MAKING BOHMIAN MECHANICS COMPATIBLE WITH RELATIVITY AND QUANTUM FIELD THEORY

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Outline:

1. Main Ideas

2. Technical Details

Based on:

H. N., Found. Phys. **39**, 1109 (2009)

H. N., Int. J. Quantum Inf. **7**, 595 (2009)

H. N., Int. J. Mod. Phys. A **25**, 1477 (2010)

H. N., arXiv:1002.3226, to appear in Int. J. Quantum Inf.

H. N., arXiv:1006.1986

H. N., arXiv:1007.4946

1. MAIN IDEAS

Qualitative non-technical arguments, relaxed discussion

It is frequently argued that:

1. Bohmian mechanics (BM) contradicts the theory of relativity (because BM is nonlocal)

2. BM based on particle trajectories is not consistent with particle creation/destruction in QFT (because particle trajectories are continuous in BM)

The purpose of this talk is to show that BM can be formulated such that:
- BM is nonlocal but relativistic covariant
- BM with continuous particle trajectories describes particle creation/destruction in QFT

1.1 Relativistic Bohmian interpretation

- Nonlocality in BM requires superluminal (faster than light) communication between particles.

- The most frequent argument that it is not compatible with relativity:

Superluminal communication

$\Rightarrow$ there is a Lorentz frame in which communication is instantaneous

$\Rightarrow$ there is a preferred Lorentz frame

$\Rightarrow$ the principle of relativity is violated.

- However, this is not a valid argument, because this is like using the following argument on *subluminal* communication:

Subluminal communication

$\Rightarrow$ there is a Lorentz frame in which particle is at rest

$\Rightarrow$ there is a preferred Lorentz frame

$\Rightarrow$ the principle of relativity is violated.

The argument on subluminal communication is wrong:

- It is the general law of motion that must have the same form in any Lorentz frame.
- A particular solution (a particle at rest with respect to some particular Lorentz frame) does not need to have the same form in all Lorentz frames.

But the argument on superluminal communication is completely analogous.

$\Rightarrow$ It is wrong for exactly the same reason:

- A particular solution (communication instantaneous with respect to some particular Lorentz frame) does not need to have the same form in all Lorentz frames.

This analogy works if one exchanges the roles of time and space.
- Is it compatible with the principle of causality?

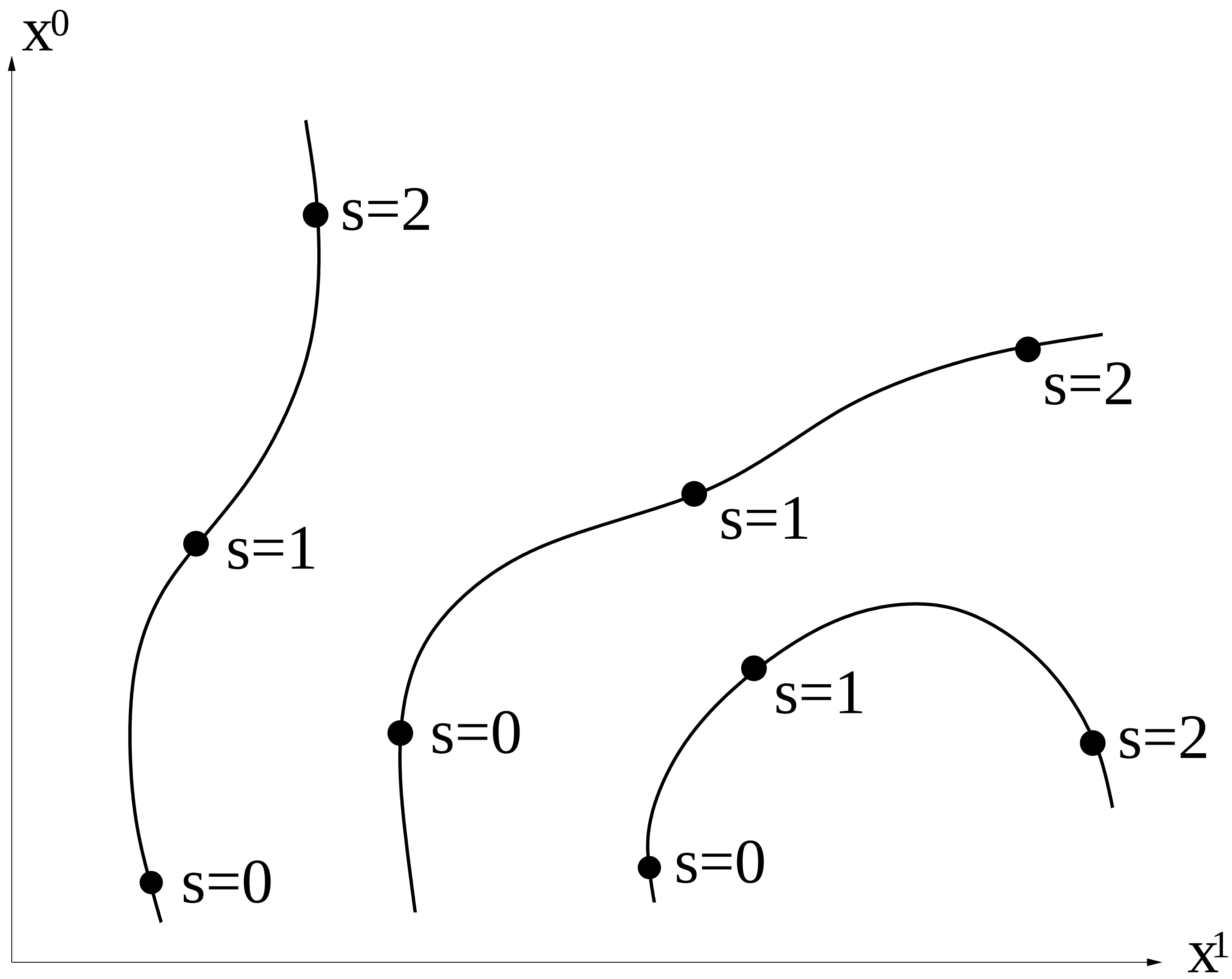
It depends on what exactly one means by principle of causality.
- It is compatible with determinism
  (all events are caused by some "prior" events).
- However, due to the superluminal influences,
  "prior" does not always need to mean "at an earlier time".

