


21st-century directions in
de Broglie-Bohm theory and beyond

De Broglie – Bohm analysis of entangled qubit pairs

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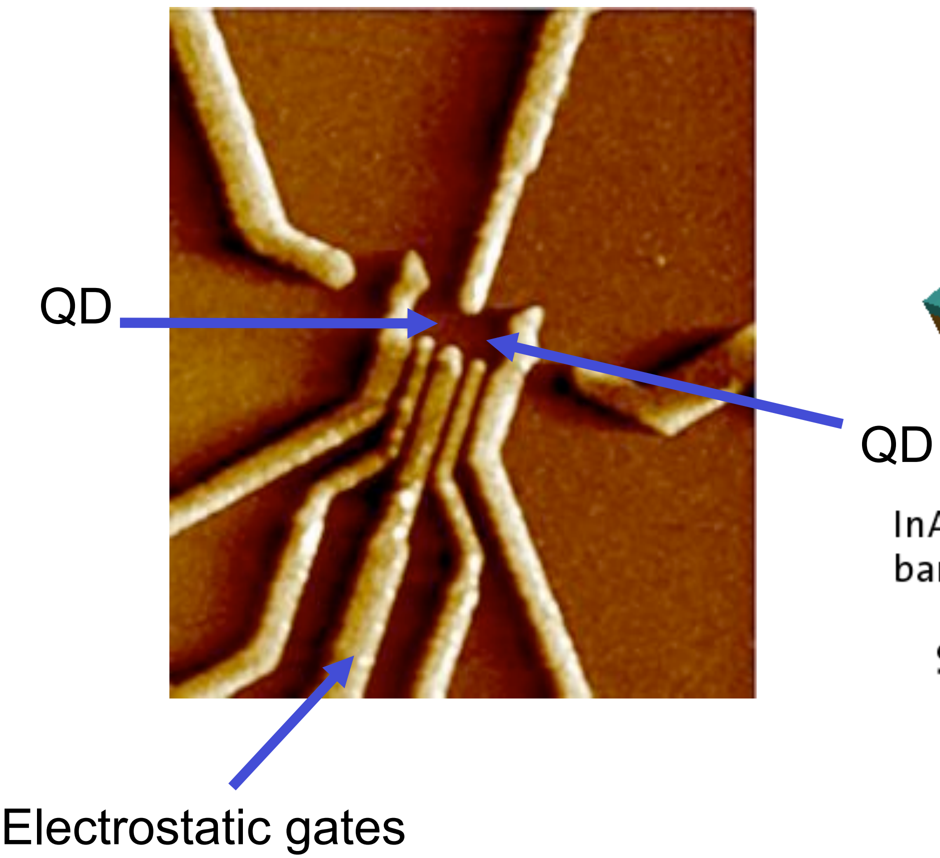
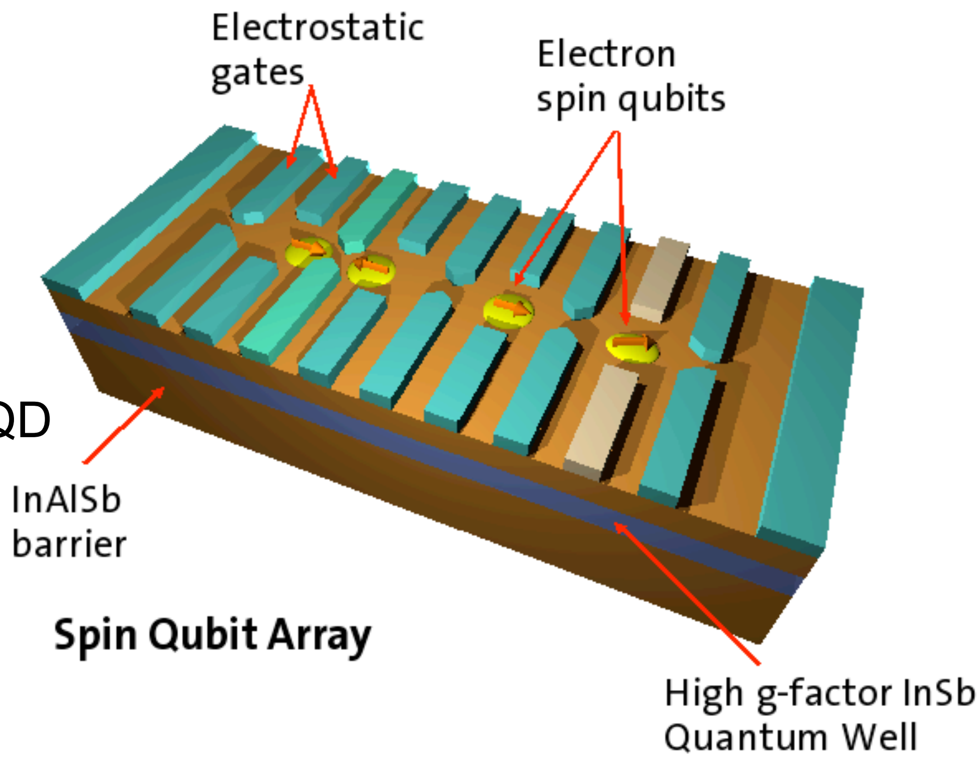
Outline

(1) entangled electrons in double quantum dots
(2) rigid rotator
(3) dBB formalism for a particle with spin ½
(4) two electrons: spin – spin correlations
(5) time dependent interaction and entanglement generation
(5) summary

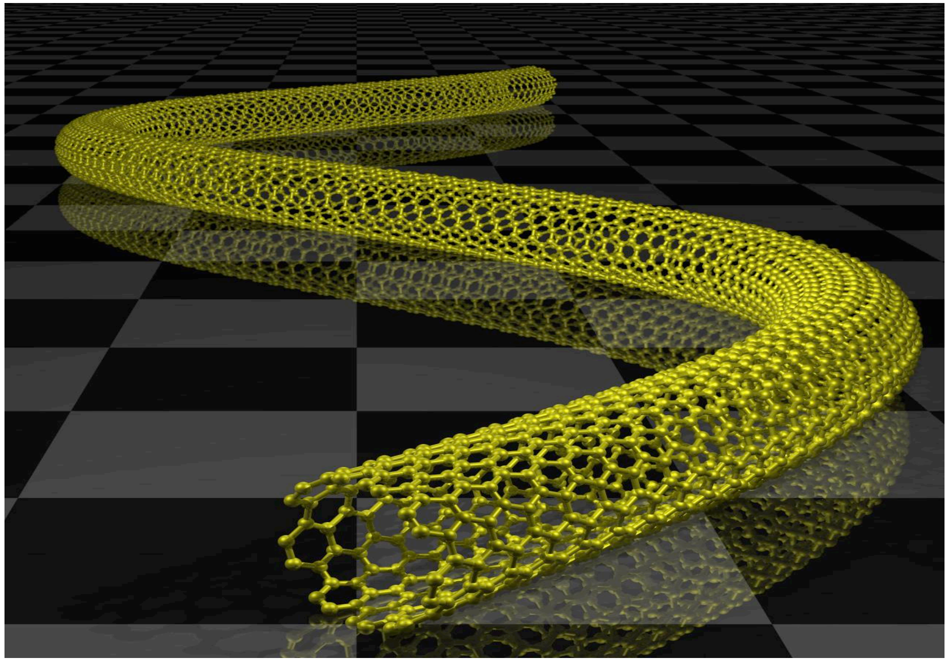
qubit pairs in double quantum dots

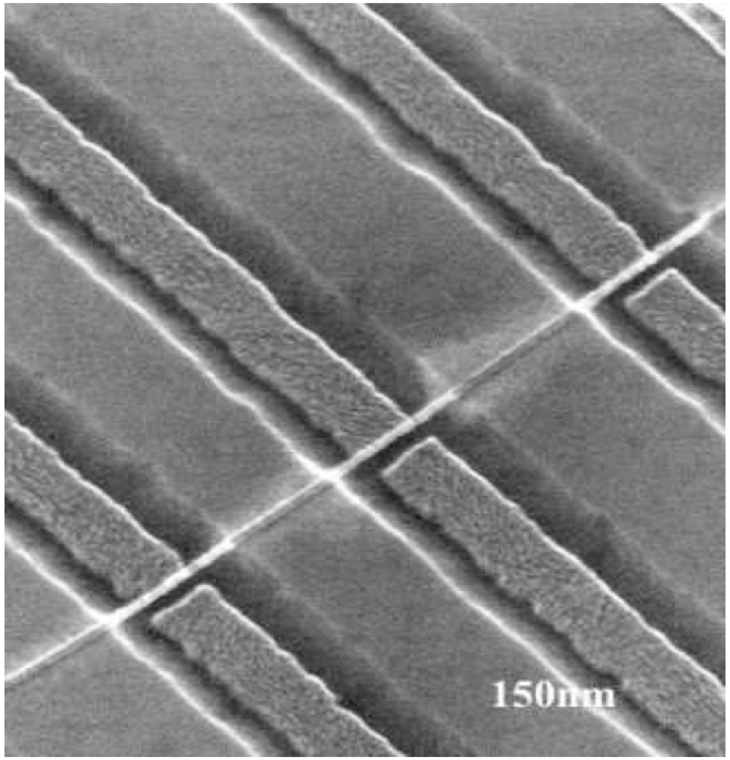
Elzerman et al, PRB **67**, 16308 (2003)

