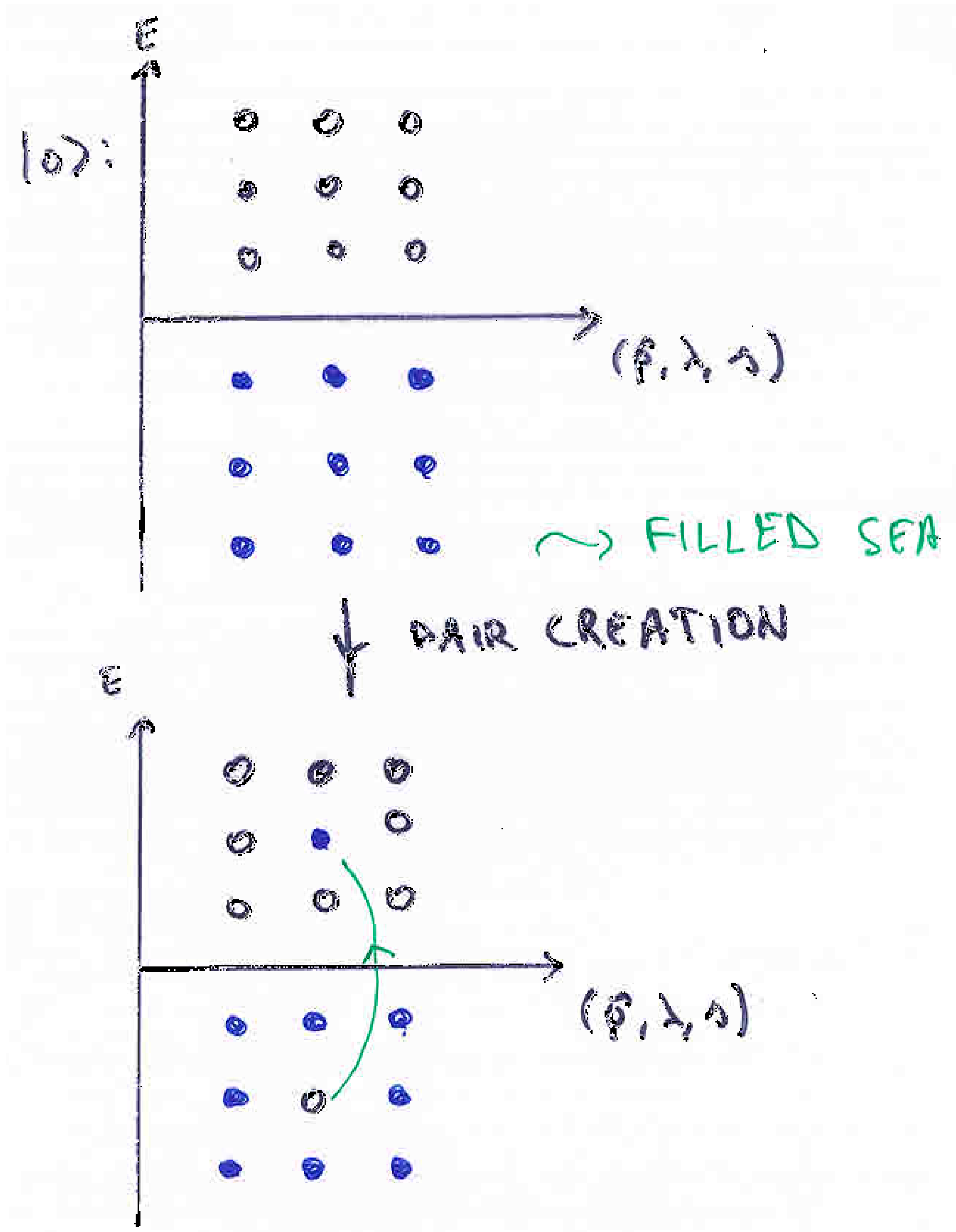
## Particle ontology

- Bell (1984)
  - Considers spatial lattice
  - Local beables: fermion numbers at the lattice points
  - Stochastic
- Continuum generalizations of Bell's model:
  1. Dürr, Goldstein, Tumulka, Zanghì (2002-2005)
     → Local beables: Particle positions
     → Explicit particle creation and annihilation
     → Stochastic model
  2. Colin (2003,2004), Colin & Struyve (2007)
     → Local beables: Particle positions
     → Dirac sea picture (number of particles does not change)
     → Deterministic