But then what "prior" means?
For n particles, the trajectories in spacetime are functions

$$X_a^\mu(s), \quad a = 1, \ldots, n, \quad \mu = 0, 1, 2, 3$$

μ - spacetime index, a - labels a particle
s - auxiliary scalar parameter that parameterizes particle trajectories
  (generalizes the notion of proper time)
$\Rightarrow$ "prior" means - at an earlier s.

Example: (3 particles, 1+1 spacetime dimensions)



$X_a^\mu(s=0)$ is arbitrary (initial condition)

"Instantaneous" means: for the same $s \Rightarrow$ no preferred Lorentz frame

Particles can move faster than light and backwards in time.
How can it be compatible with observations?

One must take into account the theory of quantum measurements:
When velocity is *measured*, then velocity cannot exceed the velocity of light.

More generally:
General theory of quantum measurements $\Rightarrow$ *all* statistical predictions coincide with those of purely probabilistic interpretation of QM.

Crucial assumptions:
- spacetime positions are preferred variables
- all measurements reduce to measurements of spacetime positions (that describe the reading of macroscopic measurement apparatus)

Relativistic probabilistic interpretation (for preferred variables):

- main idea: *treat time on an equal footing with space*

1. Generalized probabilistic interpretation:

Space probability density

$$dP_{(3)} \propto |\psi(\mathbf{x},t)|^2 d^3x$$

generalized to spacetime probability density

$$\boxed{dP_{(4)} = |\psi(\mathbf{x},t)|^2 d^3x \, dt}$$

The usual space probability density recovered as conditional probability.

Relativistic notation: $t \equiv x^0$, $\mathbf{x} \equiv (x^1, x^2, x^3)$

$$x \equiv \{x^\mu\} \quad \Rightarrow \quad \psi(\mathbf{x},t) \equiv \psi(x)$$

2. Many-time formalism:

n-particle wave function with a single time

$$\psi(\mathbf{x}_1, \ldots, \mathbf{x}_n, t)$$

generalized to

$$\boxed{\psi(\mathbf{x}_1, t_1, \ldots, \mathbf{x}_n, t_n) \equiv \psi(x_1, \ldots, x_n)}$$

The single-time formalism recovered in the coincidence limit $t_1 = \cdots = t_n = t$.

Spacetime probability resolves the equivariance problem:

Equivariance problem (Berndl, Dürr, Goldstein, Zanghì, 1996):

- Nonrelativistic equivariance equation:

$$\frac{\partial|\psi|^2}{\partial t} + \partial_i(|\psi|^2 u^i) = 0, \quad i = 1, 2, 3$$

(here $u^i(x)$ is the Bohmian 3-velocity at x)

- Not satisfied for relativistic ψ satisfying Klein-Gordon equation.
- From this, they conclude that $|\psi|^2$ cannot be probability density in relativistic BM.