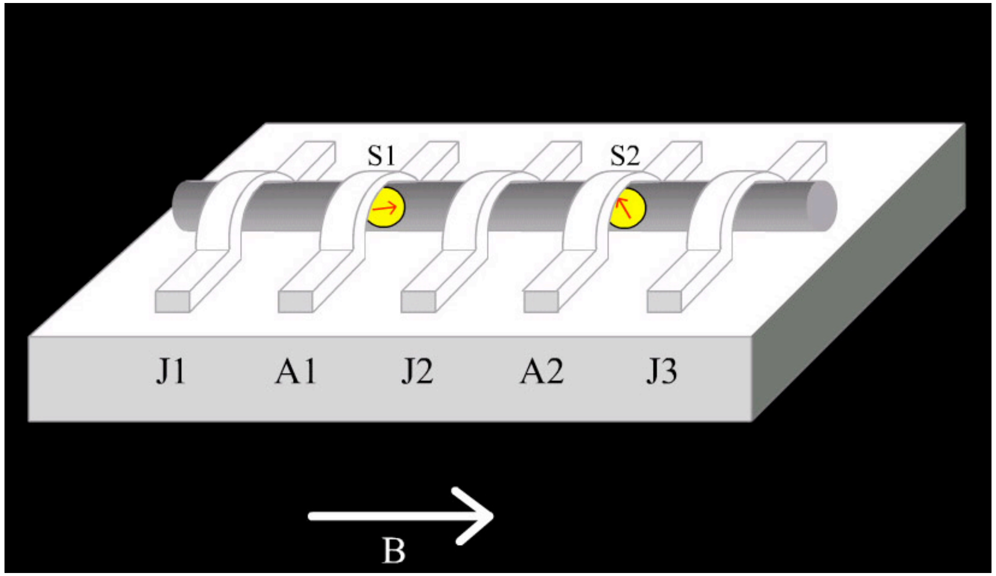




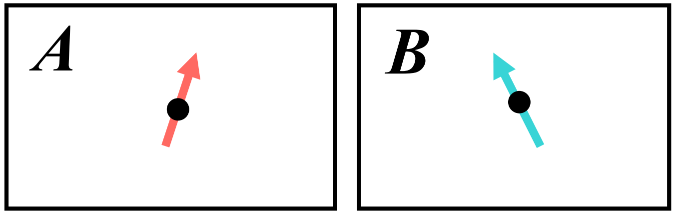
Motivation: entanglement on demand







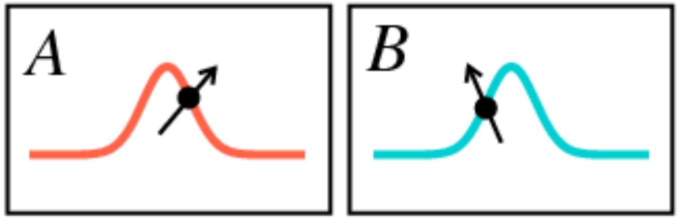
Entanglement measure for two d e l o c a l i s e d electrons



two ‘spins’ $$|\Psi_{AB}\rangle = \sum_{ss'} \alpha_{ss'} |s\rangle_A |s'\rangle_B$$
$$C = 2|\alpha_{\uparrow\uparrow}\alpha_{\downarrow\downarrow} - \alpha_{\uparrow\downarrow}\alpha_{\downarrow\uparrow}|.$$

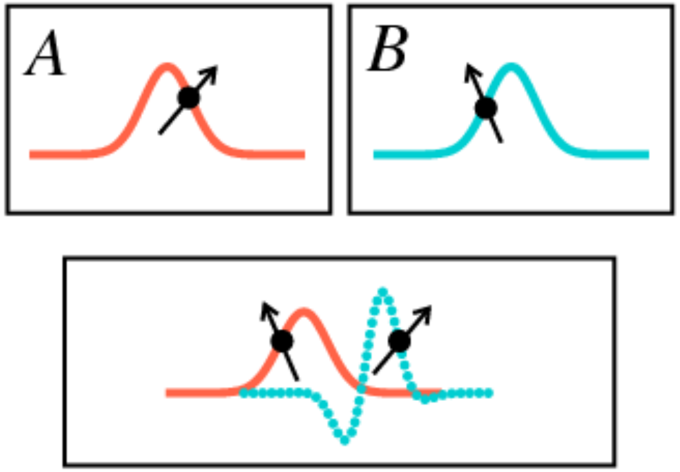
C.H. Bennett *et al*., Phys. Rev. A **54**, 3824 (1996);
S. Hill and W.K. Wootters, PRL **78**, 5022 (1997).

Entanglement measure for two d e l o c a l i s e d electrons



initial state

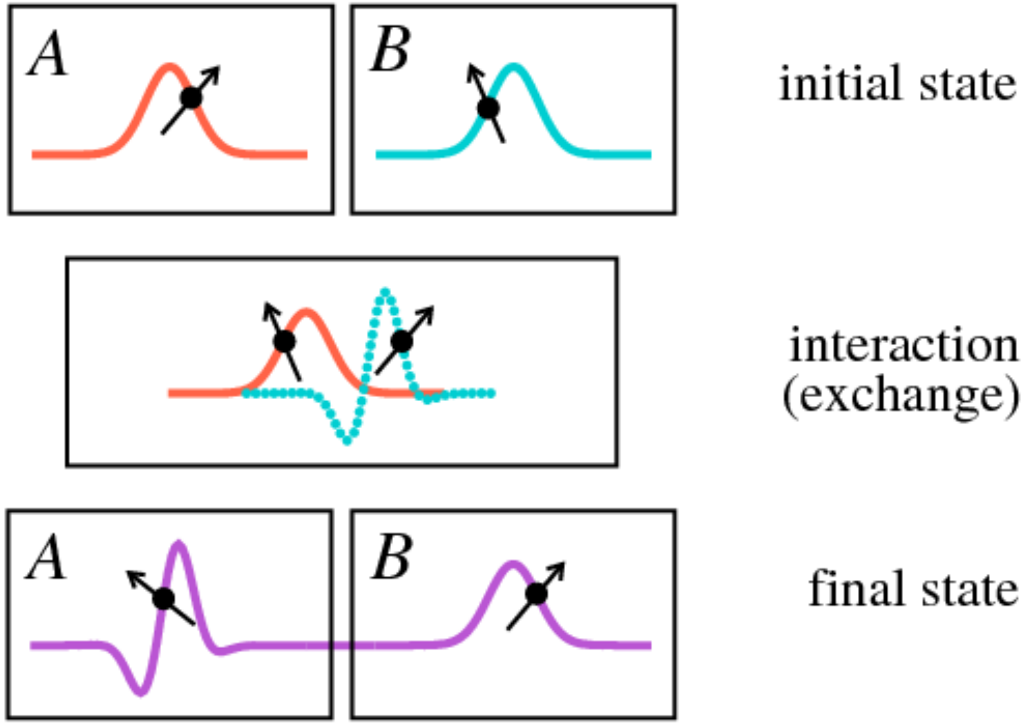
Entanglement measure for two d e l o c a l i s e d electrons



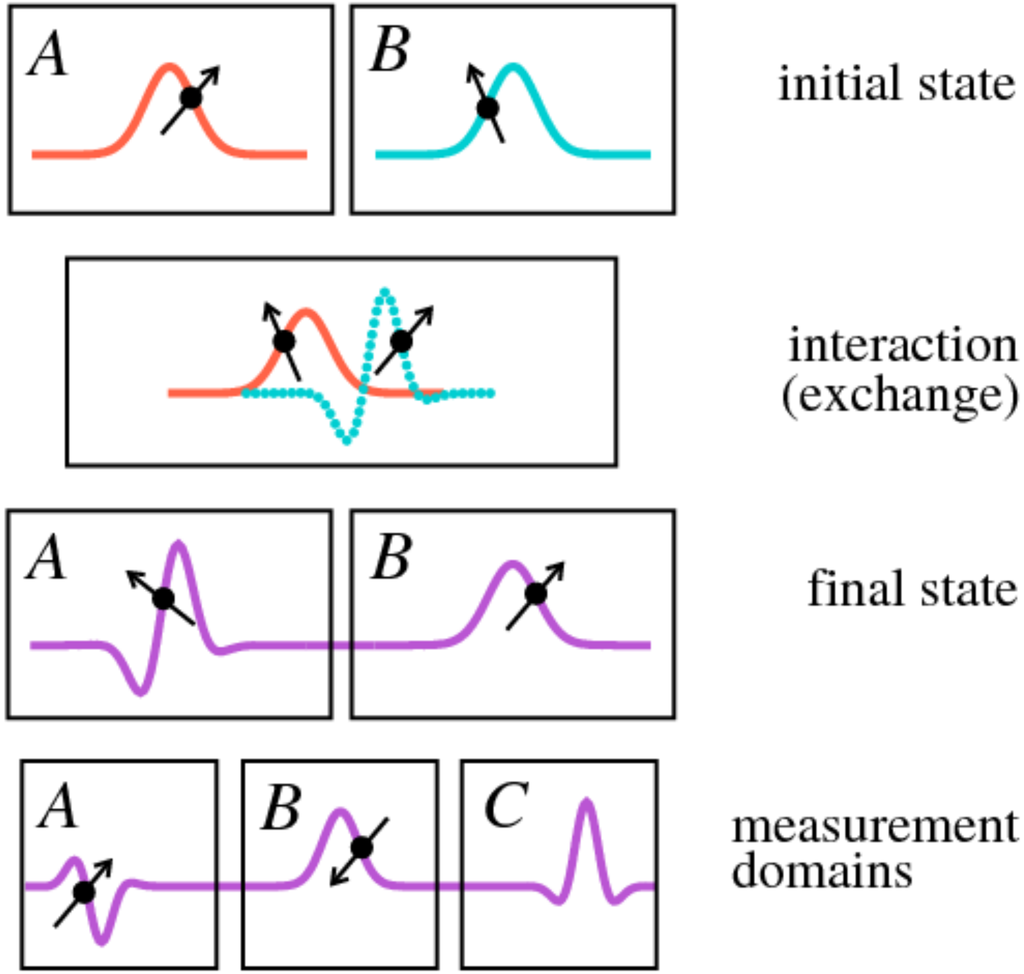
initial state

interaction
(exchange)

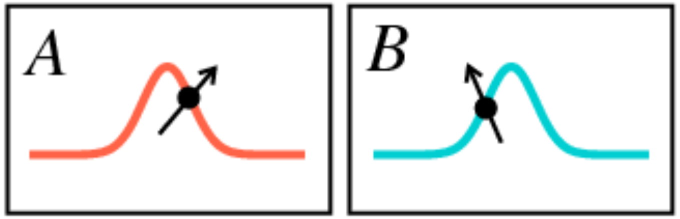
Entanglement measure for two d e l o c a l i s e d electrons



Entanglement measure for two d e l o c a l i s e d electrons



Entanglement measure for two d e l o c a l i s e d electrons



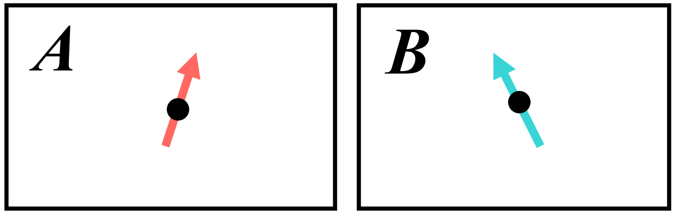
$$C = 2|\alpha_{\uparrow\uparrow}\alpha_{\downarrow\downarrow} - \alpha_{\uparrow\downarrow}\alpha_{\downarrow\uparrow}|.$$

$$|\Psi\rangle = \sum_{i,j=1}^{N}[\psi_{ij}^{\uparrow\downarrow} c_{i\uparrow}^{\dagger} c_{j\downarrow}^{\dagger} + \frac{1}{2}(\psi_{ij}^{\uparrow\uparrow} c_{i\uparrow}^{\dagger} c_{j\uparrow}^{\dagger} + \psi_{ij}^{\downarrow\downarrow} c_{i\downarrow}^{\dagger} c_{j\downarrow}^{\dagger})]|0\rangle,$$

$$\boxed{\begin{aligned}
C &= \max(0, C_{\uparrow\downarrow}, C_{||}),\\
C_{\uparrow\downarrow} &= 2|\langle S_A^+ S_B^- \rangle| - 2\sqrt{\langle P_A^\uparrow P_B^\uparrow\rangle\langle P_A^\downarrow P_B^\downarrow\rangle},\\
C_{||} &= 2|\langle S_A^+ S_B^+ \rangle| - 2\langle P_A^\uparrow P_B^\downarrow\rangle,\\
\langle S_A^+ S_B^- \rangle &= \sum_{[ij]} \psi_{ij}^{\uparrow\downarrow *}\psi_{ji}^{\uparrow\downarrow},\\
\langle S_A^+ S_B^+ \rangle &= \sum_{[ij]} \psi_{ij}^{\uparrow\uparrow *}\psi_{ij}^{\downarrow\downarrow},\\
\langle P_A^s P_B^{s'} \rangle &= \sum_{[ij]} |\psi_{ij}^{ss'}|^2,
\end{aligned}}$$

