

About Pilot Wave Field Theory

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Why de Broglie-Bohm field theory?

Because it needs a preferred frame

Horrible. Why this?

Special reason: I am an **ether** theorist. Doing dBB theory is for me like going mainstream.

- dBB theory gives arguments in favour of a preferred frame, thus, supports ether theory.
- Ether theory makes a different proposal for fundamental beables – the state of the ether. Fermions, gauge fields and gravity appear as effective fields.

The "Many Worlds in Denial" - Attack

Part of attack: BM has to **postulate \hat{q} as preferred**, while decoherence allows to **derive a preferred basis**.

How? Given $Q \cong \mathbb{R}^n$, $H = \sum \hat{p}_i^2 + V(\hat{q}_1, \dots, \hat{q}_n)$, then decoherence allows to identify the \hat{q}_i as decoherence-preferred.

Counterattack: They also have to **postulate something else**.

$$1D: H = \hat{p}_1^2 + V_1(\hat{q}_1) = \hat{p}_2^2 + V_2(\hat{q}_2), V_1 \neq V_2$$

2D: Two different decompositions into systems p_i, q_i so that H has same form $\sum \hat{p}_i^2 + V(\hat{q}_1, \hat{q}_2)$ with different but equally nice $V(.,.)$

arXiv:0901.3262: Why the Hamilton operator alone is not enough

arXiv:0903.4657: Pure quantum interpretations are not viable

Wallace's thesis

Wallace: Pilot wave beables have to be decoherence-preferred

Schmelzer: Pilot wave beables may be even unobservable

Fact: Field beables not decoherence-preferred (particles are).
⇒ Pilot wave field theory invalid?

Effective field theory: Fields not even fundamental!

Pilot wave beables will be some yet unknown fundamental things.

Reconsideration of the equivalence proof

$$\mathbf{QM} : \quad \psi_m(x, t_0) \sum \alpha_k \phi_k(y, t_0) \Rightarrow \sum \alpha_k \psi_m^k(x, t_1) \phi_k(y, t_1)$$

$$\mathbf{BM} : \quad x \in \text{sup } \psi_m^k \quad \Rightarrow \quad \psi(x, y, t_1) \approx \psi_m^k(x, t_1) \phi_k(y, t_1)$$

Common: We **observe position of device pointer** $x \in \text{sup } \psi_m^k$;

Better: Our **own configuration** is described by $x \in \text{sup } \psi_m^k$;

\Rightarrow No necessity for observable beables!

\Rightarrow No problem with “fooled particle detectors”!

But **macroscopic configurations** ψ_m^k **should not overlap**

Overlaps of n-particle states in field theory

Overlap:

Simple definition: no overlap if $\forall \phi |\Psi_1(\phi)||\Psi_2(\phi)| = 0$

Better: probability $\rho(\Psi_1|\Psi_2) = \frac{\int |\Psi_1(\phi)|^2 d\phi}{|\Psi_1(\phi)| < |\Psi_2(\phi)|} \ll 1$

States in field theory:

vacuum state: $\Psi_{vac}(\phi) = \prod_{k=1}^{\infty} \frac{1}{\sqrt{\pi}} e^{-\frac{1}{2}\phi_k^2}$, ϕ_k real coords in $\mathcal{L}^2(\mathbb{R})$

n-particle state: $\Psi_n(\phi) = \prod_{k=1}^n \sqrt{2}\phi_k \cdot \Phi_{vac}$

Overlap between two orthogonal n-particle states:

$$\rho_n = \frac{2^n}{\pi^{2n}} \int_{\prod_k \phi_{2k-1}^2 < \prod_k \phi_{2k}^2} \left(\prod_k \phi_{2k-1}^2 \right) e^{-\sum_k (\phi_{2k-1}^2 + \phi_{2k}^2)} \prod_k d\phi_{2k-1} d\phi_{2k}.$$

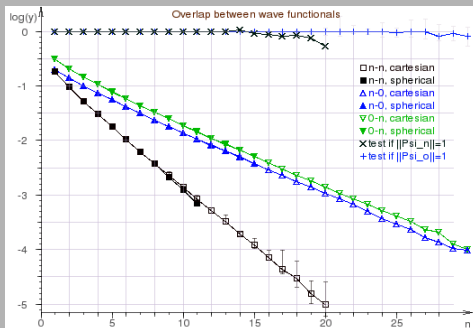
Overlaps of n-particle states in field theory

$$\rho_n = \frac{2^n}{\pi^{2n}} \int_{\prod_k \phi_{2k-1}^2 < \prod_k \phi_{2k}^2} \left(\prod_k \phi_{2k-1}^2 \right) e^{-\sum_k (\phi_{2k-1}^2 + \phi_{2k}^2)} \prod_k d\phi_{2k-1} d\phi_{2k}.$$

Overlap of one-particle states in field theory $0.18169(\pm 1)$;

Monte Carlo simulation

- Cartesian coords
- spherical coords
- overlap (Ψ_{vac}, Ψ_n)
- security check $1 = 1$
- error bounds



What if there is no connection?