- However, this equivariance equation does not treat time on an equal footing with space.
- Violation of this equivariance equation only shows that $|\psi|^2$ cannot be probability density *in space*.

$\Rightarrow$ Instead of nonrelativistic equivariance equation

$$\frac{\partial|\psi|^2}{\partial t} + \partial_i(|\psi|^2 u^i) = 0, \quad i = 1,2,3$$

probability density in *spacetime* requires
Relativistic equivariance equation:

$$\boxed{\frac{\partial|\psi|^2}{\partial s} + \partial_\mu(|\psi|^2 v^\mu) = 0, \quad \mu = 0,1,2,3}$$

(here $v^\mu(x)$ is the Bohmian 4-velocity at x), which
- is satisfied when ψ satisfies Klein-Gordon equation
- treats time on an equal footing with space.

Physically, it means that probability is not conserved in t, but is conserved in s.

But what is s physically?

Can it be measured by a "clock"?

Analogy between nonrelativistic t and relativistic s:

t - external parameter in nonrelativistic (3-dimensional) mechanics
s - external parameter in relativistic (4-dimensional) mechanics.

s can be measured indirectly by "clock" in relativistic mechanics
in the same sense as
t can be measured indirectly by "clock" in nonrelativistic mechanics.

A "clock" is a physical process periodic in "time" (t or s):
- One measures the number of periods,
and then interprets it as a measure of elapsed "time".

$\Rightarrow$ s is a relativistic analogue of Newton absolute time.

1.2 Particle creation and destruction

Main idea:
Even if particle trajectories never begin or end, the measuring apparatus behaves as if particles are created or destructed.

$\psi_1(x_1) = 1$-particle wave function
$\psi_2(x_2, x_3) = 2$-particle wave function

Superposition:

$$\psi(x_1, x_2, x_3) = \psi_1(x_1) + \psi_2(x_2, x_3)$$

How can one detect a definite number of particles (either 1 or 2)?
- When the number of particles is *measured*,
  then the system is entangled with the measuring apparatus $\Rightarrow$

Total wave function:

$$\Psi(x_1, x_2, x_3, y) = \psi_1(x_1)E_1(y) + \psi_2(x_2, x_3)E_2(y)$$

Apparatus wave functions do not overlap:

$$E_1(y)E_2(y) \simeq 0$$

$\Rightarrow Y$ is either in the support of $E_1(Y)$ or in the support of $E_2(Y)$.

In other words, the detector either says:

- There is 1 particle.

or says:

- There are 2 particles.

But what happens with undetected particles?
Can they be detected later?

Assume that Y is in the support of $E_2(Y)$
$\Rightarrow$ effective collapse:

$$\Psi(x_1, x_2, x_3, y) \to \psi_2(x_2, x_3)E_2(y)$$

- The trajectory of the undetected particle $X_1(s)$ has a negligible influence on the detector particles $Y(s)$, as well as on detected particles $X_2(s)$, $X_3(s)$.

$\Rightarrow$ For all practical purposes, the particle $X_1(s)$ behaves as if it does not exist.

But can it be detected later?
- In principle yes, if $E_1(y)$ and $E_2(y)$ overlap later.
- Yet, once $E_1(y)$ and $E_2(y)$ cease to overlap, it is extremely unlikely that they will overlap later.
- This is because disorder increases with time (2nd law of thermodyn.)
- Essentially, this is the same mechanism that is responsible for irreversibility of decoherence.

The general mechanism of particle creation/destruction:

Step 1: Deterministic evolution of the state in interacting QFT:

$$|n_{\text{initial}}\rangle \longrightarrow \sum_n c_n |n\rangle$$

Step 2: Entanglement with the environment:

$$\left[\sum_n c_n |n\rangle\right] |E_{\text{initial}}\rangle \longrightarrow \sum_n c_n |n\rangle |E_n\rangle$$