A. Ramšak, I. Sega, and J.H. Jefferson PRA **74**, 010304(R) (2006)

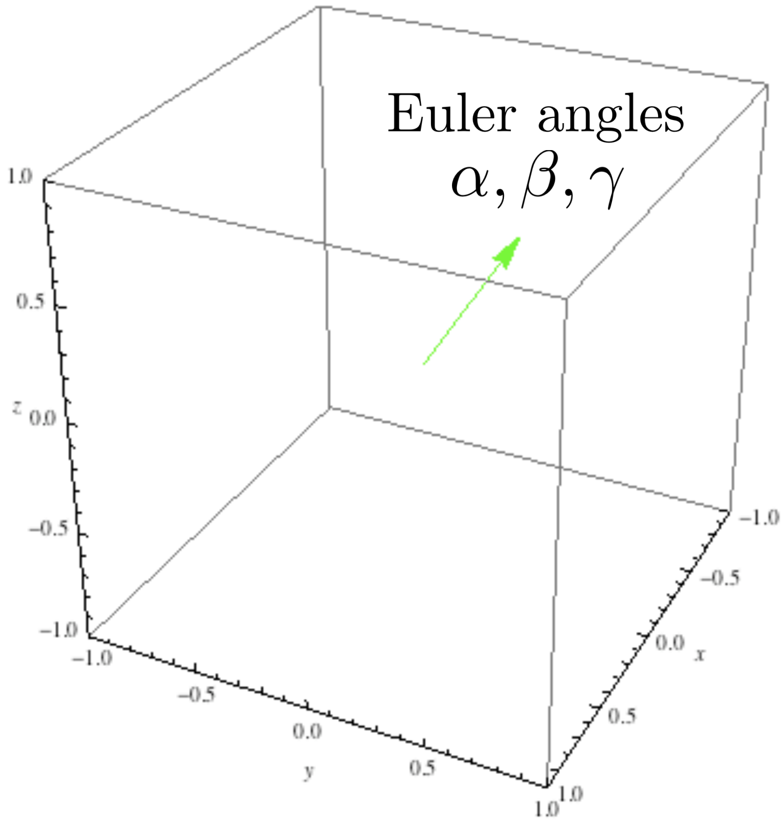
Conserved S_z case



$$C_{\uparrow\downarrow} = 2|\langle S_A^+ S_B^- \rangle|$$

Rigid rotator

P. R. Holland. Phys. Rep. 169, 293 (1988) & The Quantum Theory … book

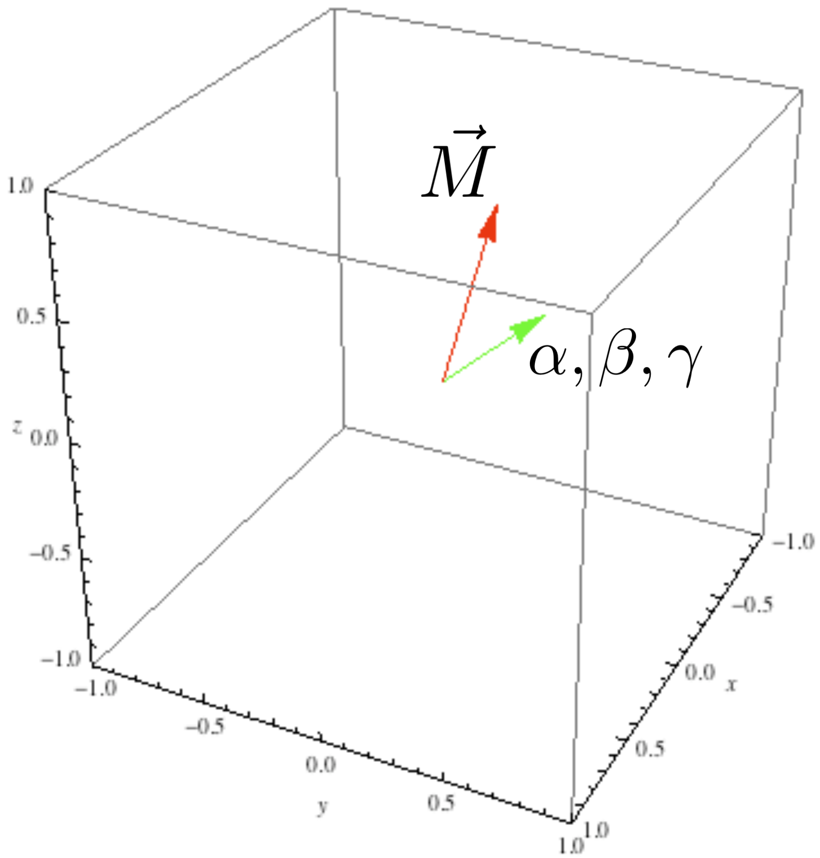


$$\psi = \psi(\alpha, \beta, \gamma)$$

$$\hat{\mathbf{M}}^2 \psi_{lm} = l(l+1)\psi_{lm}$$

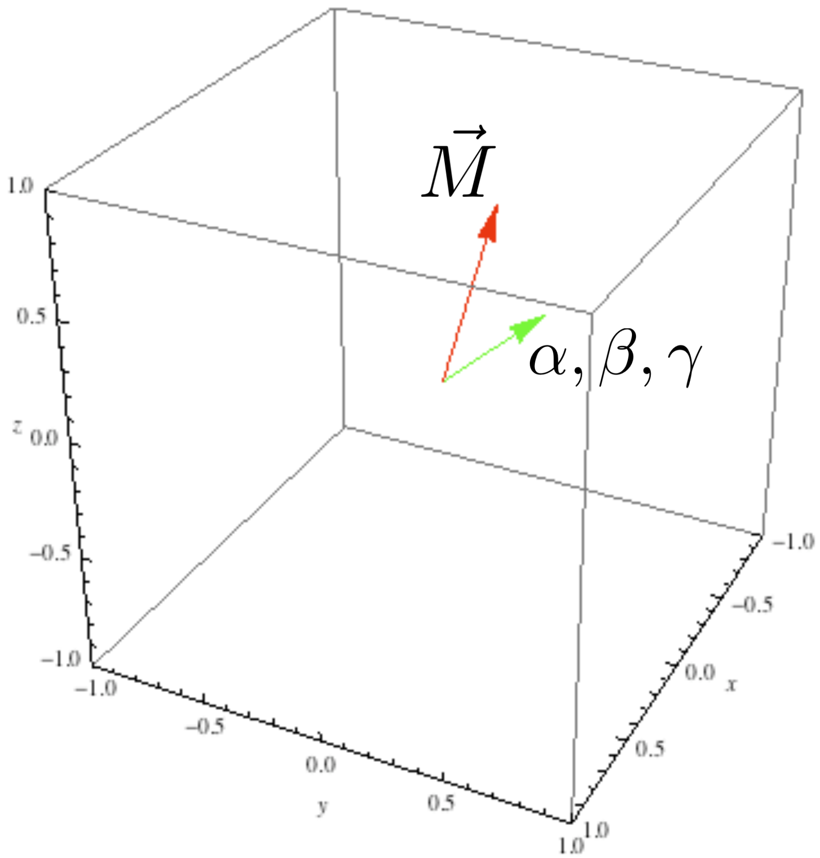
$$\hat{M}_z \psi_{lm} = m\psi_{lm}$$

$$\hat{H} = \frac{\hat{\mathbf{M}}^2}{2I}$$



$$\vec{M} = \vec{M}(\alpha\beta\gamma) = I\vec{\omega}$$

$$H = \frac{1}{2}I\vec{\omega}^2$$



$$\vec{M} = \vec{M}(\alpha\beta\gamma) = I\vec{\omega}$$

$$H = \frac{1}{2} I\vec{\omega}^2 + Q_s$$

$$Q_s = -\frac{1}{2I} \frac{\hat{\mathbf{M}}^2 R}{R}$$

$$\psi(\alpha, \beta, \gamma) = Re^{iS}$$

De Broglie – Bohm

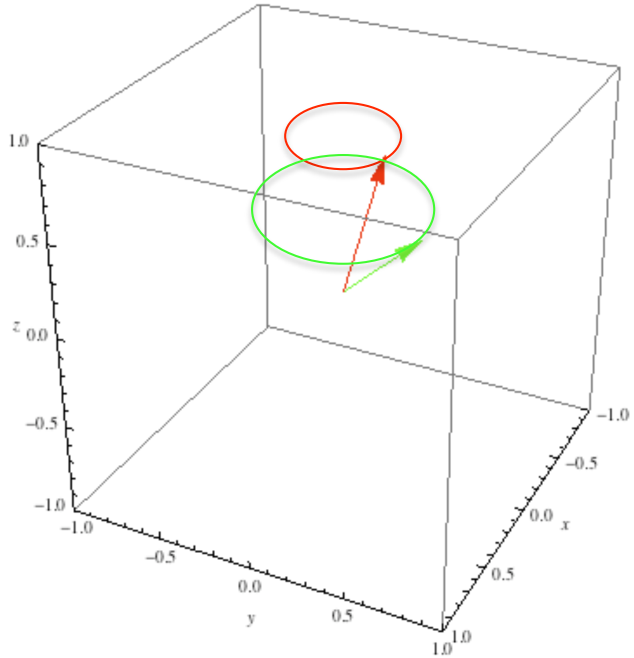
$$\vec{\omega} = \frac{1}{I}\hat{\mathbf{M}}S$$

$$\vec{M}(\alpha\beta\gamma) = I\vec{\omega}$$

$$\begin{aligned}
M_x &= -\cos\beta \;\; \partial S/\partial\alpha + \sin\beta \;\; \cot\alpha \;\; \partial S/\partial\beta - \sin\beta \;\; \mathrm{cosec}\;\alpha \;\; \partial S/\partial\gamma \\
M_y &= \sin\beta \;\; \partial S/\partial\alpha + \cos\beta \;\; \cot\alpha \;\; \partial S/\partial\beta - \cos\beta \;\; \mathrm{cosec}\;\alpha \;\; \partial S/\partial\gamma \\
M_z &= -\partial S/\partial\beta
\end{aligned}$$

