Decoherence and the Emergence of Classical Behavior in de Broglie-Bohm Theory

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Outline of Talk

Goal: To explain the role of decoherence in the emergence of macroscopic Newtonian behavior in pilot wave theory.

Outline:

- Formulating the problem: What do we mean by "classical behavior"?
- Existing Accounts of Emergent Classicality in Pilot Wave Theory
- Classicality in Isolated Subsystems
- The Need to Consider the Environment
- An Alternative Account of Classicality in Pilot Wave Theory

Formulating the Problem: What do we mean by "classical behavior"?

Formulating the Problem

- We restrict our attention here to a small but important subset of the systems that one might wish to call "classical."
- These systems can be characterized as centers of mass of macroscopic bodies such as planets and projectiles, which contain on the order of $\approx 10^{23}$ quantum particles and which obey Newtonian equations of motion.
- We seek to explain, on the basis of pilot wave theory, why the Bohmian configurations associated with these centers of mass follow Newtonian trajectories.

Formulating the Problem

I model classical behavior with a closed system comprised of two subsystems:

- A subsystem S consisting of macroscopic degrees of freedom - i.e., centers of mass
- A subsystem E consisting of residual microscopic degrees of freedom (including the bodies' internal degrees of freedom)

Formulating the Problem

I characterize the evolution of the quantum state as follows:

- Hilbert space: $\mathcal{H}_{SE} = \mathcal{H}_S \otimes \mathcal{H}_E$
  $\mathcal{H}_S$: Hilbert space for macroscopic degrees of freedom (e.g., centers of mass of a group of planets)
  $\mathcal{H}_E$: Hilbert space for microscopic, environmental degrees of freedom (e.g. interstellar dust, radiation, internal degrees of freedom of the planets)
- Wave function: $\Psi = \Psi(X_1, ..., X_N, y_1, ..., y_n) \equiv \Psi(X, y)$.
- Schrodinger Equation: $\imath\hbar\frac{\partial\Psi}{\partial t} = \hat{H}\Psi$
- Hamiltonian: $\hat{H} = \hat{H}_S(\hat{X}_1, ..., \hat{X}_N) + \hat{H}_E(\hat{y}_1, ..., \hat{y}_n) + \hat{H}_I(\hat{X}, \hat{y})$,
  $\hat{H}_S(\hat{X}_1, ..., \hat{X}_N, \hat{P}_1, ..., \hat{P}_N) = \sum_{i=1}^N \frac{\hat{P}_i^2}{2M_i} + \sum_{i<j} V(|\hat{X}_i - \hat{X}_j|)$

Formulating the Problem

- Bohmian configuration: $Q_{S,i} = \sum_j \frac{m_j q_j}{\sum_j m_j}$,
- Bohmian configuration space: $\mathbb{Q}_{SE} = \mathbb{Q}_S \times \mathbb{Q}_E$,
  $\mathbb{Q}_S$: center of mass configuration space
  $\mathbb{Q}_E$: environment configuration space
- Bohmian guidance equations:
  $\dot{Q}_{S,i} = \frac{1}{M_i} \nabla_i S(X, y)|_{Q_S, q_E}$,
  $\dot{q}_{E,j} = \frac{1}{m_j} \nabla_j S(X, y)|_{Q_S, q_E}$,
  where $\Psi(X, y) = R(X, y) e^{iS(X,y)}$, $Q_{S,i}$ is the Bohmian configuration of the i^{th} center of mass in S, and $q_{E,j}$ is the Bohmian configuration of the j^{th} microscopic particle in E (see Rimini and Peruzzi, 2000).

Formulating the Problem

Goal: *Explain why the trajectory of the Bohmian configuration* Q_S *for the centers of mass is approximately Newtonian* (Given certain physically motivated assumptions about $\hat{H}$).

Existing Accounts of Emergent Classicality in Pilot Wave Theory

Existing Accounts of Emergent Classicality in Pilot Wave Theory: Brief Survey of the Literature

- Most accounts of classical behavior rely essentially on the vanishing of the quantum potential or quantum force. The alternative account that I offer here makes no use of the quantum potential, but only of the constraint of equivariance.
- The quantum potential approach to classical behavior has only been developed in any detail for the case where the system S is isolated, so that $\hat{H}_I = 0$ and effects of the environment are ignored (e.g., Holland). As a description of actual macroscopic classical systems, the assumption of isolation is highly unrealistic, since such systems are strongly affected by the interaction with their evnironment.
- Some of these accounts briefly mention the role of decoherence, but only briefly at the end of the analysis, and not in much detail (e.g. Allori et al, Bohm, Durr and Teufel).