N-dimensional Hilbert space (\mathbb{R}^N or \mathbb{C}^N);

A basis ψ^i connected with the beables;

Case 1 — beables are decoherence-preferred:

The ψ_k are localized in this basis; $\|\psi_k\|_\infty \approx \|\psi_k\|_2$

Case 2 — no connection between beables and decoherence:

The ψ_k are homogeneously distributed on the sphere $|\psi_k| = 1$
(S^{N-1} or $\mathbb{C}P^{N-1}$)

The different ψ_k are independent.

Upper bound for the overlap

$$E = \int_{S^{N-1}} \frac{d\Omega_0}{A_{S^{N-1}}} \int_{S^{N-1}} \frac{d\Omega_1}{A_{S^{N-1}}} \sum_{i=1}^N \chi_{|\psi_0^i| < |\psi_1^i|} |\psi_0^i|^2.$$

$$\int_{S^{N-1}} \frac{d\Omega_\psi}{A_{S^{N-1}}} f(\psi) = \int_{-1}^1 \frac{d\psi_0}{2} \cdots \int_{-1}^1 \frac{d\psi_N}{2} f\left(\frac{\psi}{\|\psi\|_2}\right) N \frac{\|\psi\|_\infty^N}{\|\psi\|_2^N}.$$

Localized part: $\|\psi_k\|_\infty \geq (1 - \varepsilon)\|\psi_k\|_2$ for above ψ_k

E_{ij} : maximal coordinates are $|\psi_0^i|$ and $|\psi_1^j|$.

$$E_{local} = \sum_i E_{ii} + \sum_{i \neq j} E_{ij} \leq \frac{1}{N} + 2\varepsilon$$

Remaining part: Say $\|\psi_0\|_\infty < (1 - \varepsilon)\|\psi_0\|_2$, $\|\psi_1\|_\infty < \|\psi_1\|_2$

$$\int_{-1}^1 \frac{d\psi_0^1}{2} \cdots \int_{-1}^1 \frac{d\psi_0^N}{2} \int_{-1}^1 \frac{d\psi_1^1}{2} \cdots \int_{-1}^1 \frac{d\psi_1^N}{2} \sum_i \chi_{|\psi_0^i| < |\psi_1^i|} \frac{|\psi_0^i|^2}{\|\psi_0\|_2^2} \leq 1$$

$$E \leq \frac{1}{N} + 2\varepsilon + N^2(1 - \varepsilon)^N$$

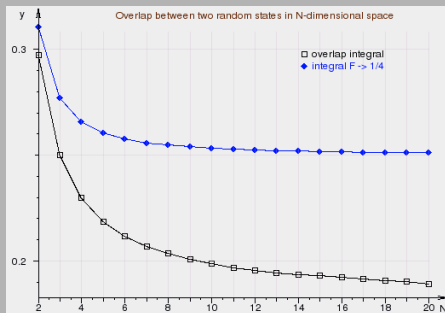
No connection – complex Hilbert space

$$E = \int_{\mathbb{C}^{PN}} \frac{d\Omega_0}{A_{\mathbb{C}^{PN}}} \int_{\mathbb{C}^{PN}} \frac{d\Omega_1}{A_{\mathbb{C}^{PN}}} \sum_{i=1}^N \chi_{|\psi_0^i| < |\psi_1^i|} |\psi_0^i|^2 \leq \frac{1}{N} + 2\varepsilon + (2N)^2(1-\varepsilon)^{2N}$$

Hopf projection

$$S^{2N-1} \rightarrow \mathbb{C}P^N:$$

$$\int_{\mathbb{C}^{PN}} \frac{d\Omega_0}{A_{\mathbb{C}^{PN}}} \Rightarrow \int_{S^{2N-1}} \frac{d\Omega_0}{A_{S^{2N-1}}}$$



But there is a connection!