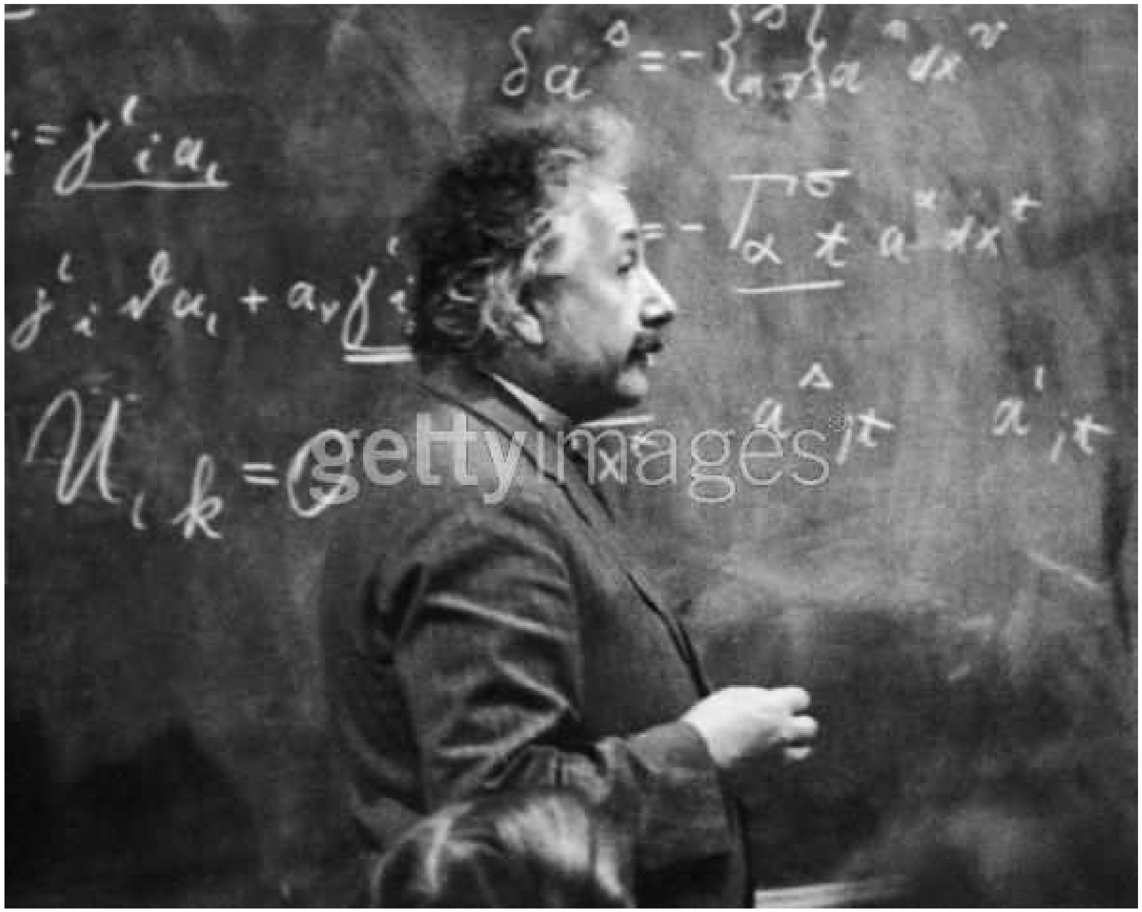
$$E_n(y) E_{n'}(y) \simeq 0 \quad \text{for} \quad n \neq n'$$

Step 3: Bohmian interpretation $\Rightarrow Y(s)$ enters only one $E_{n_{\text{final}}}(y)$
$\Rightarrow$ effective collapse:

$$\sum_n c_n |n\rangle |E_n\rangle \longrightarrow |n_{\text{final}}\rangle |E_{n_{\text{final}}}\rangle$$

Summary: $|n_{\text{initial}}\rangle |E_{\text{initial}}\rangle \longrightarrow |n_{\text{final}}\rangle |E_{n_{\text{final}}}\rangle$

2. TECHNICAL DETAILS



To fill the gaps in the preceding qualitative arguments

2.1 Relativistic probabilistic interpretation

Spacetime point: $x = \{x^\mu\} = (x^0, x^1, x^2, x^3)$

$[x^0 \equiv t, \;\; (x^1, x^2, x^3) \equiv \mathbf{x}]$

Units: $\hbar = c = 1$

Spacetime scalar product:

$$\langle \psi | \psi' \rangle = \int d^4x\, \psi^*(x) \psi'(x)$$

Normalization: $\langle \psi | \psi \rangle = 1$

$\longrightarrow$ probabilistic interpretation:

$$\boxed{dP = |\psi(x)|^2 d^4x}$$

Is it compatible with the usual probabilistic interpretation

$$dP_{(3)} \propto |\psi(\mathbf{x}, t)|^2 d^3x \text{ ?}$$

It is compatible:

- If dP is fundamental *a priori* probability

$\Rightarrow dP_{(3)}$ is conditional probability

(when one knows that the particle is detected at time t):

$$dP_{(3)} = \frac{|\psi(\mathbf{x},t)|^2 d^3x}{N_t}$$

$$N_t = \int d^3x |\psi(\mathbf{x},t)|^2$$

N_t = *marginal* probability that the particle will be found at time t

- If t is not known $\Rightarrow \int d^3x\, |\psi|^2 =$ probability *per unit time*.
- That's how QM is interpreted in *standard* calculations of cross sections (scattering) and lifetimes (spontaneous decay).

Generalization to many particles:

$$\boxed{dP = |\psi(x_1, \ldots, x_n)|^2 d^4x_1 \cdots d^4x_n}$$

If first particle detected at t_1, second particle at t_2, ...
$\Rightarrow$ conditional probability:

$$dP_{(3n)} = \frac{|\psi(\mathbf{x}_1, t_1, \ldots, \mathbf{x}_n, t_n)|^2 d^3x_1 \cdots d^3x_n}{N_{t_1,\ldots,t_n}}$$

$$N_{t_1,\ldots,t_n} = \int |\psi(\mathbf{x}_1, t_1, \ldots, \mathbf{x}_n, t_n)|^2 d^3x_1 \cdots d^3x_n$$

Usual single-time probabilistic interpretation in the limit
$t_1 = \cdots = t_n \equiv t$.

2.2 Quantum theory of measurements

Measured system:

$$\psi(x) = \sum_b c_b \psi_b(x)$$

$\psi_b(x) =$ eigenstates of some measured observable (hermitian operator)

- normalization: $\int d^4x\, \psi_b^*(x)\psi_b(x) = 1$

$|c_b|^2 =$ probability that the observable will have the value b.

Probability in b-space can be derived from probability in position-space.

- Reason: every macroscopic measurement apparatus actually determines position y of some macroscopic variable.

States $E_b(y)$ of measuring apparatus do not overlap:

$$E_b(y)E_{b'}(y) \simeq 0 \quad \text{for} \quad b \neq b'$$

Normalized: $\int d^4y\, E_b^*(y)E_b(y) = 1$.

Measurement (deterministic evolution):

$$\psi_b(x)E_0(y) \to \psi_b(x)E_b(y)$$

Linearity $\Rightarrow$ entanglement with the measuring apparatus:

$$\sum_b c_b\psi_b(x)E_0(y) \to \sum_b c_b\psi_b(x)E_b(y) \equiv \psi(x,y)$$

Marginal probability for finding apparatus-particle at the position y:

$$\rho(y) = \int d^4x\, \psi^*(x,y)\psi(x,y) \simeq \sum_b |c_b|^2|E_b(y)|^2$$

$\Rightarrow$ probability that y will be in the support of $E_b(y)$:

$$\boxed{p_b = \int_{\text{supp}\, E_b} d^4y\, \rho(y) \simeq |c_b|^2}$$

Q.E.D.

2.3 Wave equation and the Bohmian interpretation

System of n (entangled) relativistic spin-0 particles:

$\Rightarrow$ n-particle Klein-Gordon equation:

$$\sum_{a=1}^{n} [\partial_a^\mu \partial_{a\mu} + m_a^2] \psi(x_1, \ldots, x_n) = 0$$

Bohmian interpretation: particles have some trajectories $X_a^\mu(s)$
$s =$ auxiliary scalar parameter.

Complex wave function: $\psi = |\psi| e^{iS}$
Klein-Gordon equation $\Rightarrow$ relativistic equivariance equation:

$$\boxed{\frac{\partial |\psi|^2}{\partial s} + \sum_{a=1}^{n} \partial_{a\mu} (|\psi|^2 v_a^\mu) = 0}$$

$$v_a^\mu(x_1, \ldots, x_n) \equiv -\partial_a^\mu S(x_1, \ldots, x_n)$$

$\Rightarrow$ consistent to postulate equation for the trajectories:

$$\boxed{\frac{dX_a^\mu(s)}{ds} = v_a^\mu(X_1(s), \ldots, X_n(s))}$$

If a statistical ensemble of particles has the spacetime distribution $|\psi|^2$ for some initial s $\Rightarrow$(due to equivariance equation):
The ensemble will have the $|\psi|^2$ distribution for any s.

Nonlocality: velocity of one particle (for some value of s) depends on the positions of all other particles (for the same value of s).