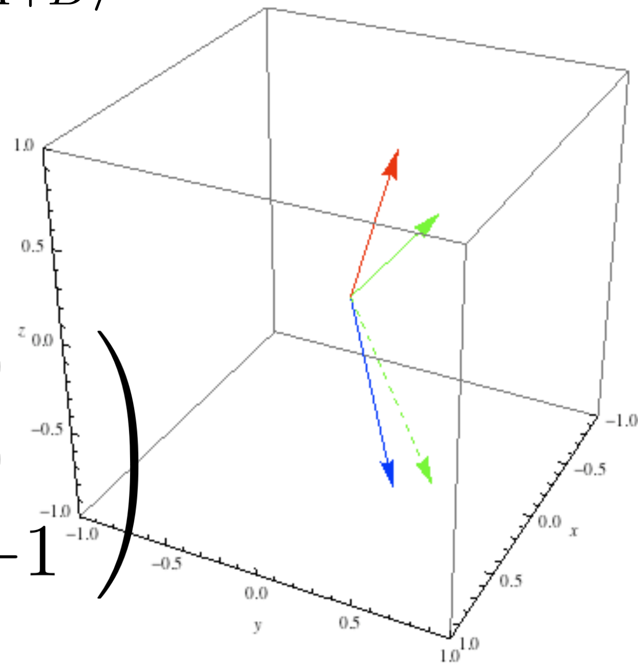
$$l = \tfrac{1}{2}$$

$$\begin{aligned}
\psi_{\frac{1}{2}\frac{1}{2}} &= u_+ \propto \cos\frac{\alpha}{2}e^{-i(\beta+\gamma)/2} &\sim& \quad \langle\alpha\beta\gamma|\uparrow\rangle \\
\psi_{\frac{1}{2},-\frac{1}{2}} &= u_- \propto \sin\frac{\alpha}{2}e^{-i(\beta-\gamma)/2} &\sim& \quad \langle\alpha\beta\gamma|\downarrow\rangle
\end{aligned}$$



De Broglie – Bohm: two spin ½ particles $(S_z = 0 \text{ case})$

$$|\Psi_{AB}\rangle = \cos\frac{\vartheta}{2}|\uparrow_A\downarrow_B\rangle + e^{i\varphi}\sin\frac{\vartheta}{2}|\downarrow_A\uparrow_B\rangle$$



standard QM:

$$\hat{M}_i = \frac{1}{2}\sigma_i, \quad i = x, y, z$$

$$\langle\sigma_i\sigma_j\rangle = \begin{pmatrix} \sin\vartheta\cos\varphi & \sin\vartheta\sin\varphi & 0 \\ -\sin\vartheta\sin\varphi & \sin\vartheta\cos\varphi & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

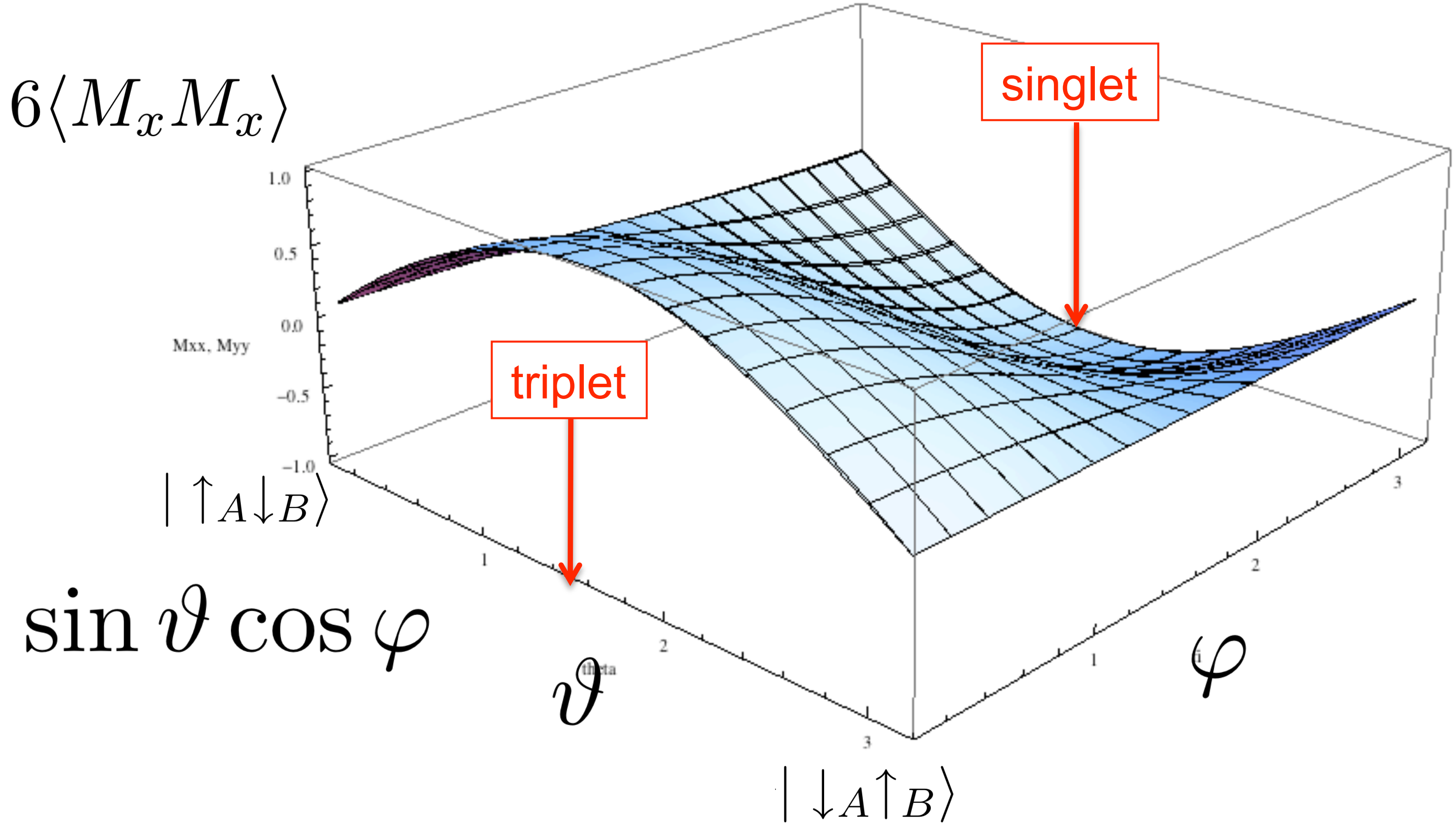
dBB:

$$\langle ...\rangle = \frac{\int\int\int\int\int\int ...|\psi|^2 \sin\alpha_A \sin\alpha_B d\alpha_A d\alpha_B d\beta_A d\beta_B d\gamma_A d\gamma_B}{\int\int\int\int\int\int |\psi|^2 \sin\alpha_A \sin\alpha_B d\alpha_A d\alpha_B d\beta_A d\beta_B d\gamma_A d\gamma_B}$$

$$\boxed{\langle M_i M_j\rangle = ?}$$

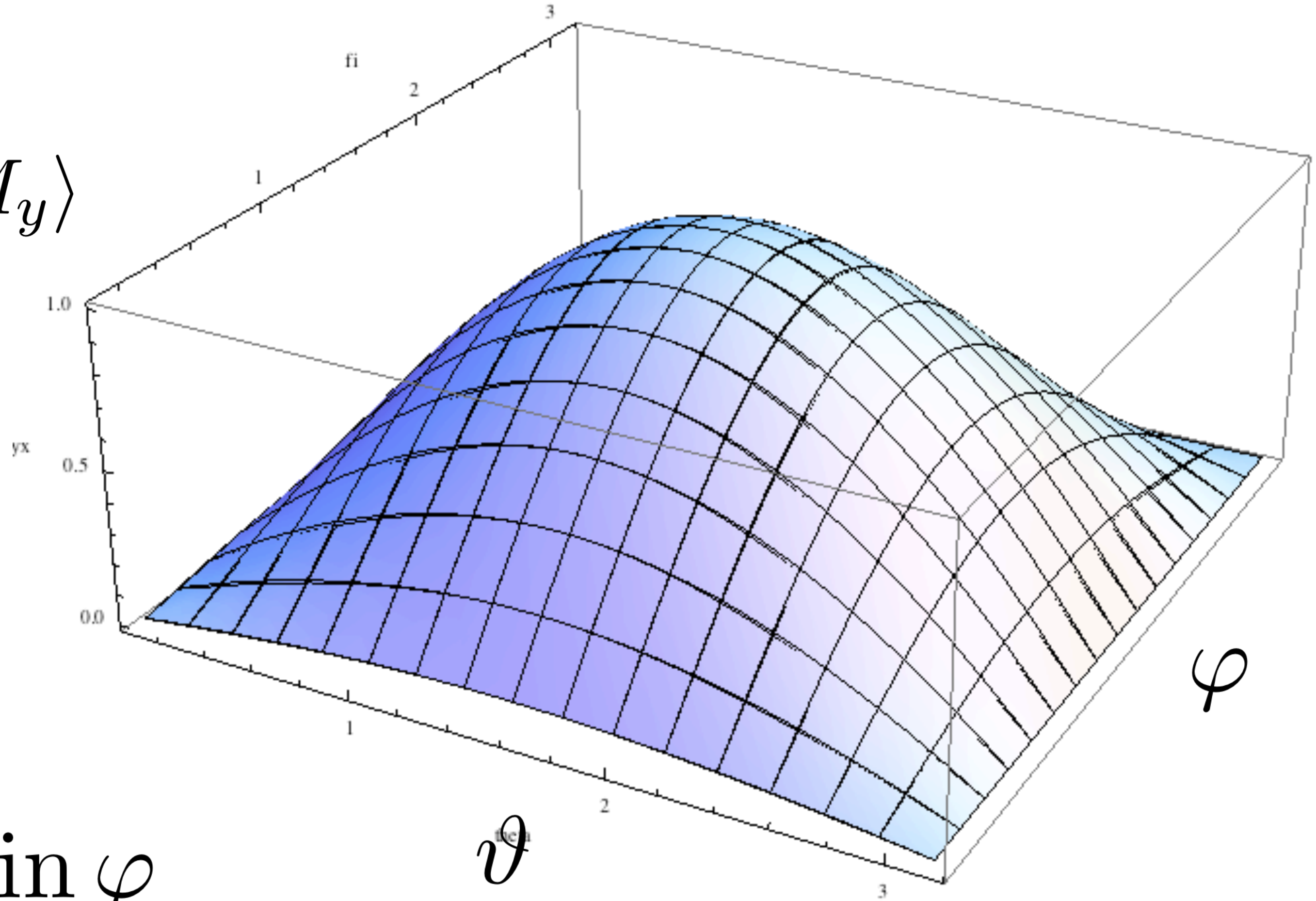
Results: static qubits

$$|\Psi_{AB}\rangle = \cos\frac{\vartheta}{2}|\uparrow_A\downarrow_B\rangle + e^{i\varphi}\sin\frac{\vartheta}{2}|\downarrow_A\uparrow_B\rangle$$