Actual configuration $q_0 = q(t) \in Q$

Linear theory in the tangent space $\mathcal{T} \cong TQ|_{q(t)}$

Systems as linear subspaces \mathcal{T}_{S_i} of \mathcal{T}

Decomposition $\mathcal{T} \cong \prod \mathcal{T}_{S_i}$ gives decomposition
 $\mathcal{L}^2(\mathcal{T}, \mathbb{C}) \cong \bigotimes \mathcal{L}^2(\mathcal{T}_{S_i}, \mathbb{C})$ of the Hilbert space $\mathcal{L}^2(\mathcal{T}, \mathbb{C})$

Approximation of $\mathcal{H} \cong \mathcal{L}^2(Q, \mathbb{C})$ by $\mathcal{L}^2(\mathcal{T}, \mathbb{C})$

Consequences of the connection

- Product states tend to remain product states.
Such a stability is reason for defining such decompositions.

- Local measurements destroy non-local superpositions

$$|\psi_1\rangle|\phi_1\rangle + |\psi_2\rangle|\phi_2\rangle \Rightarrow |\psi_1\rangle|\phi_1\rangle|\theta_1\rangle + |\psi_2\rangle|\phi_2\rangle|\theta_2\rangle.$$

q^θ not in overlap \Rightarrow resulting effective wave function is
or $|\psi_1\rangle|\phi_1\rangle|\theta_1\rangle$ or $|\psi_2\rangle|\phi_2\rangle|\theta_2\rangle$

\Rightarrow Product states preferred;

Plausibility arguments for product states

- Distance between the maxima increases like $\Delta^2 = \sum \Delta_i^2$
- Case of identical systems: Maximum of overlap ψ_{max} for one system, then maximum of overlap on line between maxima, with value ψ_{max}^N for N identical systems.
- Case of functions with values only in $\{0, 1\}$: For one system p , for N identical systems p^N .
- Case of n-particle states in field theory;

⇒ Strong plausibility arguments for exponential decrease in dependence on the number of systems.

⇒ Much more than we need, given macroscopic numbers of systems.

Fermions in Pilot Wave Theory

General scheme of de Broglie-Bohm pilot wave theory works for **canonical quantum theories**.

Necessity to obtain fermions from canonical quantization in another context: Ether interpretation for standard model fermions.

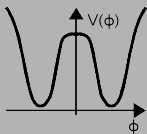
- Gives pairs of Dirac fermions (electroweak doublets);
- Gives heavy bosonic partners;

Canonical Fermion Quantization

Classical lattice theory: $\psi_n = \{\pi_n, \varphi_n\}, \varphi_n \in \mathbb{R}$.

Canonical quantization gives **bosonic field**.

We want **fermionic field**;



We use a \mathbb{Z}_2 -degenerated potential $V(\varphi_n)$ with two vacuum states \implies The ground states define a \mathbb{Z}_2 -valued or “spin field” theory.

Problem: **Spin field** operators σ_n^i of different nodes **commute**.
But **fermion** operators ψ_n, ψ_n^* of different nodes **anticommute**.

⊕ We nonetheless **find an isomorphism**.

⊖ It is not natural, nonlocal, **depends on some order**.

\implies We **choose an order**, transform $H(\sigma_n^i) \rightarrow H(\psi_n, \psi_n^*)$.

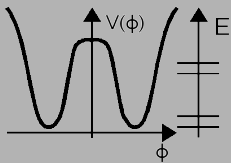
\implies For $dim > 1$ we need a **local projection** $H \rightarrow \pi_\psi H$

\implies We obtain **our lattice Dirac operator**

How to get a \mathbb{Z}_2 -valued field from a real field

Lagrangian for a relativistic scalar field with \mathbb{Z}_2 -degenerated $V(\varphi)$:

$$\mathcal{L} = \frac{1}{2}((\partial_t\varphi)^2 - (\partial_i\varphi)^2) + \frac{\mu^2}{2}\varphi^2 - \frac{\lambda}{4!}\varphi^4$$



Regularization: Lattice $\mathbb{Z}^3 \subset \mathbb{R}^3$.

$$\partial_i\varphi(x) \rightarrow \frac{1}{h}(\varphi_{n+1} - \varphi_n)$$

$$\partial_t\varphi(x) \rightarrow \frac{1}{2}(\partial_t\varphi_{n+1} + \partial_t\varphi_n)$$

Canonical quantization!

$$\langle \Psi_1 H \Psi_1 \rangle - \langle \Psi_0 H \Psi_0 \rangle \sim e^{-\frac{\mu^3}{\lambda}} \ll \sqrt{2}\mu \sim \langle \Psi_2 H \Psi_2 \rangle - \langle \Psi_0 H \Psi_0 \rangle$$