- Appleby also has calculated Bohmian velocties for relevant degrees of freedom in the Caldeira-Legett model for a specific set of initial conditions and spectral density. The calculations suggest a classical trajectory for relevant degrees of freedom in S, but his results lack generality because of the specificity of the parameters of his model. Appleby does not propose a set of general conditions for classical behavior.
- Bowman is an exception. The account that I provide below elaborates some aspects of the approach that he suggests: namely, to use decoherence to explain the existence of wave packets which are effectively narrow and which cause the Bohmian configuration to evolve classically. However, Bowman does not recognize the need for a very specific kind of decoherence in pilot wave theory, or describe the behavior of the wave function for the entire closed system consisting of both S and E.

Existing Accounts of Emergent Classicality in Pilot Wave Theory: Brief Survey of the Literature

While the assumption of isolation is unrealistic, it is instructive to consider how classical Bohmian trajectories emerge in isolated systems before moving on to consider the effects of interaction with the environment.

Classicality in Isolated Subsystems

Classicality in Isolated Subsystems

In the case of isolation, we assume that the central system's dynamics is described by

- Wavefunction
  - Hilbert space: $\mathcal{H}_S$
  - Wave function: $\Psi = \Psi(X)$
  - Schrodinger Equation: $\imath\hbar\frac{\partial\Psi}{\partial t} = \hat{H}_S\Psi$
    $\hat{H}_S(\hat{X}_1, ..., \hat{X}_N, \hat{P}_1, ..., \hat{P}_N) = \sum_{i=1}^{N} \frac{\hat{P}_i^2}{2M_i} + \sum_{i<j} V(|\hat{X}_i - \hat{X}_j|)$
- Bohmian Configuration
  - Configuration space: $\mathbb{Q}_S$
  - Configuration: Q_S
  - Dynamics: $\dot{Q}_{S_i} = \nabla_i S(X)|_{Q_S}$

Classicality in Isolated Subsystems

There are two popular ways to regain classical Bohmian trajectories in isolated systems:

- **Narrow wave packets**
- **The Quantum Potential Approach: Q, Q' $\rightarrow$ 0**

Both approaches strongly constrain the form of the wave function, but in opposite ways, since the former requires that the wave function be narrow and the latter that it be spread out.

Classicality in Isolated Subsystems

The Narrow Wave Packet Approach

- For *any* admissible wave function, Ehrenfest's Theorem holds:

$$m\frac{d^2 < \hat{x} >}{dt^2} = -\langle \frac{\partial V(\hat{x})}{\partial \hat{x}} \rangle \tag{1}$$

- For narrow wave packets, a stronger condition holds:

$$m\frac{d^2 < \hat{x} >}{dt^2} \approx -\frac{\partial V(\langle \hat{x} \rangle)}{\partial \langle \hat{x} \rangle} \tag{2}$$

- This second condition implies that a narrow wave packet follows an approximately Newtonian trajectory as long as it remains narrow (this is the reason that we have imposed the no-spreading condition). By equivariance, we can expect Bohmian trajectories to follow the wave packet they're in. Thus, if the wave function of the system remains in the form of a narrow wave packet, its Bohmian trajectories will be approximately Newtonian (to within an error given by the width of the wave packet).

Classicality in Isolated Subsystems

The Narrow Wave Packet Approach

- Assume that wave packet spreading in S can be ignored over the length and time scales for which we seek to approximate Newtonian behavior.
- If $\hat{H}_S$ is regular (i.e., not chaotic), the M_i being macroscopically large ($\sim 1kg.$) will suffice to enforce this assumption. However, if $\hat{H}_S$ is chaotic, then a wave packet a few Angstroms in width may, even in spite of macroscopically large mass, spread to macroscopic width within a short time span (days or weeks). In such a case, the wave packet's expectation value will no longer follow a Newtonian trajectory.
- Here, I restrict my attention to $\hat{H}_S$ for which substantial spreading (whether due to chaos or other factors) occurs on time scales much longer than the timescales over which we expect Newton's laws to hold. This will occur if the mass is sufficiently large and, in the case of chaotic H_S, if the Lyapunov exponent is sufficiently small.