Results

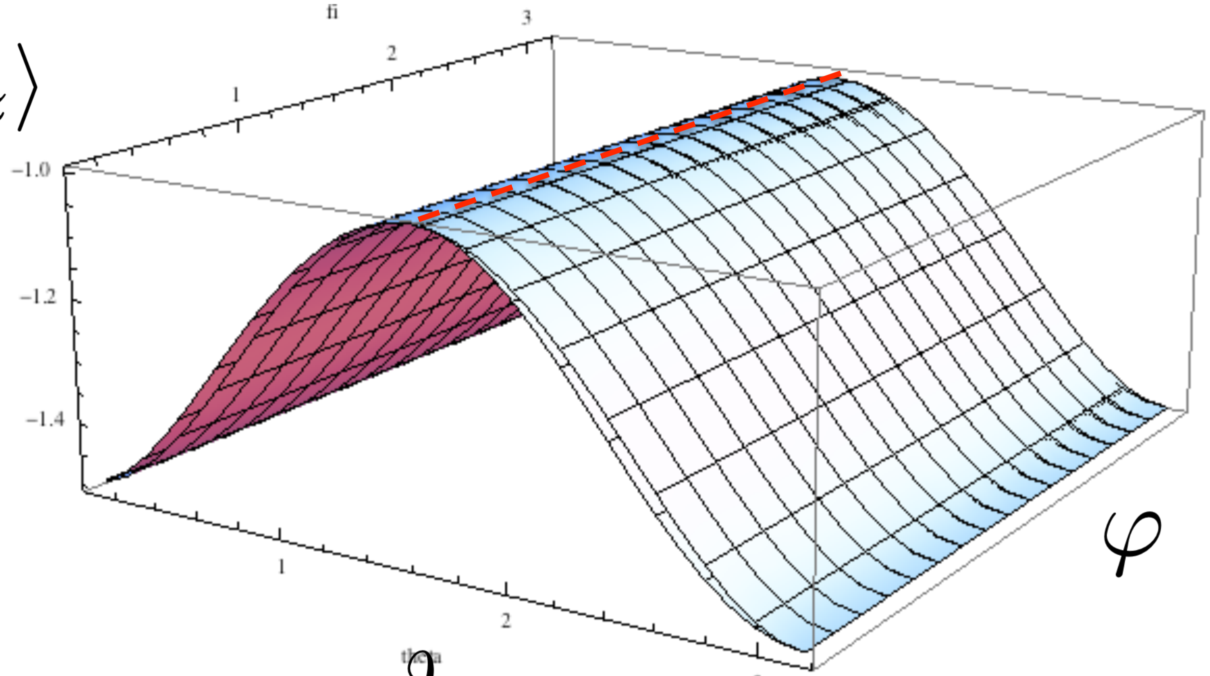
$6\langle M_x M_y \rangle$



$\sin\vartheta \sin\varphi$

Results

$6\langle M_z M_z \rangle$



ϑ

φ

-1

Results

$$
\begin{aligned}
\langle M_x M_x \rangle &\sim \frac{1}{6}\langle \sigma_x \sigma_x \rangle = \frac{2}{3}\langle \hat{M}_x \hat{M}_x \rangle \\
\langle M_y M_y \rangle &\sim \frac{1}{6}\langle \sigma_y \sigma_y \rangle = \frac{2}{3}\langle \hat{M}_y \hat{M}_y \rangle \\
\langle M_x M_y \rangle &\sim \frac{1}{6}\langle \sigma_x \sigma_y \rangle = \frac{2}{3}\langle \hat{M}_x \hat{M}_y \rangle \\
\langle M_x M_z \rangle &\sim \langle M_y M_z \rangle = \ldots = 0 \\
\langle M_z M_z \rangle &\sim -(5 + \cos 2\vartheta)/24 = \frac{1}{6}(5 + \cos 2\vartheta)\langle \hat{M}_z \hat{M}_z \rangle
\end{aligned}
$$

2/3 factor due to quantum torque and quantum potential

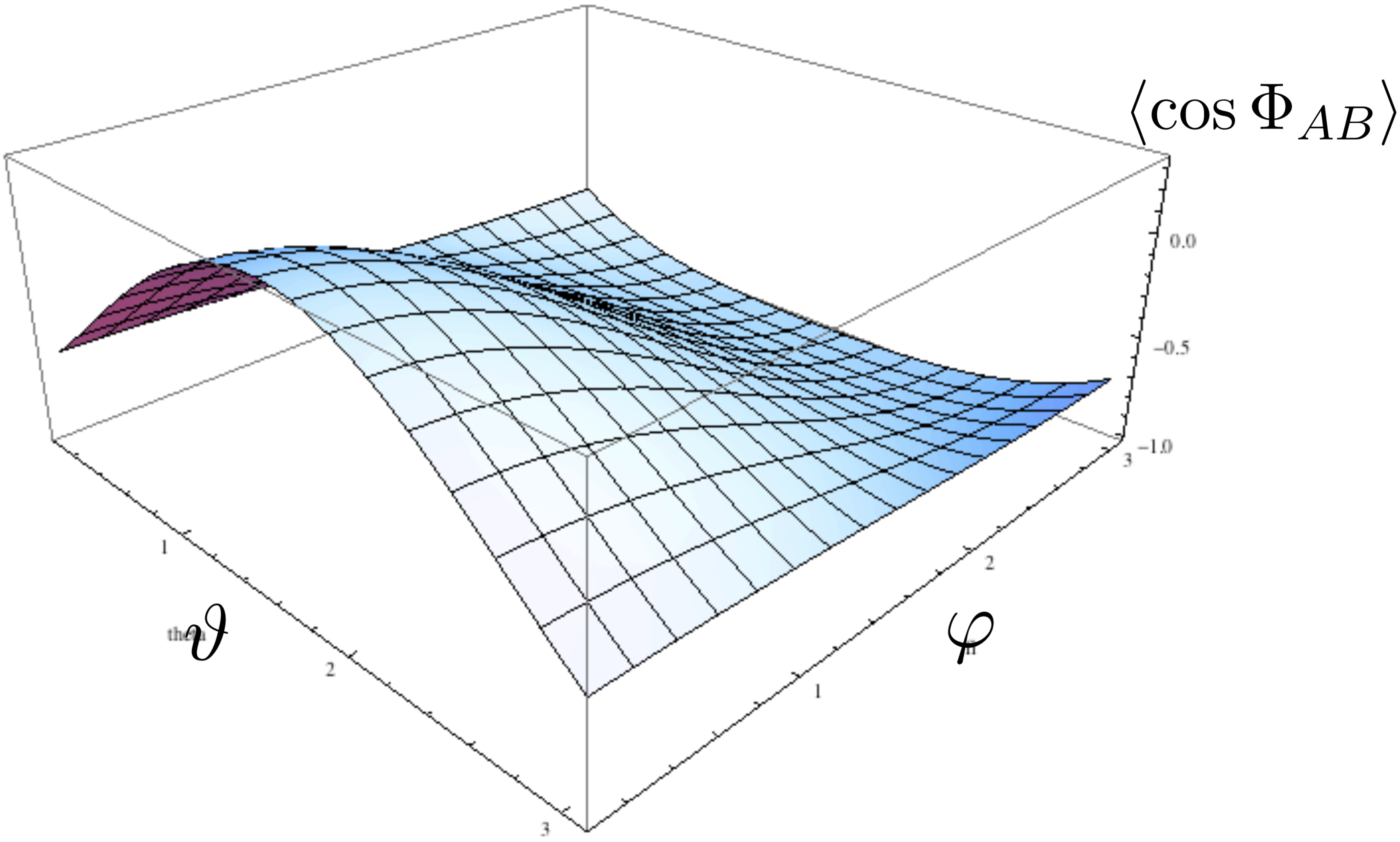
Example: 1 electron case

$$
H = \frac{1}{2} I \vec{\omega}^2 + Q_s = \frac{\vec{M}^2}{2I} + Q_s \qquad \hat{H} = \frac{\hat{\mathbf{M}}^2}{2I}
$$

$$
\boxed{\langle H \rangle \equiv \langle \hat{H} \rangle}
$$

Angles between dBB spins

$$\langle \cos \Phi_{AB} \rangle = \langle \frac{\vec{M}_A \cdot \vec{M}_B}{|\vec{M}_A||\vec{M}_B|} \rangle = \langle \frac{M_x M_x + M_y M_y + M_z M_z}{|\vec{M}_A||\vec{M}_B|} \rangle$$