Low energy domain generated by $\Psi_{0/1}(\varphi_n)$ in each node:

$$\text{Pauli matrices: } (\sigma_n^i)^2 = 1, \quad [\sigma_m^i, \sigma_n^j] = 2i\delta_{mn}\epsilon_{ijk}\sigma_n^k.$$

$$H = c_0 \sum_n \sigma_n^3 + c_1 \sum_{n,i} \sigma_n^1 \sigma_{n+h_i}^1 + c_2 \sum_{n,i} \sigma_n^2 \sigma_{n+h_i}^2$$

Spin fields are not fermion fields!

Spin field operators on different nodes commute.

Fermion operators on different nodes anticommute:

$$\{\psi_m, \psi_n^*\} = \delta_{mn}, \quad \{\psi_m^*, \psi_n^*\} = \{\psi_m, \psi_n\} = 0.$$

But isomorphism exist!

$$\psi_n^1 = \psi_n + \psi_n^*, \quad \psi_n^2 = -i(\psi_n - \psi_n^*), \quad \psi_n^3 = -i\psi_n^1\psi_n^2.$$

$$\psi_n^{1/2} = \sigma_n^{1/2} \prod_{m>n} \sigma_m^3, \quad \psi_n^3 = \sigma_n^3,$$

$$\sigma_n^{1/2} = \psi_n^{1/2} \prod_{m>n} \psi_m^3, \quad \sigma_n^3 = \psi_n^3.$$

This is known in Clifford algebra theory: $Cl^{N,N}(\mathbb{R}) \cong M_{2^N}(\mathbb{R})$.

Transformation of the Hamilton operator

$$H = c_0 \sum_n \sigma_n^3 + c_1 \sum_{n,i} \sigma_n^1 \sigma_{n+h_i}^1 + c_2 \sum_{n,i} \sigma_n^2 \sigma_{n+h_i}^2$$

Special case: $c_0 = \frac{m}{2}$, $c_1 = -c_2 = -\frac{1}{4h}$.

$$\pi_\psi H = \frac{1}{2h} \sum_{n,i} \alpha_{n+h_i}^n (\psi_n \psi_{n+h_i} - \psi_n^* \psi_{n+h_i}^*) + \frac{m}{2} \sum_n \psi_n^* \psi_n - \psi_n \psi_n^*$$

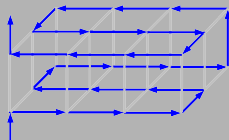
$\pi_\psi H$ is a lattice Dirac operator!

$$i\partial_t \psi_n = [\pi_\psi H, \psi_n] = \frac{1}{2h} \sum_i \alpha_{n+h_i}^n (\psi_{n+h_i}^* - \psi_{n-h_i}^*) - m\psi_n$$

The lattice Dirac equation

$$i\partial_t\psi_n = \frac{1}{2h} \sum_i \alpha_{n+h_i}^n (\psi_{n+h_i}^* - \psi_{n-h_i}^*) - m\psi_n$$

$$\alpha_{n+h_i}^n = \begin{cases} 1 & \text{if } n < n + h_i; \\ -1 & \text{if } n > n + h_i; \end{cases}$$



$$\alpha_m^n = \alpha_{m+2h_i}^{n+2h_i}; \quad \alpha_{n+h_i}^n \alpha_{n+2h_i}^{n+h_i} = 1; \quad \alpha_{n+h_i}^n \alpha_{n+h_i+h_j}^{n+h_i} = -\alpha_{n+h_j}^n \alpha_{n+h_i+h_j}^{n+h_j}$$

$$\partial_t^2\psi_n = \sum_i \frac{\psi_{n+2h_i} - 2\psi_n + \psi_{n-2h_i}}{(2h)^2} - m^2\psi_n = -(\Delta_{2h} + m^2)\psi_n$$

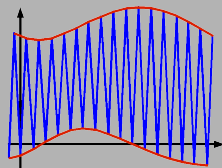
The doubling effect

Low energy solutions of

$$-\partial_t^2 \psi = (\Delta_{2h} + m^2) \psi$$

smooth only on **mod 2** sublattices!