Relativistic covariance: no *a priori* preferred coordinate frame is involved.

Compatibility between Bohmian interpretation
and probabilistic interpretation of QM:

- both have same statistical predictions in *spacetime-position* space.

- general theory of quantum measurements - reduces all measurements to *spacetime-position* measurements.

$\Rightarrow$ both have same statistical predictions for any measurement.

2.4 Particles with spin

Wave function has many components.

For 1 particle:

$$\psi(x) \to \psi_l(x), \quad l = \text{discrete label}$$

For n particles:

$$\psi_{l_1 \ldots l_n}(x_1, \ldots, x_n) \equiv \psi_L(x_1, \ldots, x_n)$$

Notation:

$$\psi = \begin{pmatrix} \psi_1 \\ \psi_2 \\ \vdots \end{pmatrix}, \quad \psi^\dagger = \begin{pmatrix} \psi_1^* & \psi_2^* & \cdots \end{pmatrix}$$

Probability density:

$$dP = \psi^\dagger(x_1, \ldots, x_n)\psi(x_1, \ldots, x_n) d^4x_1 \cdots d^4x_n$$

Example: 1 particle with spin-$\frac{1}{2}$

$$dP = \psi^\dagger(x)\psi(x)d^4x$$

$\Rightarrow \psi^\dagger\psi$ must be a scalar.

On the other hand, it is well-known that

$$\psi^\dagger\psi = \bar{\psi}\gamma^0\psi \quad \text{is a time component of a vector}$$

where $\gamma^\mu =$ Dirac matrices, $\bar{\psi} \equiv \psi^\dagger\gamma^0$.

Is it consistent?
Yes, because there is some freedom in the *definition* of transformations of spinors under change of spacetime coordinates.

The most frequent definition:
ψ - transforms as a spinor, γ^μ does not transform.
- Suitable only for Lorentz transformations in flat spacetime.

We adopt a less frequent, but more suitable definition:
ψ - does not transform (scalar), γ^μ transforms as a vector

- the usual fixed Dirac matrices denoted by $\gamma^{\bar{\mu}} \Rightarrow \bar{\psi} \equiv \psi^\dagger \gamma^{\bar{0}}$ is scalar
- suitable for arbitrary coordinate transformations in arbitrary spacetime
- widely used in curved spacetime
- in flat spacetime there is a Lorentz frame in which $\gamma^\mu = \gamma^{\bar{\mu}}$
- $\bar{\psi}\psi$ and $\psi^\dagger\psi$ are both scalars under coordinate transformations
- the usual spinor transformations of ψ reinterpreted as transformations under *internal* group SO(1,3)

Dirac current:

$$j^\mu_{\text{Dirac}} = \bar{\psi}(x)\gamma^\mu\psi(x)$$

Klein-Gordon current:

$$j^\mu = \frac{i}{2}\psi^\dagger(x)\overleftrightarrow{\partial^\mu}\psi(x)$$

- both transform as vectors

Bohmian interpretation for particles with spin:

For spin-0, Bohmian velocity

$$\frac{dX_a^\mu(s)}{ds} = -\partial_a^\mu S(X_1(s),\ldots,X_n(s))$$

equivalent to

$$\frac{dX_a^\mu(s)}{ds} = V_a^\mu(X_1(s),\ldots,X_n(s))$$

where

$$V_a^\mu = \frac{j_a^\mu}{\psi^*\psi}\,, \quad j_a^\mu = \frac{i}{2}\psi^* \overleftrightarrow{\partial_a^\mu}\psi$$

$$\psi_1 \overleftrightarrow{\partial_a^\mu}\psi_2 \equiv \psi_1(\partial_a^\mu\psi_2) - (\partial_a^\mu\psi_1)\psi_2$$

Generalization for particles with spin: $\psi^* \to \psi^\dagger$

$$\frac{dX_a^\mu(s)}{ds} = V_a^\mu(X_1(s), \ldots, X_n(s))$$

$$V_a^\mu = \frac{j_a^\mu}{\psi^\dagger \psi}, \quad j_a^\mu = \frac{i}{2} \psi^\dagger \overleftrightarrow{\partial_a^\mu} \psi$$

For *any* spin: ψ satisfies the (n-particle) Klein-Gordon equation $\Rightarrow$

$$\sum_{a=1}^{n} \partial_{a\mu} j_a^\mu = 0$$

$\psi(x_1, \ldots, x_n)$ does not explicitly depend on s $\Rightarrow$

$$\frac{\partial \psi^\dagger \psi}{\partial s} = 0$$

$\Rightarrow$ Equivariance:

$$\frac{\partial \psi^\dagger \psi}{\partial s} + \sum_{a=1}^{n} \partial_{a\mu}(\psi^\dagger \psi V_a^\mu) = 0$$

$\Rightarrow$ Bohmian trajectories consistent with probabilistic interpretation.