Classicality in Isolated Subsystems

The Quantum Potential Approach

If one plugs in the polar decomposition of the wave function $\Psi = Re^{iS}$ in the Schrodinger equation, one obtains the familiar pair of coupled differential equations

$$\frac{\partial S}{\partial t} + \frac{1}{2m}(\nabla S)^2 + V - \frac{\hbar^2}{2m}\frac{\nabla^2 R}{R} = 0, \qquad (3)$$

$$\frac{\partial R^2}{\partial t} + \nabla \cdot (R^2 \frac{\nabla S}{m}) = 0. \qquad (4)$$

where $\oint \nabla S \cdot ds = n\hbar$.

- The first is the Hamilton-Jacobi equation, but with an additional "quantum potential" term, $Q \equiv -\frac{\hbar^2}{2m}\frac{\nabla^2 R}{R}$.
- The second is a continuity equation for the probability distibution R^2.

Bohmian trajectories become Newtonian in form when $Q \to 0$, or when $\nabla Q \to 0$.

Classicality in Isolated Subsystems

Problems with the narrow wave packet approach

- Even assuming that the system's dynamics are such that narrow wave packets remain narrow over appropriate timescales, narrow wave packets constitute only a small subset of possible solutions to the Schrodinger equation.
- The most general solution will, rather, be a superposition of narrow wave packets of the form

$$|\Psi >= \int \, dq \; dp \; \alpha(q,p) \; |q,p>, \tag{5}$$

  where $\alpha(q,p)$ is some complex coefficient and each $|q,p>$ traverses its own classical trajectory.
- While each wave packet $|q,p>$ in the superposition traverses a classical trajectory, such a solution will **not** generally yield a classical trajectory for the Bohmian configuration, as we demonstrate on the next slide.

Classicality in Isolated Subsystems

Problems with the narrow wave packet approach

Example

- Let S consist of a single center of mass governed by a free Hamiltonian $\hat{H}_S = \frac{\hat{P}^2}{2m}$. If we like, m can be macroscopically large.
- Let the wave function of this system take the form of two initially separated wave packets passing through each other:

$$|\Psi>_S = \frac{1}{\sqrt{2}}[|q_1, p> + |q_2, -p>]. \tag{6}$$

- Bohmian trajectories associated with each packet will initially follow the same classical path that their wave packets follow.
- However, Bohmian trajectories associated with a single pure state can never cross. When the packets overlap, the trajectories will reverse direction and leave the region of overlap in the packet in which they did *not* begin. This reversal is highly non-classical.

Problems with the narrow wave packet approach

More generally, this sort of non-classical behavior on the part of the Bohmian trajectory will occur whenever the expansion of the wave function contains wave packets that are initially separated and later come to intersect as a result of the dynamics - *even if the mass is macroscopically large*.

Classicality in Isolated Subsystems

Problems with the quantum potential approach

- One might expect, because of the m in the denominator of $Q \equiv -\frac{\hbar^2}{2m}\frac{\nabla^2 R}{R}$, that large m will be sufficient to guarantee that $Q \to 0$.
- However, even for very large m, interference may cause the curvature term $\frac{\nabla^2 R}{R}$ to trump m and make Q sizable.
- For example, a single WKB solution satisfies the condition $Q \approx 0$.
- However, a superposition of WKB solutions will **not** generally satisfy the condition $Q \approx 0$ (see Holland, 1993) because the two solutions interfere.

Classicality in Isolated Subsystems

Summary

- Superpositions of narrow wave packets, or of solutions for which $Q \approx 0$, can yield highly non-classical Bohmian trajectories even for large m. This is because interference makes Q large.
- Thus, we cannot explain the emergence of classical trajectories at the macroscopic scale by taking S as isolated without excluding from consideration a very broad class of solutions to the Schrodinger equation.
- The set of wave functions that one must throw away will depend heavily on the dynamics of the particular system, making such an exclusion seem especially *ad hoc*.
- We would like to explain the emergence of classicality for some much more general class of initial wave functions.