Angles between dBB spins

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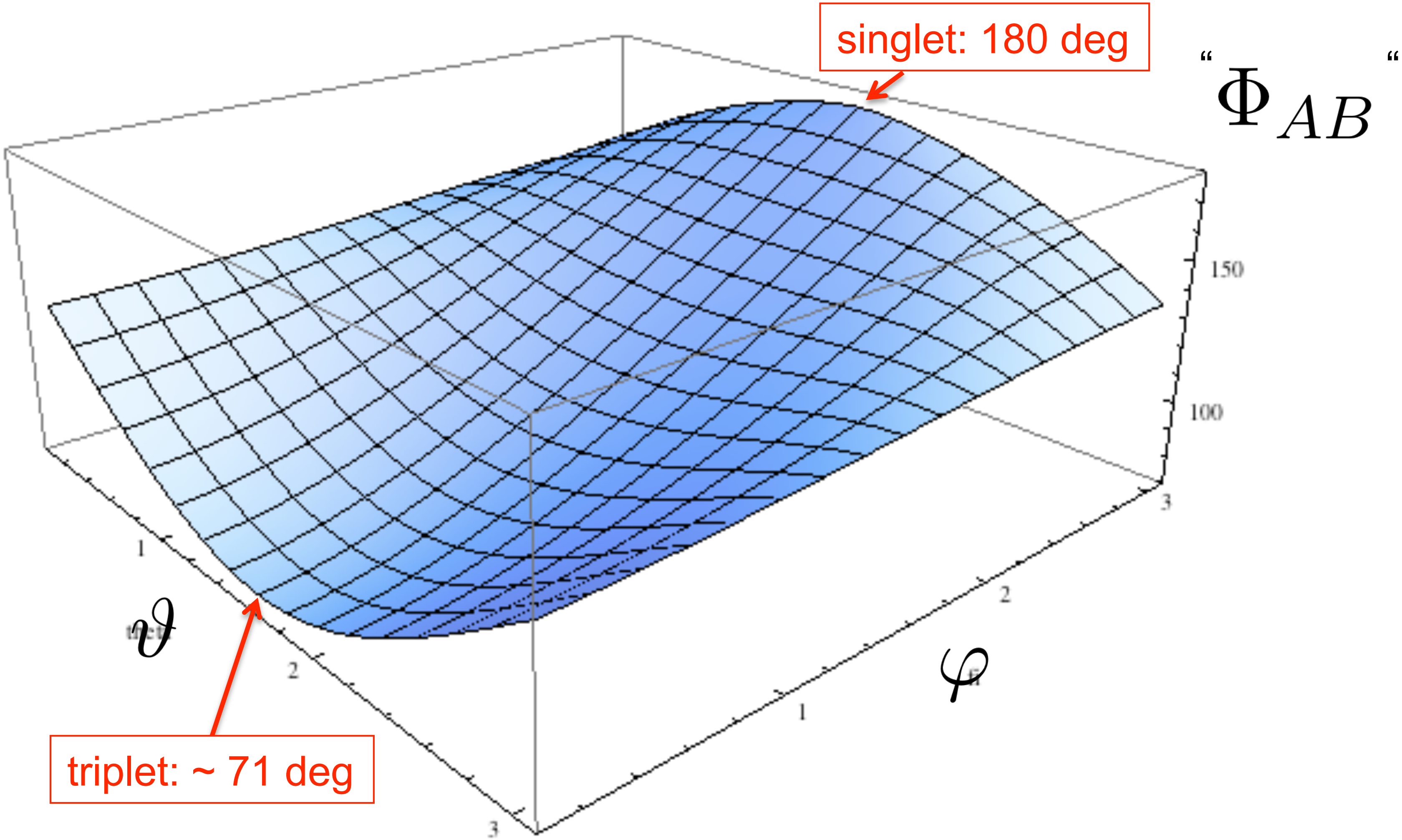
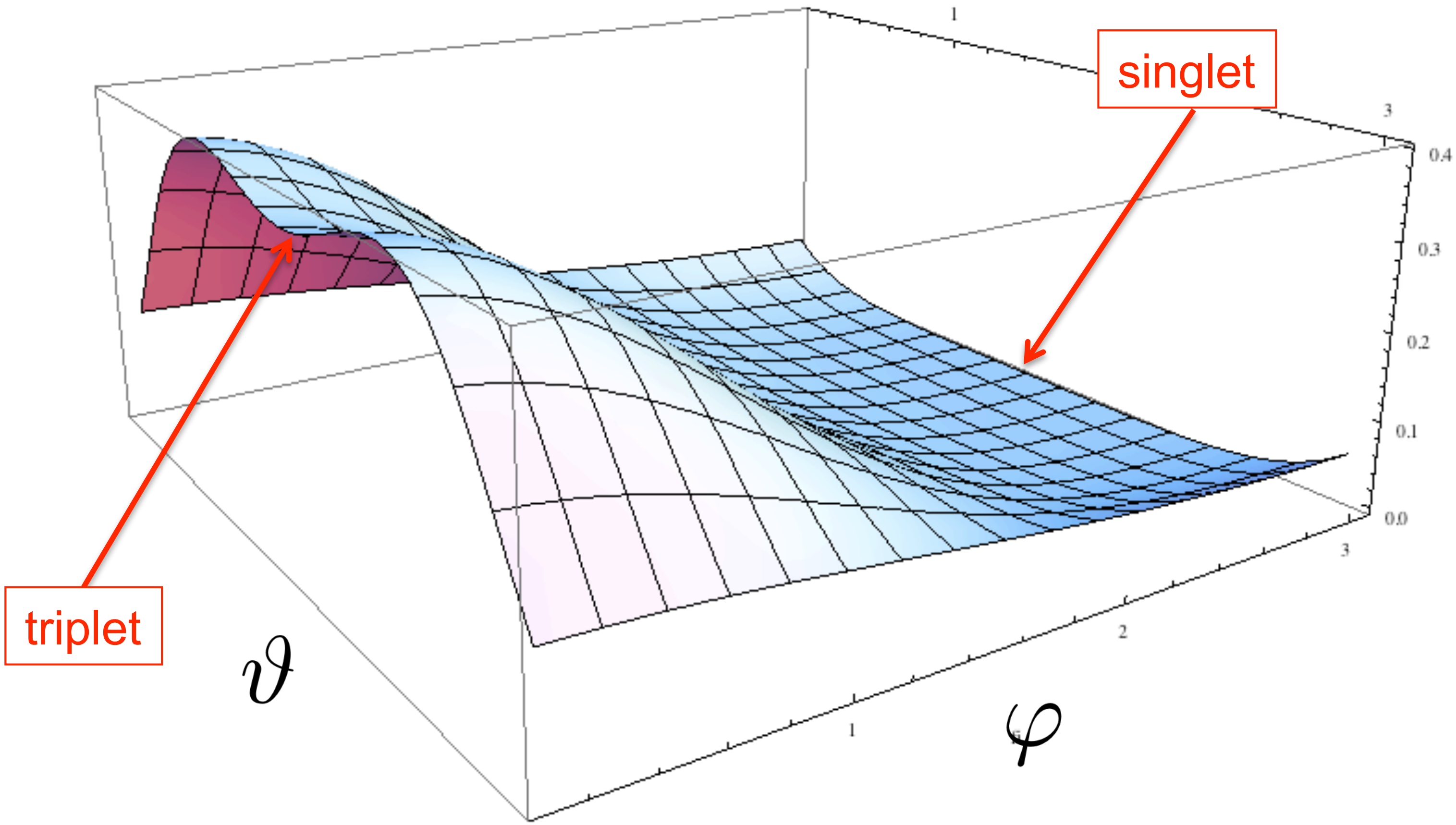


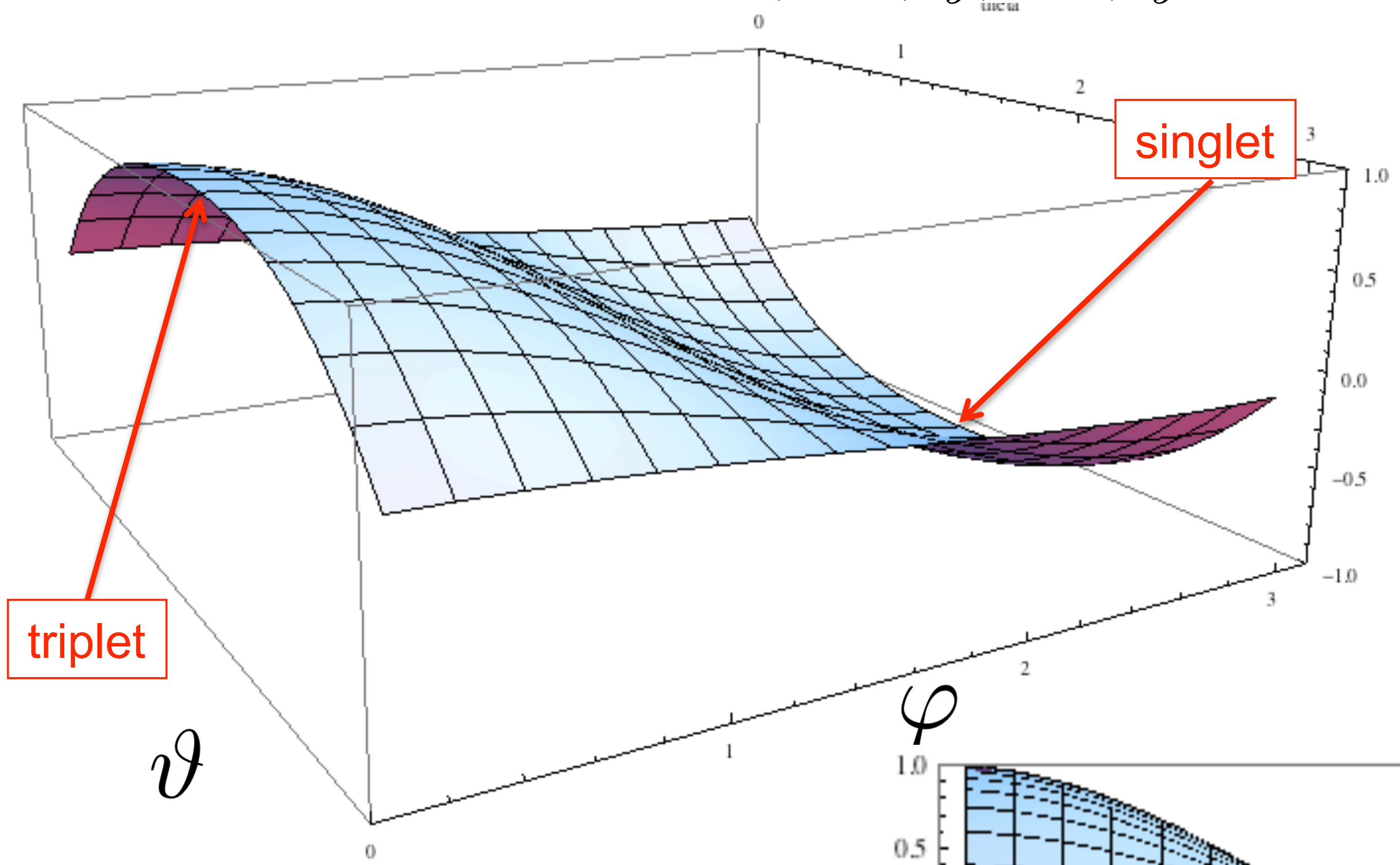
Fluctuations of angles between dBB spins

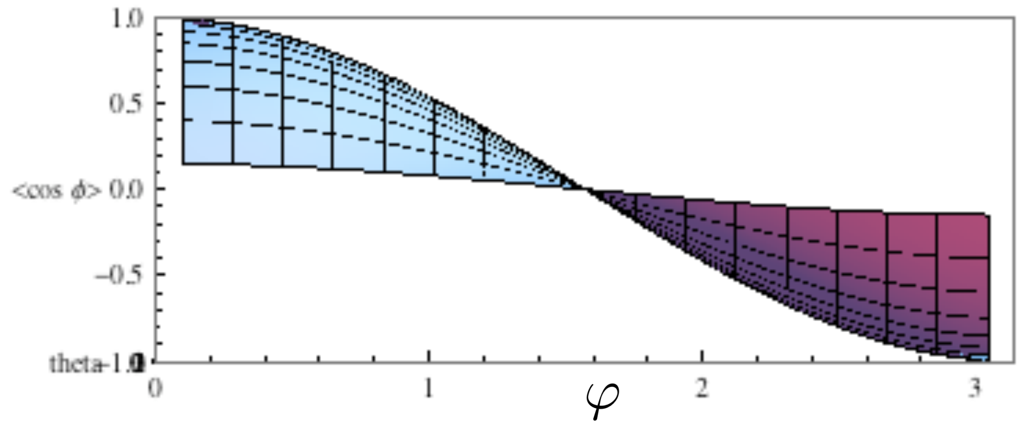
$$\Delta(\cos\Phi_{AB})^2 = \langle \cos\Phi_{AB}^2 \rangle - \langle \cos\Phi_{AB} \rangle^2$$