2.5 Free QFT states represented by wave functions

For simplicity, discuss only hermitian fields without spin (generalization to other fields is straightforward).

Klein-Gordon equation for field operator:

$$\partial^\mu \partial_\mu \hat{\phi}(x) + m^2 \hat{\phi}(x) = 0$$

General solution:

$$\hat{\phi}(x) = \hat{\psi}(x) + \hat{\psi}^\dagger(x)$$

$$\hat{\psi}(x) = \int d^3k\, f(\mathbf{k})\, \hat{a}(\mathbf{k}) e^{-i[\omega(\mathbf{k})x^0 - \mathbf{k}\mathbf{x}]}$$

$$\hat{\psi}^\dagger(x) = \int d^3k\, f(\mathbf{k})\, \hat{a}^\dagger(\mathbf{k}) e^{i[\omega(\mathbf{k})x^0 - \mathbf{k}\mathbf{x}]}$$

$$\omega(\mathbf{k}) = \sqrt{\mathbf{k}^2 + m^2}$$

$\hat{a}^\dagger(\mathbf{k})$ - creation operator
$\hat{a}(\mathbf{k})$ - destruction operator

Operator that destroys n particles:

$$\widehat{\psi}_n(x_{n,1},\ldots,x_{n,n}) = d_n S_{\{x_{n,1},\ldots,x_{n,n}\}} \widehat{\psi}(x_{n,1}) \cdots \widehat{\psi}(x_{n,n})$$

d_n - normalization

$S_{\{x_{n,1},\ldots,x_{n,n}\}}$ - symmetrization

The n-particle states in the basis of particle spacetime positions:

$$|x_{n,1},\ldots,x_{n,n}\rangle = \widehat{\psi}_n^\dagger(x_{n,1},\ldots,x_{n,n})|0\rangle$$

For arbitrary n-particle state $|\Psi_n\rangle$, the wave function is

$$\psi_n(x_{n,1},\ldots,x_{n,n}) = \langle x_{n,1},\ldots,x_{n,n}|\Psi_n\rangle$$

Normalization:

$$\int d^4x_{n,1} \cdots \int d^4x_{n,n}\, |\psi_n(x_{n,1},\ldots,x_{n,n})|^2 = 1$$

Problem: Wave functions for different n normalized in different spaces. We want them all to live in the same space $\longrightarrow$ different normalization:

$$\Psi_n(x_{n,1},\ldots,x_{n,n}) = \sqrt{\frac{\mathcal{V}^{(n)}}{\mathcal{V}}}\,\psi_n(x_{n,1},\ldots,x_{n,n})$$

where

$$\mathcal{V}^{(n)} = \int d^4x_{n,1}\cdots\int d^4x_{n,n}, \qquad \mathcal{V} = \prod_{n=1}^{\infty}\mathcal{V}^{(n)}$$

In particular, the wave function of the vacuum is constant:

$$\Psi_0 = \frac{1}{\sqrt{\mathcal{V}}}$$

Condensed notation:

$$\vec{x} = (x_{1,1}, x_{2,1}, x_{2,2}, \ldots), \quad \mathcal{D}\vec{x} = \prod_{n=1}^{\infty}\prod_{a_n=1}^{n} d^4x_{n,a_n}$$

$\Rightarrow$ all wave functions normalized in the same space:

$$\int \mathcal{D}\vec{x}\,|\Psi_n(x_{n,1},\ldots,x_{n,n})|^2 = 1$$

To further simplify notation:

Condensed label: $A = (n, a_n) \Rightarrow \vec{x} = (x_1, x_2, x_3, \ldots)$

$$\mathcal{D}\vec{x} = \prod_{A=1}^{\infty} d^4x_A, \quad \mathcal{V} = \int \prod_{A=1}^{\infty} d^4x_A$$

n-particle state:

$$\Psi_n(\vec{x}) = (\vec{x}|\Psi_n\rangle$$

General state:

$$\Psi(\vec{x}) = (\vec{x}|\Psi\rangle = \sum_{n=0}^{\infty} c_n \Psi_n(\vec{x}) \equiv \sum_{n=0}^{\infty} \tilde{\Psi}_n(\vec{x})$$

Normalization:

$$\int \mathcal{D}\vec{x}\, |\Psi(\vec{x})|^2 = 1$$

$$\sum_{n=0}^{\infty} |c_n|^2 = 1$$

$\Rightarrow$ In QFT, general wave function depends on an *infinite* number of coordinates.