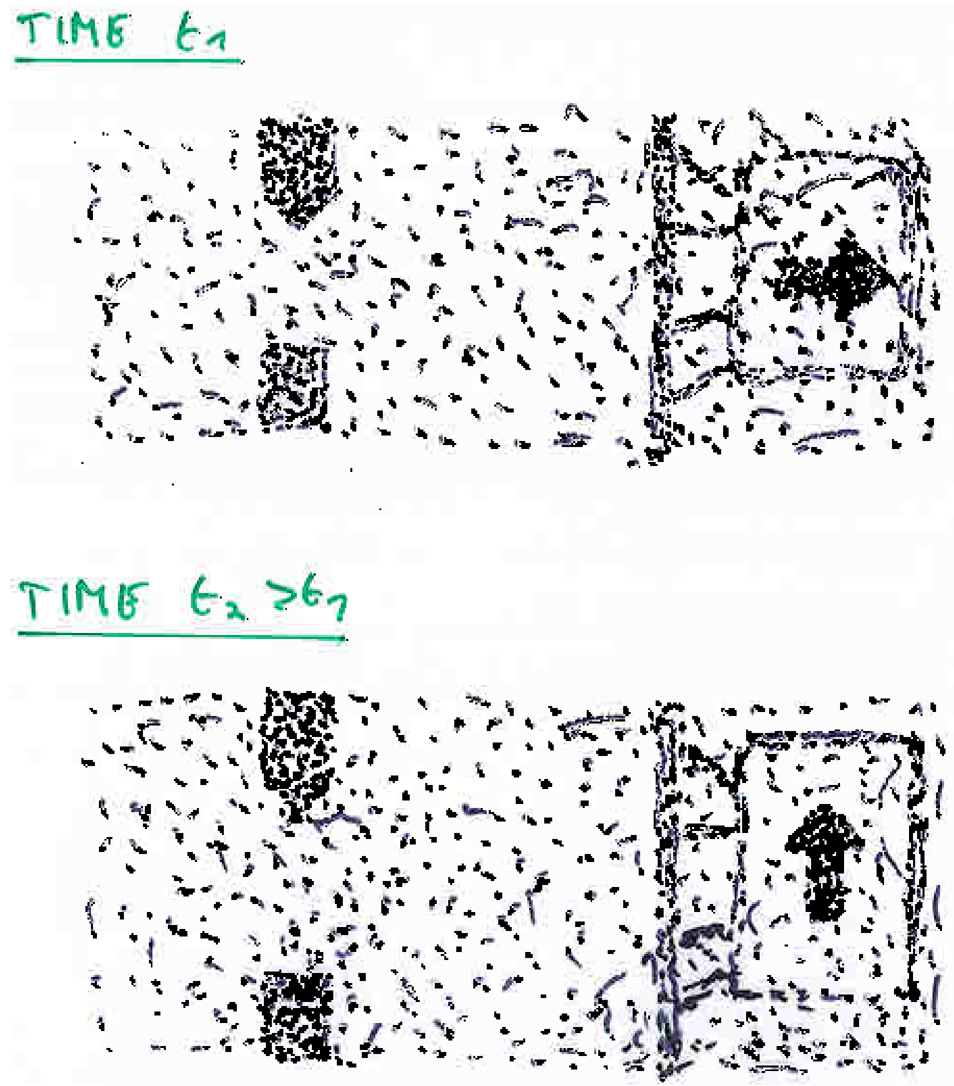
Pair creation in the standard picture:



Pair creation in the Dirac sea picture:



Stern-Gerlach experiment in the Dirac sea - pilot-wave picture:



## Field ontology (see Struyve (2007) for a review)

### Bosonic fields

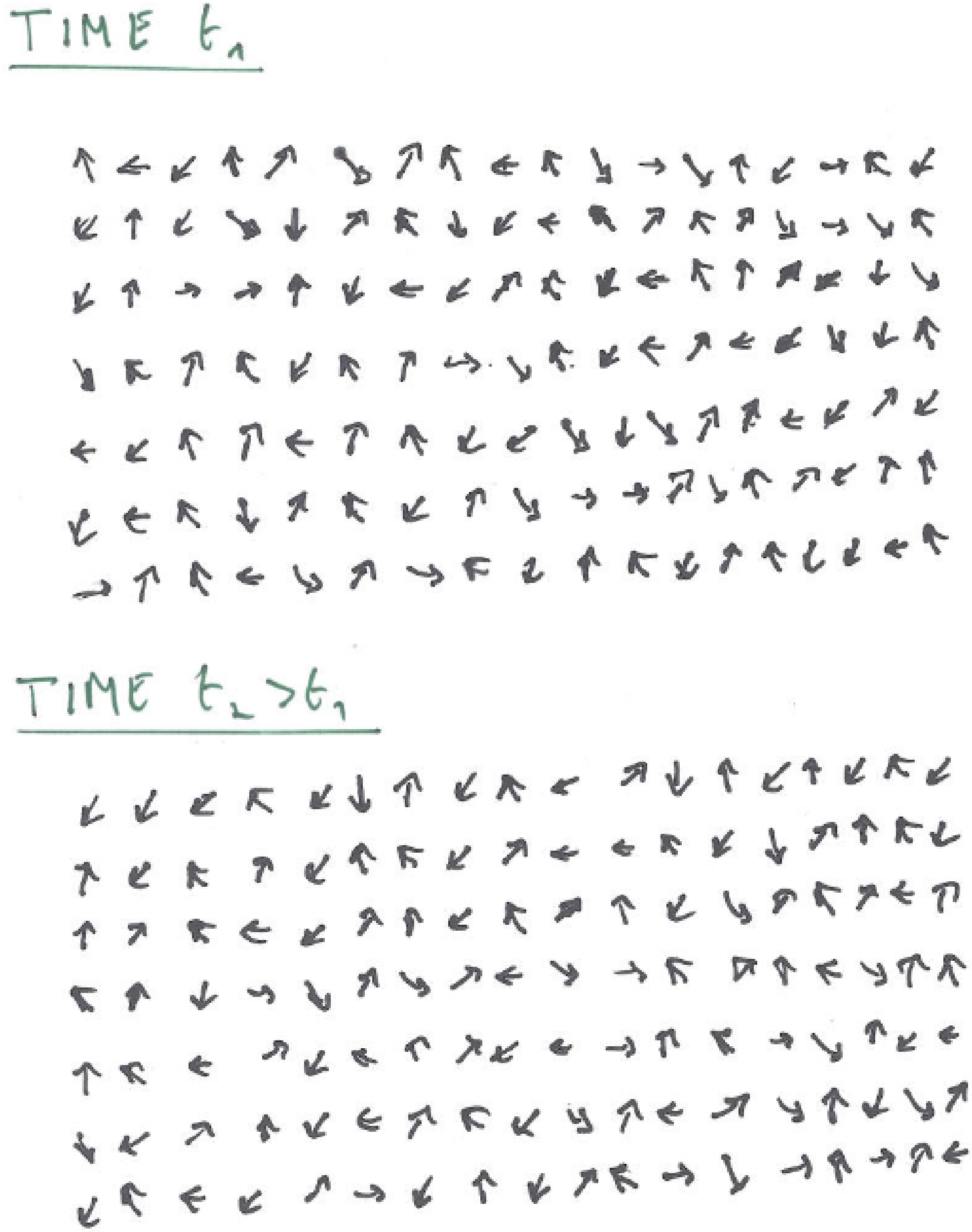
- Free electromagnetic field
  - Bohm (1952)
  - Wave function $|\Psi\rangle$ in the field representation: $\Psi(A_i^T) = \langle A_i^T|\Psi\rangle$, $A_i^T(\mathbf{x})$ is tranverse part of the vector potential
  - Functional Schrödinger equation:
$$\mathrm{i}\frac{\partial \Psi}{\partial t} = \frac{1}{2}\int d^3x \left(-\frac{\delta^2}{\delta A_i^T \delta A_i^T} - A_i^T \nabla^2 A_i^T\right)\Psi \,.$$
  - Local beable is field in space: $A_i^T(\mathbf{x})$
  - Guidance equation
$$\dot{A}_i^T(\mathbf{x}) = \frac{\delta S}{\delta A_i^T(\mathbf{x})}, \qquad \Psi = |\Psi| e^{\mathrm{i}S}$$
  - Equilibrium distribution $|\Psi(A_i^T)|^2$
- Similarly for other bosonic fields, e.g. scalar field $(\phi(\mathbf{x}), \Psi(\phi))$