x-y plane angles between dBB spins

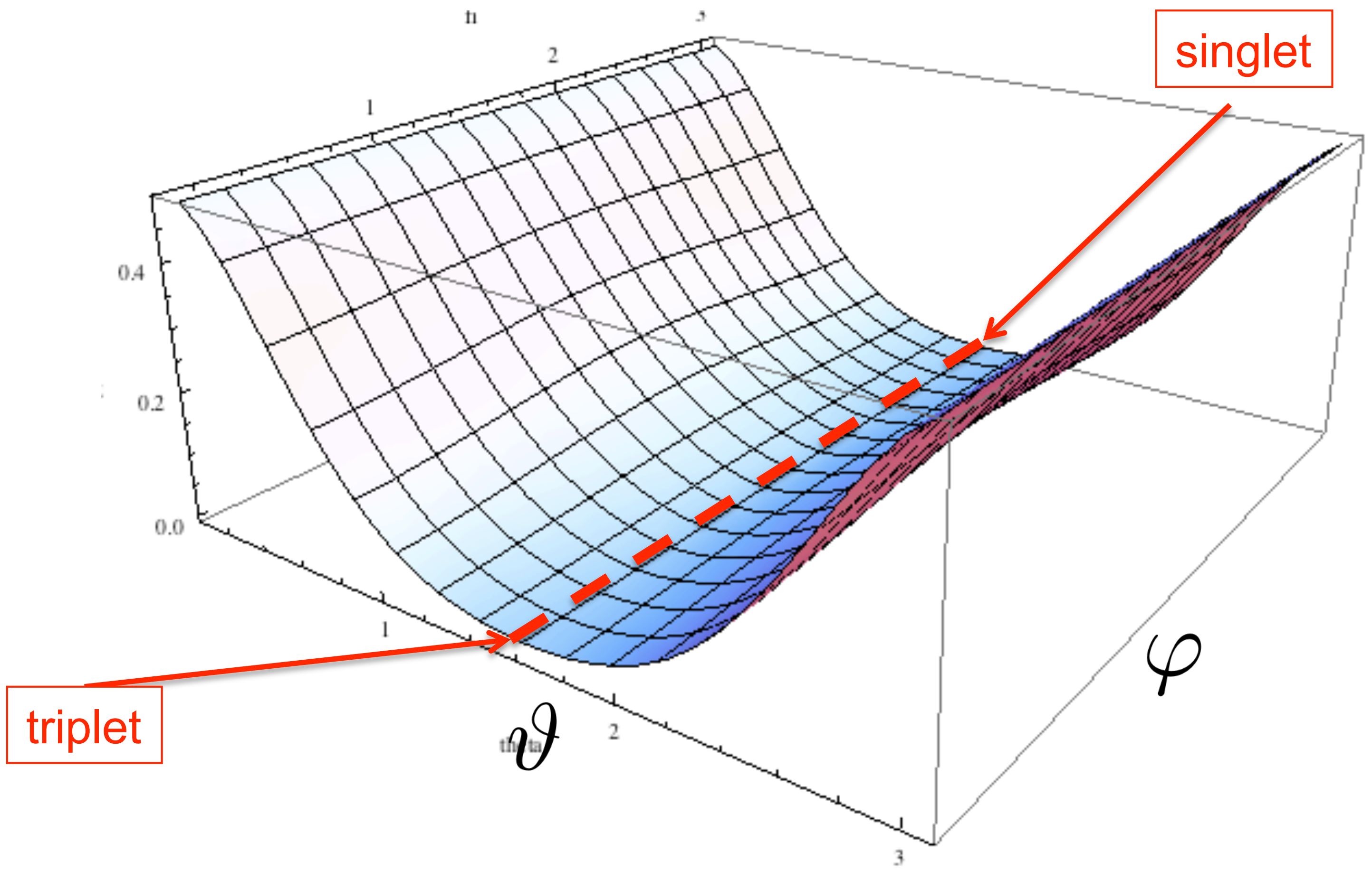
$$\langle \cos\phi_{AB}\rangle = \langle \frac{M_x M_x + M_y M_y}{|\vec{M}_A|_{xy}|\vec{M}_B|_{xy}}\rangle$$





Fluctuations of x-y plane angles between dBB spins

$$\Delta(\cos\phi_{AB})^2 = \langle \cos\phi_{AB}^2 \rangle - \langle \cos\phi_{AB} \rangle^2$$

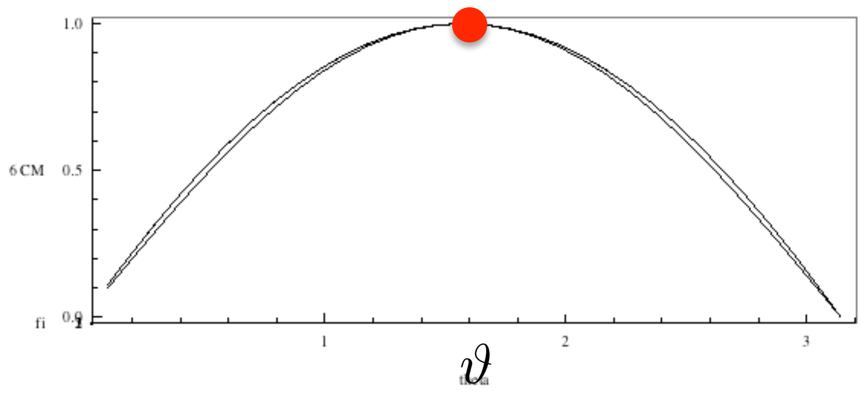


Entanglement

$$|\Psi_{AB}\rangle = \cos\vartheta|\uparrow_A\downarrow_B\rangle + e^{i\varphi}\sin\vartheta|\downarrow_A\uparrow_B\rangle$$

$$C = 2|\langle S_A^+ S_B^-\rangle| = |\sin\vartheta|$$

$$\boxed{\Delta(\cos\phi_{AB})^2 = 0 \rightarrow \text{perfect entanglement}, \; C = 1.}$$



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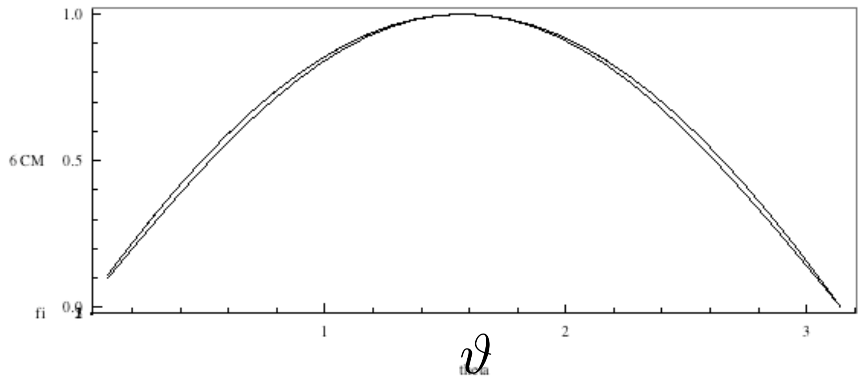
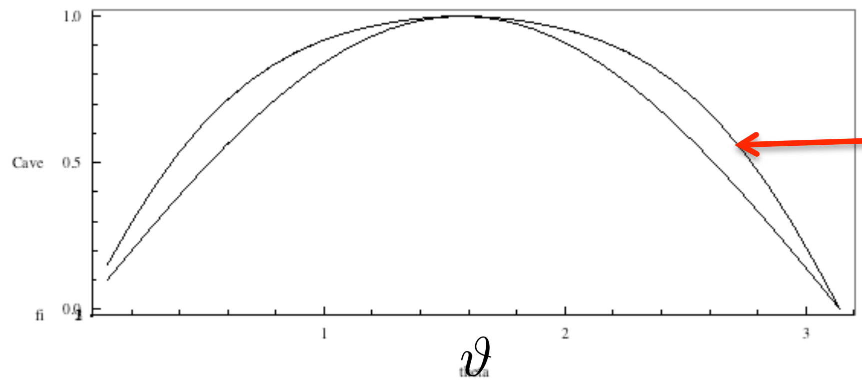