The Need to Consider the Environment

The Need to Consider the Environment

Reconsider the example of two narrow wave packets above. Suppose now that $\hat{H}_I \neq 0$ and the central system S becomes entangled with the environment E. Suppose also that at every time the wave function of the closed system SE has the form

$$|\Psi>_{SE} = \frac{1}{\sqrt{2}}[|q_1, p> \otimes |\phi_1> + |q_2, -p> \otimes |\phi_2>], \tag{7}$$

for some q_1, q_2, and p, where $|\phi_1> \in \mathcal{H}_E$ and $|\phi_2> \in \mathcal{H}_E$ and

Config Space Decoherence $: <\phi_1|y><y|\phi_2> \approx 0$ for all $y \in \mathbb{Q}_E$. (8)

This last condition amounts to a special kind of decoherence, which we call "configuration space decoherence." It is the condition of disjoint configuration space supports.

In S's configuration space, the Bohmian trajectories of the center of mass associated with the two wave packets pass through each other and continue classically in a straight line.

The Need to Consider the Environment

In more general systems where the macro d.o.f's in S are governed by some non-zero potential $V(x)$, and where the total wave function takes the form

$$|\Psi> = \sum_i \alpha(q_i, p_i)\ |q_i, p_i> \otimes |\phi_i>, \tag{9}$$

where

$$\textbf{Config Space Decoherence} :< \phi_i|y>< y|\phi_j> \approx 0 \text{ for all } y \in \mathbb{Q}_E, \text{ for } i \neq j, \tag{10}$$

the Bohmian trajectory of S will follow a single wave packet and therefore evolve classically, even when packets overlap in S's configuration space.

An Alternative Approach

An Alternative Approach

In pilot wave theory, the wave function determines the evolution of the Bohmian configuration, but the Bohmian configuration has no effect on the wave function. Relying on this observation, I divide my analysis of emergent classicality into two stages:

- The first stage of the analysis closely mimics an Everettian account of classicality since it is concerned exclusively with the unitary evolution of the wave function. It closely follows the analysis given by David Wallace in Chapter 3 of his recent book.
- The second stage considers the effect that the wave function has on the evolution of the Bohmian configuration. In doing so, my account makes no reference to the quantum potential or force, but instead determines the wave function's effect on the Bohmian configuration entirely through the constraint of equivariance. However, in order to make use of equivariance, we must invoke a well-motivated but unproven conjecture discussed below.

An Alternative Approach: Part 1, the Wavefunction

First, consider the evolution of the wave function for the closed system SE, assuming only that the decoherence-preferred states of S (i.e., those which suffer least entanglement with E) are the coherent states $|q, p>$ (see Zurek, 1993) and for the moment allowing for the possibility of significant spreading in S. For purposes of illustration, I consider the evolution in discrete time intervals.

We can, in full generality, write the total state of SE (here, the state at $t = 0$) as

$$|\Psi_0 >= \int \, dq_0 dp_0 \; \alpha(q_0, p_0) \; |q_0, p_0 > \otimes |\phi(q_0, p_0) >, \tag{11}$$

Interaction between S and E will quickly cause decoherence. In pilot wave theory, we require a stronger decoherence condition than in the Everett interpretation, corresponding to the condition of disjoint supports:
Decoherence (Everett): $< \phi(q_0', p_0')|\phi(q_0, p_0) > \approx 0$ for (q_0, p_0) and (q_0', p_0') sufficiently different.
Config Decoherence (Pilot Wave):
$< \phi(q_0', p_0')|y >< y|\phi(q_0, p_0) > \approx 0 \; \forall y \in \mathbb{Q}_E$, for (q_0, p_0) and (q_0', p_0') sufficiently different.

An Alternative Approach: Part 1, the Wavefunction

After time Δt, an individual component of the wave function $|q_0, p_0 > \otimes |\phi(q_0, p_0 >$ will evolve into some linear combination of wave packets:

$$|q_0, p_0 > \otimes |\phi(q_0, p_0 > \xrightarrow{\Delta t} \int dq_1 dp_1 \, \beta(q_1, p_1; q_0, p_0) \, |q_1, p_1 > \otimes |\phi(q_1, p_1; q_0, p_0) > . \tag{12}$$

By linearity of the equations of motion, the total state at time Δt will then be

$$|\Psi(\Delta t) >= \int dq_0 dp_0 dq_1 dp_1 \, \beta(q_1, p_1; q_0, p_0) \, \alpha(q_0, p_0) \, |q_1, p_1 > \otimes |\phi(q_1, p_1; q_0, p_0) > . \tag{13}$$

An Alternative Approach: Part 1, the Wavefunction

Again, the interaction between the system and the environment will cause decoherence, but the particular decoherence condition that we require will depend on the interpretation that we assume.