  (Bohm, Dewdney, Hiley, Holland, Kaloyerou, Nikolić, Struyve, Valentini, ...)

# Fermionic fields

- Holland (1988,1993)
  - Wave function $\Psi(\boldsymbol{\alpha})$,
    $\boldsymbol{\alpha}(\mathbf{k}) = (\alpha(\mathbf{k}), \beta(\mathbf{k}), \gamma(\mathbf{k}))$ set of Euler angles for each point in momentum space
  - Local beable: Isn't specified by Holland, but could be constructed from beables $\boldsymbol{\alpha}(\mathbf{k})$ and the wave function.
  - Hard to see whether it is empirically adequate
- Variant of Holland's approach, Struyve (2007)
  - Wave function $\Psi(\boldsymbol{\alpha})$,
    $\boldsymbol{\alpha}(\mathbf{x}) = (\alpha(\mathbf{x}), \beta(\mathbf{x}), \gamma(\mathbf{x}))$ set of Euler angles for each point in physical space
  - Local beable: $\boldsymbol{\alpha}(\mathbf{x})$
  - Seems empirically inadequate!

Stern-Gerlach experiment:



- Valentini (1992)
  - Wave function $\Psi(\eta)$,

    $\eta(\mathbf{x})$ is a Grassmannian field,

    $\Psi$ is an element of a Grassmann algebra (not $\mathbb{C}$)
  - Local beable: $\eta(\mathbf{x})$
  - Problem: what would be an adequate guidance equation?

# Minimalist model

- Struyve, Westman (2006,2007)
  - Wave function: $\Psi_f(A_i^T) = \langle f, A_i^T | \Psi \rangle$,
    $\rightarrow$ $f$ labels the fermionic degrees of freedom
    $\rightarrow$ $A_i^T(\mathbf{x})$ is tranverse part of the vector potential
  - Local beable: $A_i^T(\mathbf{x})$
    $\rightarrow$ None for fermionic degrees of freedom
  - Equilibrium distribution: $\sum_f |\Psi_f(A_i^T)|^2$

Stern-Gerlach experiment:

TIME $t_1$

TIME $t_2 > t_1$

## Minimalist model + extras

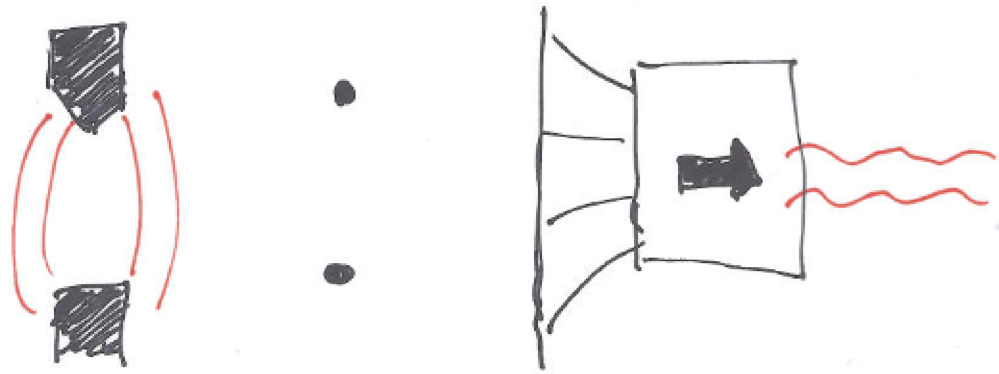
- Struyve, Westman (2007)
  - Wave function: $\Psi_f(A_i^T) = \langle f, A_i^T | \Psi \rangle$,
  - Local beables:
    - $\rightarrow$ transverse potential $A_i^T(\mathbf{x})$
    - $\rightarrow$ charge distribution $\rho(\mathbf{x})$, given by

$$\rho(\mathbf{x}, t) = \left. \frac{\sum_{f,f'} \Psi_f^*(A_i^T, t) \widehat{\rho}_{ff'}(\mathbf{x}) \Psi_{f'}(A_i^T, t)}{\sum_f |\Psi_f(A_i^T, t)|^2} \right|_{A_i^T = A_i^T(t)}$$

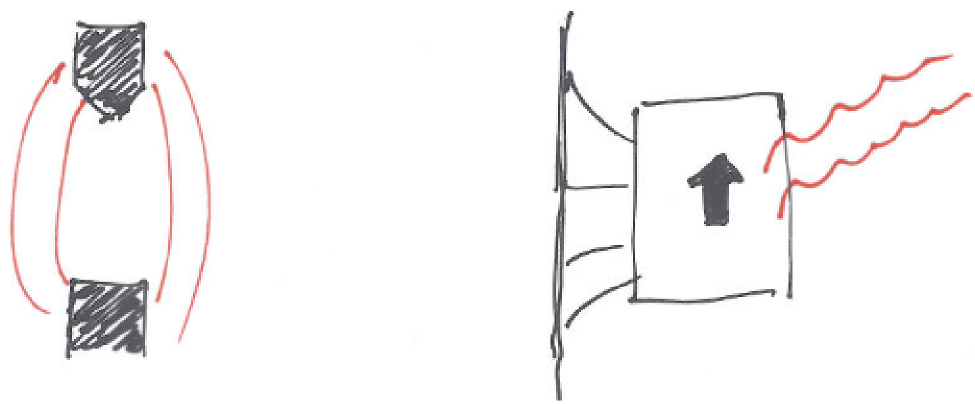
where $\widehat{\rho}_{ff'}(\mathbf{x}) = \langle f | \widehat{\rho}(\mathbf{x}) | f' \rangle$, with $\widehat{\rho}(\mathbf{x})$ the charge density operator

Stern-Gerlach experiment:

TIME $t_1$



TIME $t_2 > t_1$

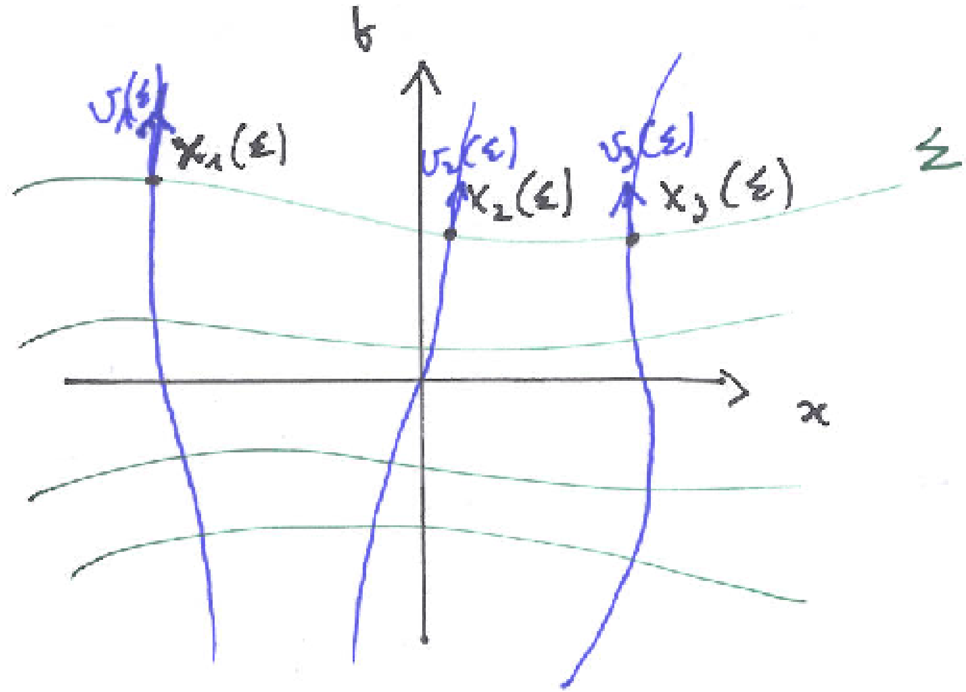


## Can the theory made Lorentz invariant?

- Dürr, Goldstein, Tausk, Struyve & Zanghì (in prep.)

  Other approaches by Dewdney & Horton, Nikolić.

  See also Valentini (1996) for an argument that the natural kinematics is Aristotelian,
- Quantum equilibrium can not hold in all reference frames. Berndl, Dürr, Goldstein, Zanghì (1996)
- Dynamics of the local beables (particles or fields) can be choosen such that equilibrium holds with respect to a particular referernce frame or foliation $\mathcal{F}$ (determined by normal $n^{\mu}$).




$$\dot{X}_k(\Sigma) = v_k^{\psi,\mathcal{F}}(X_1(\Sigma), \ldots, X_N(\Sigma))$$

→ velocity $v^{\psi,\mathcal{F}}$ depends on wavefunction and foliation

- Lorentz invariant law for foliation $\mathcal{F}$. E.g.

  1 The foliation is a hyperplane foliation ($\partial_\nu n^\mu = 0$).

  2 The foliation is arbitrary.

$\rightarrow$ Not very serious.

- Instead of foliation as extra structure, let foliation be completely determined by the wave function, i.e. $\mathcal{F} = \mathcal{F}(\psi)$

  Covariant map $\psi \to \mathcal{F} = \mathcal{F}(\psi)$:

$$\begin{array}{ccc} \psi & \longrightarrow & \mathcal{F}(\psi) \\ {\scriptstyle U(\Lambda)}\big\downarrow & & \big\downarrow{\scriptstyle \Lambda} \\ \psi' & \longrightarrow & \mathcal{F}(\psi') \end{array} \tag{1}$$

  For example:

  $n^\mu \sim P^\mu$, $P^\mu$ the total expected energy momentum four vector,

$$P^\mu = \Big( \langle\psi|\widehat{H}|\psi\rangle, \langle\psi|\widehat{\mathbf{p}}|\psi\rangle \Big) = \langle\psi| \int_\sigma d\sigma_\nu \widehat{T}^{\nu\mu}(x) |\psi\rangle \tag{2}$$

- Is this proposal serious Lorentz invariant?

  Compare with debate on general invariance. E.g. John Norton's review:

  General covariance and the foundations of general relativity: eight decades of dispute

  (Reports on Progress in Physics **56**, 791, 1993)