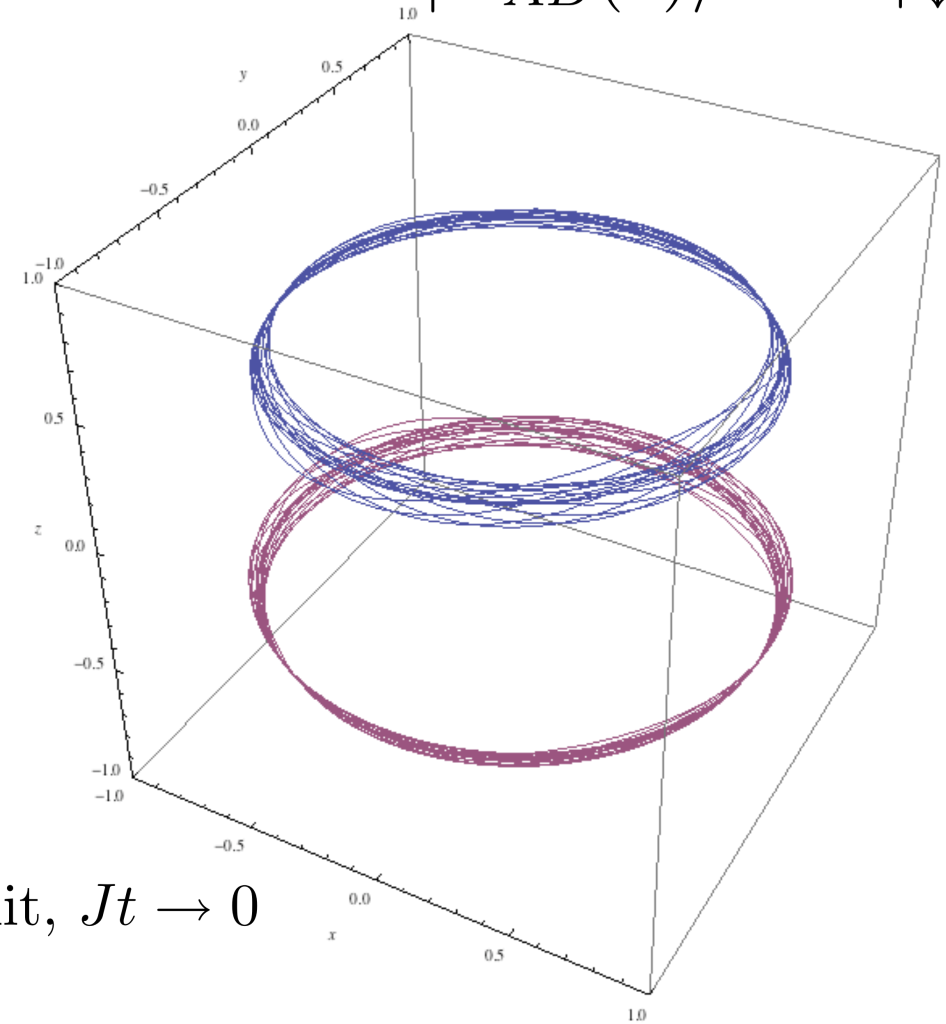


$$C \leq \sqrt{\langle\cos\phi_{AB}\rangle^2 + \langle\sin\phi_{AB}\rangle^2}$$

$$H_{int} = J(t)\mathbf{S}_A \cdot \mathbf{S}_B$$

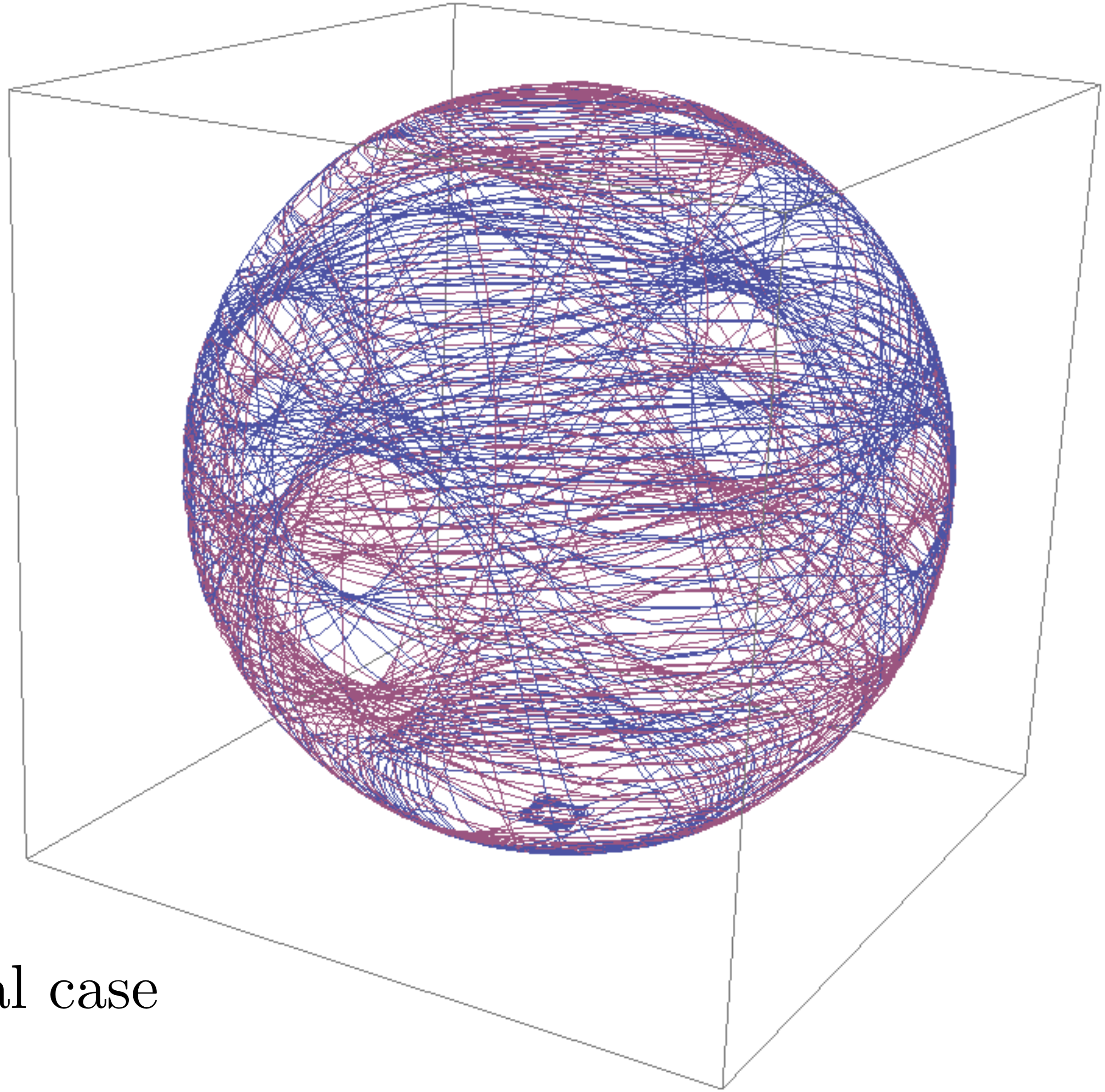
Example: $J(t)$ is step function and $|\Psi_{AB}(0)\rangle \sim \uparrow\downarrow$

$$C(t) = |\sin Jt|$$



weak interaction limit, $Jt \to 0$

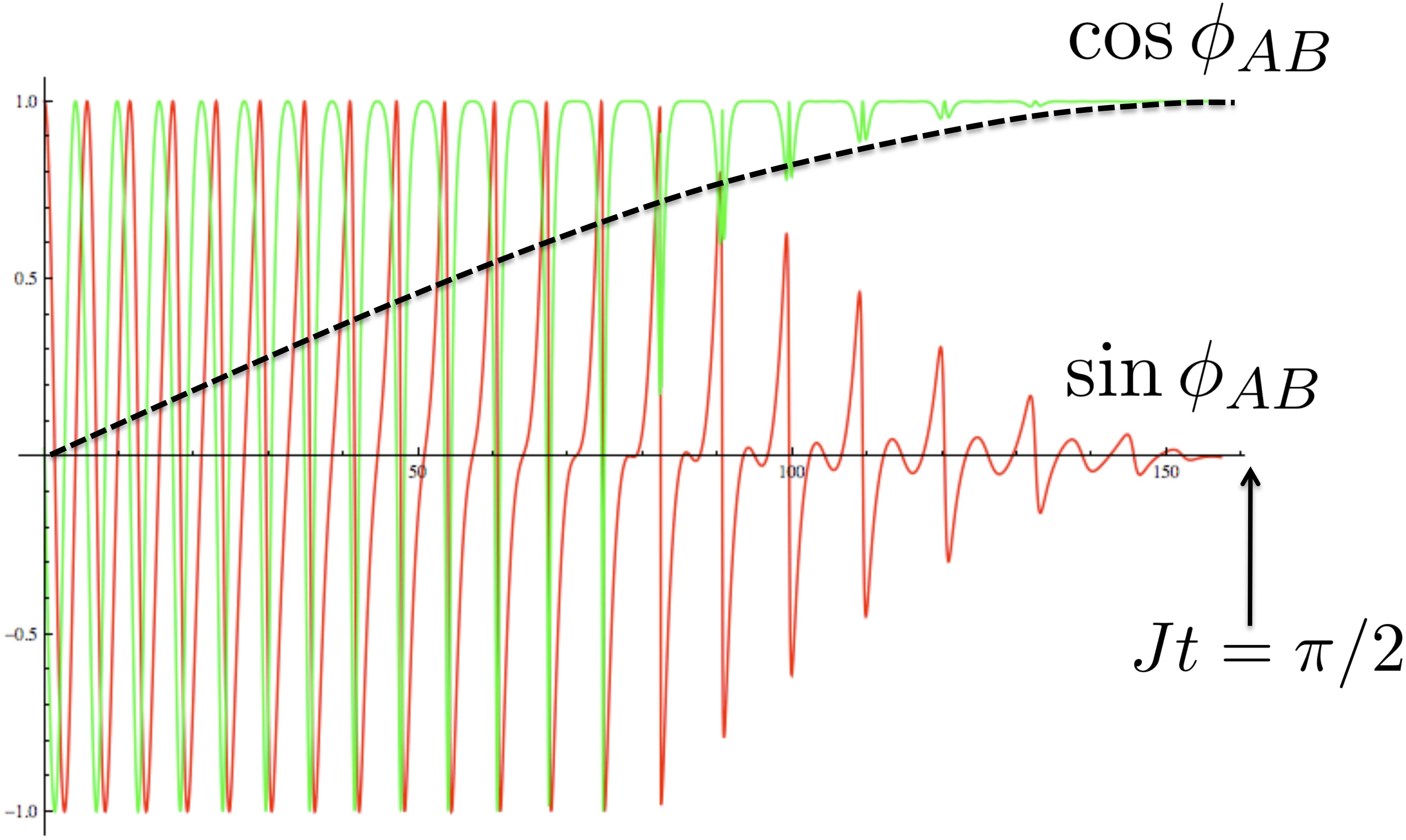
Dynamics



typical general case

Dynamics

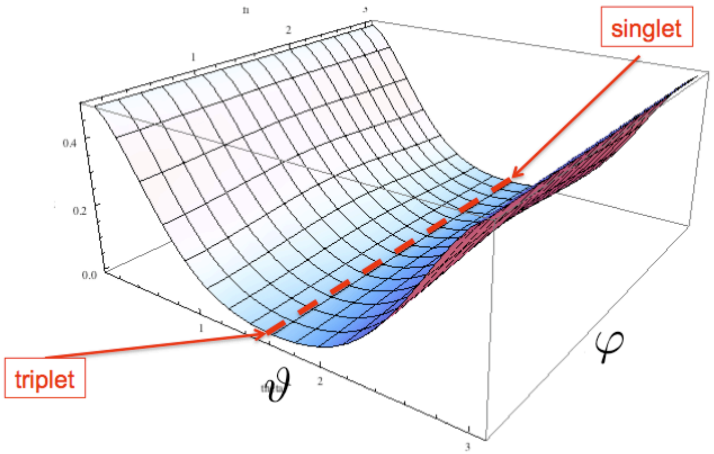
$$C(t) = |\sin Jt|$$

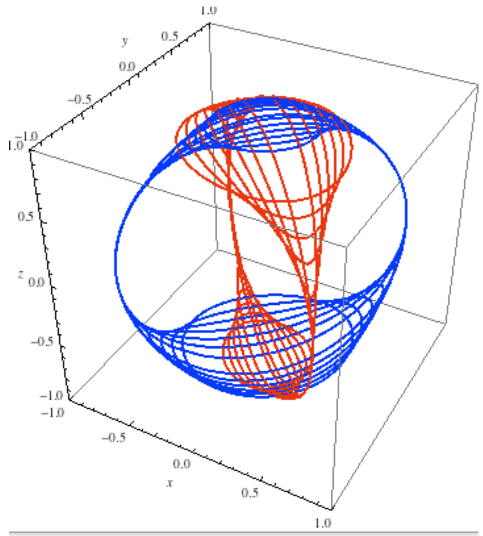


$$C \leq \sqrt{\langle\cos\phi_{AB}\rangle^2 + \langle\sin\phi_{AB}\rangle^2}$$

Summary

$$\langle M_x M_x \rangle = \frac{1}{6}\langle \sigma_x \sigma_x \rangle = \frac{2}{3}\langle \hat{M}_x \hat{M}_x \rangle$$





$$C \leq \sqrt{\langle \cos\phi_{AB}\rangle^2 + \langle \sin\phi_{AB}\rangle^2}$$