Decoherence (Everett):

$< \phi(q_1', p_1'; q_0', p_0') | \phi(q_1, p_1; q_0, p_0) > \approx 0$ if (q_i, p_i) and (q_i', p_i') are sufficiently different for either $i = 0$ or $i = 1$.

Config Decoherence (Pilot Wave):

$< \phi(q_1', p_1'; q_0', p_0') | y >< y | \phi(q_1, p_1; q_0, p_0) > \approx 0 \; \forall y \in \mathbb{Q}_E$ if (q_i, p_i) and (q_i', p_i') are sufficiently different for either $i = 0$ or $i = 1$.

An Alternative Approach: Part 1, the Wavefunction

Iterating this dynamics up to time $t = N\Delta t$, we find

$$|\Psi(N\Delta t) >= \int \; d\mathbf{q} \; d\mathbf{p} \; C_N(\mathbf{q}, \mathbf{p}) \; |q_N, p_N > \otimes |\phi(\mathbf{q}, \mathbf{p}) >, \quad (14)$$

where $\int \; d\mathbf{q} \; d\mathbf{p} \equiv \int dq_N dp_N \ldots dq_0 dp_0$,
$C_N(\mathbf{q},\mathbf{p}) \equiv \beta(q_N, p_N; q_{N-1}, p_{N-1}) \beta(q_{N-1}, p_{N-1}; q_{N-2}, p_{N-2}) \ldots \beta(q_1, p_1; q_0, p_0) \alpha(q_0, p_0)$,
and $|\phi(\mathbf{q},\mathbf{p}) > \equiv |\phi(q_N, p_N; \ldots; q_0, p_0) >$.
The interaction between system and environment will cause decoherence, and the particular kind of decoherence that we insist upon will depend on the interpretation that we assume.
Decoherence (Everett):
$< \phi(q'_N, p'_N; \ldots; q'_0, p'_0) | \phi(q_N, p_N; \ldots; q_0, p_0) > \approx 0$ if (q_i, p_i) and (q'_i, p'_i) are sufficiently different for any $0 \leq i \leq N$.
Config Decoherence (Pilot Wave): $<$
$\phi(q'_N, p'_N; \ldots; q'_0, p'_0) | y >< y | \phi(q_N, p_N; \ldots; q_0, p_0) > \approx 0 \; \forall y \in \mathbb{Q}_E$,
if (q_i, p_i) and (q'_i, p'_i) are sufficiently different for any $0 \leq i \leq N$.

An Alternative Approach: Part 1, the Wavefunction

Note that the configuration decoherence requirement defines disjoint regions of $\mathbb{Q}_E$ corresponding to different sequences $(q_N, p_N; \ldots; q_0, p_0)$.

An Alternative Approach: Part 1, the Wavefunction

Now, assume $\hat{H}_S$ is such that wave packets in S don't spread significantly over relevant time scales. In this case, the expression for the evolution of the state up to some time $t = N\Delta t$ simplifies to

$$\boxed{|\Psi(N\Delta t) >= \int dq_0 \; dp_0 \; \alpha(q_0, p_0)|\Psi_{q_0,p_0}(N\Delta t) >} \tag{15}$$

where

$$|\Psi_{q_0,p_0}(N\Delta t) >\equiv |q(N\Delta t), p(N\Delta t) >_{q_0,p_0} \otimes|\phi(\mathbf{q}, \mathbf{p}) > . \tag{16}$$

and $|\phi(\mathbf{q},\mathbf{p})>\equiv|\phi(q_N,p_N;...;q_0,p_0)>$, with $(q_i,p_i)\equiv(q(i\Delta t),p(i\Delta t))_{q_0,p_0}$ for $0<i\leq N$.

An Alternative Approach: Part 1, the Wavefunction

The two decoherence conditions are
Decoherence (Everett):
$< \phi(q'_N, p'_N; ...; q'_0, p'_0)|\phi(q_N, p_N; ...; q_0, p_0) > \approx 0$ if (q_i, p_i) and (q'_i, p'_i) are sufficiently different for any $0 \leq i \leq N$.
Config Decoherence (Pilot Wave): $< \phi(q'_N, p'_N; ...; q'_0, p'_0)|y >< y|\phi(q_N, p_N; ...; q_0, p_0) > \approx 0 \; \forall y \in \mathbb{Q}_E$, if (q_i, p_i) and (q'_i, p'_i) are sufficiently different for any $0 \leq i \leq N$.