\Rightarrow we need **eight** continuous functions
to describe **one** oscillating lattice function



$$\kappa = (\kappa_1, \kappa_2, \kappa_3), \quad \kappa_i \in \{0, 1\}$$

$$\psi(2n_1 + \kappa_1, 2n_2 + \kappa_2, 2n_3 + \kappa_3) = \psi_\kappa(n_1 h, n_2 h, n_3 h)$$

Electroweak doublets: $\mathbb{C}(\mathbb{Z}^3) \cong \{\psi(n_1, n_2, n_3)\} \rightarrow \mathbb{C}^8(\mathbb{R}^3)$

Comparison with relativistic approach

dim = 3: Spatial lattice \Rightarrow 8 staggered fields $\psi_{\kappa_1\kappa_2\kappa_3}(\mathbf{n}_i, t)$.

dim = 4: Spacetime lattice \Rightarrow 16 staggered fields $\psi_{\kappa_0\kappa_1\kappa_2\kappa_3}(n_\mu)$.

Kogut-Susskind staggered fermions.

dim = 3: $i\partial_t\psi(\mathbf{x}, t) = -i\alpha^i\partial_i\psi(\mathbf{x}, t) + m\beta\psi(\mathbf{x}, t); \quad \psi(\mathbf{x}, t) \in \mathbb{C}^8$

dim = 4: $i\gamma^\mu\partial_\mu\psi(x) = m\psi(x); \quad \psi(x) \in \mathbb{C}^{16}$

dim = 3: **geometric Dirac operator** on $\mathbb{C} \otimes \Lambda(\mathbb{R}^3)$ with metric δ_{ij}

dim = 4: **Dirac-Kähler equation** on $\mathbb{C} \otimes \Lambda(\mathbb{R}^4)$ with metric $\eta_{\mu\nu}$

dim = 3: Two Dirac fermions \cong **Electroweak doublets** of SM.

dim = 4: Four Dirac fermions \Rightarrow **No interpretation in SM.**

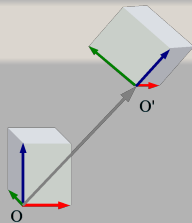
The lattice of cells

The New Ether: A Lattice of Cells

State of cell: affine deformation $y^i = \varphi_j^i x^j + \varphi_0^i$
from standard reference cell at origin O.

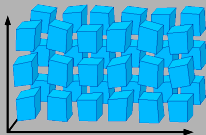
$$\text{Aff}(3) \cong \{\varphi_\mu^i \in \mathbb{R}, 1 \leq i \leq 3, 0 \leq \mu \leq 3\}.$$

i: generation; $\mu > 0$ quark color; $\mu = 0$ leptons;



Configuration space of the whole lattice:

$$\text{Aff}(3)(\mathbb{Z}^3): \{\varphi_\mu^i(n_1, n_2, n_3) : \mathbb{Z}^3 \rightarrow \text{Aff}(3)\}$$



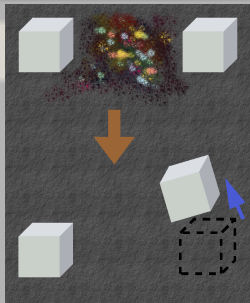
Phase space: $\text{Aff}(3) \otimes \mathbb{C}(\mathbb{Z}^3): \{\psi_\mu^i(n_1, n_2, n_3) \in \mathbb{C}\}$

Doubling: Each complex function \cong one electroweak doublet

The SM Gauge Fields

- They commute with rotations: preserve/act identical on all generations.
- They commute with translations: leaves translational direction fixed – right-handed neutrinos inert.
- They preserve symplectic structure: compact \Rightarrow unitary.
- Wilson gauge fields: have same charge on whole doublets $\Rightarrow U(4) \Rightarrow U(3)_c \supset SU(3)_c$
- Lattice deformations: generated by lattice shifts $2\gamma^5 l_i$ $\Rightarrow U(2)_L \times U(2)_R \Rightarrow U(2)_L \times U(1)_R \supset SU(2)_L$
- EM field as combination: $U(3)_c \times U(2)_L \times U(1)_R \supset U(1)_\gamma$
- Dirac sea neutral: $\Rightarrow S(U(3)_c \times U(2)_L \times U(1)_R)$
- $\Rightarrow G_{SM} \times U(1)_{upper\ axial}$ anomal; Anomaly-freedom $\Rightarrow G_{SM}$

Wilson gauge fields



Imagine irregularities between the cells.
This has some influence on the equation.
We compensate it by some correction term,
which acts on the phase space of one node.

⇒ **Wilson gauge field.**

Group G acts pointwise. But electroweak doublets are $\psi(n) \in \mathbb{C}$.

Thus, G has to have the same charge on the whole doublet.