Note: if (q_0, p_0) and (q'_0, p'_0) differ significantly, then so will (q_i, p_i) and (q'_i, p'_i), for all $0 \leq i \leq N$. At the level of the wave function, this evolution describes a linear superposition of parallel classical evolutions for S, in each of which the state of the environment is constantly becoming correlated with the evolution of S.

An Alternative Approach: Part 1, the Wavefunction

Since the dynamics is in fact continuous in time, taking the limit $\Delta t \to 0$, we can write this last state as

$$\boxed{|\Psi(t) >= \int dq_0 \; dp_0 \; \alpha(q_0, p_0) \; |\Psi_{q_0,p_0}(N\Delta t) >} \tag{17}$$

where

$$|\Psi_{q_0,p_0}(N\Delta t) >\equiv |q(t), p(t) >_{q_0,p_0} \otimes |\phi[q(t), p(t)] >, \tag{18}$$

and $|\phi[q(t), p(t)] >$ is a functional of the trajectory $(q(t), p(t))_{q_0,p_0}$.

The two decoherence conditions are

Decoherence (Everett): $< \phi[q'(t), p'(t)]|\phi[q(t), p(t)] >\approx 0$ if $(q(t), p(t))$ and $(q'(t), p'(t))$ are sufficiently different for any t.

Config Decoherence (Pilot Wave): $< \phi[q'(t), p'(t)]|y >< y|\phi[q(t), p(t)] >\approx 0 \; \forall y \in \mathbb{Q}_E$, if $(q(t), p(t))$ and $(q'(t), p'(t))$ are sufficiently different for any t.

Note: If (q_0, p_0) and (q_0', p_0') differ significantly, then so will $(q(t), p(t))$ and $(q'(t), p'(t))$, for all $t \geq 0$.

An Alternative Approach: Part 2, the Configuration

Assuming that the wave function is as given in (15), and satisfies the configuration space decoherence condition, what can we infer about the evolution of the Bohmian configuration? (For purposes of illustration, we choose here to work with the discrete time expression for the wave function, rather than the continuous time one.)

An Alternative Approach: Part 2, the Configuration

The wave function components $|\Psi_{q_0,p_0}(N\Delta t) >$ have disjoint supports for sufficiently different $(q_N, p_N; ...; q_0, p_0)$. Denote the region of support by the subset $SE_{q_N,p_N;...;q_0,p_0} \subset \mathbb{Q}_{SE}$ of the total system's configuration space:

$$SE_{q_N,p_N;\ldots;q_0,p_0} \equiv \mathrm{supp}_\epsilon[\Psi_{q_0,p_0}(X, y, N\Delta t)] \subset \mathbb{Q}_{SE}, \tag{19}$$

where

$$\Psi_{q_0,p_0}(X, y, N\Delta t) \equiv < X, y|\Psi_{q_0,p_0}(N\Delta t) > . \tag{20}$$

The disjointness of these regions can be expressed as the condition

$$SE_{q_N,p_N;\ldots;q_0,p_0} \cap SE_{q'_N,p'_N;\ldots;q'_0,p'_0} = \varnothing \tag{21}$$

if (q_i, p_i) and (q'_i, p'_i) are sufficiently different for any $0 \le i \le N$ (where, again, (q_i, p_i) and (q'_i, p'_i) will only differ substantially if (q_0, p_0) and (q'_0, p'_0) do.)

An Alternative Approach: Part 2, the Configuration

Let us now assume that if q_{SE} begins in one of the disjoint packets, then it will remain in that packet as long as the future evolutions of the different packets remain disjoint. That is, if $q_{SE}(t=0)$ is in the support of $\Psi_{q_0,p_0}(X,y,t=0)$, then it will be in the support of $\Psi_{q_0,p_0}(X,y,N\Delta t)$ for all $N>0$, for some fixed (q_0,p_0). This claim can be formulated more precisely as follows:

> **Conjecture:** *Assume that the wave function of SE takes the form given in (15) and satisfies the associated configuration space decoherence requirement at all times* $N\Delta t$. *If* $q_{SE}(t=0)\in SE_{q_0,p_0}$, *then* $q_{SE}(N\Delta t)\in SE_{q_N,p_N;\ldots;q_0,p_0}$ *for all* $N>0$ *and for some fixed* (q_0,p_0).

Note: "For all $N>0$" means for all $N>0$ such that wave packet spreading can be ignored.

An Alternative Approach: Part 2, the Configuration

Note that disjointness of wave packet supports for sufficiently different $(\mathbf{q}, \mathbf{p})$ does not obviously guarantee that the configuration q_{SE} is guided by only a single wave packet.

In general in pilot wave theory, when the wave function is expanded in terms of disjoint wave packets that are **discretely** indexed, there are regions between packets where $|\Psi| \approx 0$ and through which the configuration therefore cannot pass. For this reason, we are assured that the configuration does not drift between packets.

However, in cases where the disjoint wave packets are **continuously** indexed, there are not necessarily regions where $|\Psi| \approx 0$ between disjoint wave packets. Thus, the conventional arguments for effective collapse do not carry over to the continuous case, and the guidance equation must be invoked in order to prove that the configuration does not drift between disjoint packets.

(See blackboard for illustration)

An Alternative Approach: Part 2, the Configuration

Define

$$E_{q_N,p_N;\ldots;q_0,p_0} \equiv \mathrm{supp}_\epsilon[< y|\phi(q_N,p_N;\ldots;q_0,p_0) >] \subset \mathbb{Q}_E, \qquad (22)$$

and

$$S_{q_N,p_N} \equiv \mathrm{supp}_\epsilon[< X|q(N\Delta t),p(N\Delta t) >_{q_0,p_0}] \subset \mathbb{Q}_S. \qquad (23)$$

Then,

$$SE_{q_N,p_N;\ldots;q_0,p_0} = S_{q_N,p_N} \times E_{q_N,p_N;\ldots;q_0,p_0}. \qquad (24)$$

An Alternative Approach: Part 2, the Configuration

Since $q_{SE} = (Q_S, q_E)$, it follows from the fact that

$$q_{SE}(N\Delta t) \in SE_{q_N,p_N;\ldots;q_0,p_0} \tag{25}$$

that

$$Q_S(N\Delta t) \in S_{q_N,p_N} \tag{26}$$

and

$$q_E(N\Delta t) \in E_{q_N,p_N;\ldots;q_0,p_0} \tag{27}$$

for all admissible N.

An Alternative Approach: Part 2, the Configuration

By Ehrenfest's Theorem, S_{q_N,p_N} follows a classical trajectory as time evolves - that is, as N increases. Since the Bohmian configuration Q_S of the macroscopic degrees of freedom lies in this region for all N, it too follows an approximately classical trajectory.

An Alternative Approach: Part 2, the Configuration

- The Bohmian configuration q_E of the environment lies in the region $E_{q_N,p_N;\ldots;q_0,p_0}$ at time $N\Delta t$, and this region in turn is associated through decoherence to the entire past trajectory of the wave packet $|q(N\Delta t), p(N\Delta t)>_{q_0,p_0}$, and therefore also to the trajectory of Q_S, which lies in the support of that wave packet.
- Thus, knowledge of the configuration of q_E at any time enables us to infer Q_S's past trajectory, to within an error determined by the configuration space width of the wave packet $< X|q(N\Delta t), p(N\Delta t)>_{q_0,p_0}$.

Conclusion: Summary

- When we examine the structure of the wave function under decoherence, we find that, for certain systems, the total wave function of SE is a superposition of states, each of which describes a classically evolving wave packet in S, tensor producted with a state for the environment which becomes correlated to the past trajectory of the wave packet.
- A natural but unproven conjecture suggests that, by the usual Bohmian effective collapse mechanism, the Bohmian configuration will follow just one of these packets.
- When we focus on the macroscopic degrees of freedom, we find that the Bohmian configuration evolves classically.
- When we focus on the environmental degrees of freedom, we find that the environmental configuration is correlated to the past history of the macroscopic degrees of freedom.

Conclusion: Loose Ends

- Future work should show that the special kind of decoherence required by pilot wave theory - decoherence with respect to configuration space - does in fact occur.
- Future work should also show that, for continuously indexed pointer bases, the presence of configuration space decoherence ensures that the configuration stays in the same packet.
- The account should be generalized to include cases in which branching due to chaos cannot be ignored.

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