⇒ **$G \subseteq \mathbf{U(3)}_c \cong \mathbf{SU(3)}_c \times \mathbf{U(1)}_B$**

Wilson gauge fields have exact $U(3)$ lattice gauge symmetry;

- Explains zero gluon ($SU(3)$) masses;
- Electrostrong $SU(3)_c \times U(1)_{em}$ action is really action of $U(3) \cong SU(3) \times U(1)/\mathbb{Z}_3$. Deformation of exact $U(3)_c$?

Lattice deformations as gauge fields

Correction terms for lattice deformations:

$$\psi_{n-h_i}\psi_n \rightarrow \psi_{n-h_i}(\psi_n + \sum_j a_j(n)\psi_{n+h_j})$$

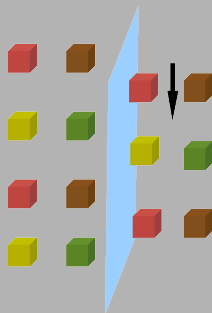
⇒ effective gauge field on doublets;

⇒ generators: $\tilde{\tau}_i \rightarrow 2\gamma^5 I_i$

Maximal gauge group is: $U(2)_L \times U(2)_R$.

Translational invariance: $U(2)_L \times U(1)_R$ or $U(1)_L \times U(2)_R$.

No lattice gauge symmetry ⇒ weak gauge fields are massive;



Consequences for pilot wave gauge theory?

Gauge fields are effective fields;

- Weak interactions do not have exact lattice gauge symmetry at all.
- Strong interactions have exact $U(3)$ lattice gauge symmetry;

In above cases:

Gauge-equivalent configurations are **different configurations**.

⇒ No BRST factorization!

⇒ Corresponding problems for pilot wave field theories irrelevant.

Ether Theory of Gravity

$$\mathcal{L} = \frac{1}{2} \gamma_{\alpha\beta} X_{,\mu}^{\alpha} X_{,\nu}^{\beta} g^{\mu\nu} \sqrt{-g} + \mathcal{L}_{GR}(g^{\mu\nu}) + \mathcal{L}_{cov}(g^{\mu\nu}, \varphi_{matter})$$

harmonic coordinates $X^{\alpha}(x)$: $\frac{\delta S}{\delta X^{\alpha}} = \gamma_{\alpha\beta} \partial_{\gamma} (g^{\beta\gamma} \sqrt{-g}) = 0$

A variant of ADM decomposition for the foliation $T(x) = X^0(x)$:

$$\begin{array}{ll}
 g^{00} \sqrt{-g} = \rho & \text{density } \rho \\
 g^{0i} \sqrt{-g} = \rho v^i & \text{velocity } v^i \\
 g^{ij} \sqrt{-g} = \rho v^i v^j - \sigma^{ij} & \text{stress tensor } \sigma^{ij} \\
 \varphi_{matter} & \text{other material properties } \varphi_{mat}
 \end{array}$$

continuity equation: $\partial_t \rho + \partial_i (\rho v^i) = 0$

Euler equations: $\partial_t (\rho v^i) + \partial_i (\rho v^i v^j - \sigma^{ij}) = 0$

Physical predictions:

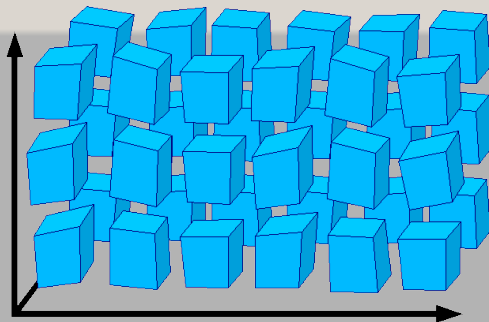
- Exact Einstein Equivalence Principle.
- The Einstein equations of GR in some natural limit.
- No black hole singularity – gravitational collapse stops before horizon formation, leading to a stable gravastar.
- No big bang singularity – big bounce before big bang.
- Flat universe preferred as the only homogeneous universe.
- $\rho > 0 \Leftrightarrow X^0(x)$ timelike \implies no closed causal loops.

Note: $\rho \rightarrow 0$ possible, but parts with $\rho < 0$ unphysical.

Consequences for Bohmian mechanics

- No problem of time;
- Different equivalent metrics describe different states of the gravitational field;
- More degrees of freedom than in canonical GR;
- Gravitational field is effective – fundamental beables are different;

Thank you very much
for your attention



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Foundations of Physics, vol. 39, 1, p. 73 (2009), arXiv:0908.0591