2.6 Generalization to interacting QFT

- operators in the Heisenberg picture:
$\hat{O}_H(x)$ - satisfy exact equations of motion

- operators in the interaction picture:
$\hat{O}(x)$ - satisfy free equations of motion

Operator that destroys n particles:

$$\widehat{\psi}_{nH}(x_{n,1},\ldots,x_{n,n}) = d_n S_{\{x_{n,1},\ldots,x_{n,n}\}} \widehat{\psi}_H(x_{n,1}) \cdots \widehat{\psi}_H(x_{n,n})$$

n-particle wave function:

$$\tilde{\psi}_n(x_{n,1},\ldots,x_{n,n}) = \langle 0|\widehat{\psi}_{nH}(x_{n,1},\ldots,x_{n,n})|\Psi\rangle$$

Represent all wave functions in the same configuration space:

$$\tilde{\psi}_n(x_{n,1},\ldots,x_{n,n}) \rightarrow \tilde{\Psi}_n(\vec{x})$$

Total state:

$$\Psi(\vec{x}) = \sum_{n=0}^{\infty} \tilde{\Psi}_n(\vec{x})$$

- Encodes complete information about properties of the interacting system.

In practice, one cannot calculate it exactly, but only perturbatively:

$$\hat{\psi}_H(x_{n,a_n}) = \hat{U}^\dagger(x^0_{n,a_n})\hat{\psi}(x_{n,a_n})\hat{U}(x^0_{n,a_n})$$

$$\hat{U}(t) = Te^{-i\int_{t_0}^{t} dt' \hat{H}_{\text{int}}(t')}$$

$\Rightarrow$ perturbation theory by expansion in the powers of $\hat{H}_{\text{int}}$.

In the coincidence limit $x^0_{n,1} = \cdots = x^0_{n,n} \equiv t \to \infty$
$\Rightarrow$ reduces to the usual S-matrix theory in QFT.

2.7 Bohmian interpretation of QFT

Probability density of particle spacetime positions:

$$\rho(\vec{x}) = \Psi^\dagger(\vec{x})\Psi(\vec{x})$$

The current

$$J_A^\mu(\vec{x}) = \frac{i}{2}\Psi^\dagger(\vec{x})\overleftrightarrow{\partial}_A^\mu\Psi(\vec{x}) \equiv \rho(\vec{x})U_A^\mu(\vec{x})$$

in general is not conserved:

$$\sum_{A=1}^{\infty} \partial_{A\mu}[\rho(\vec{x})U_A^\mu(\vec{x})] = J(\vec{x})$$

For Bohmian trajectories

$$\frac{dX_A^\mu(s)}{ds} = V_A^\mu(\vec{X}(s))$$

we need equivariance

$$\frac{\partial\rho(\vec{x})}{\partial s} + \sum_{A=1}^{\infty} \partial_{A\mu}[\rho(\vec{x})V_A^\mu(\vec{x})] = 0$$

$\Rightarrow$ we need conservation

$$\sum_{A=1}^{\infty} \partial_{A\mu}[\rho(\vec{x})V_A^\mu(\vec{x})] = 0$$

The solution is

$$V_A^\mu(\vec{x}) = U_A^\mu(\vec{x}) + \rho^{-1}(\vec{x})[e_A^\mu + E_A^\mu(\vec{x})]$$

where

$$e_A^\mu = -\mathcal{V}^{-1} \int \mathcal{D}\vec{x}\, E_A^\mu(\vec{x})$$

$$E_A^\mu(\vec{x}) = \partial_A^\mu \int \mathcal{D}\vec{x}'\, G(\vec{x},\vec{x}')J(\vec{x}')$$

$$G(\vec{x},\vec{x}') = \int \frac{\mathcal{D}\vec{k}}{(2\pi)^{4\aleph_0}} \frac{e^{i\vec{k}(\vec{x}-\vec{x}')}}{\vec{k}^2}$$

CONCLUSIONS

- The usual formulation of BM is not relativistic covariant because it is based on *standard QM which is also not relativistic covariant*.

- To make BM covariant $\Rightarrow$ first reformulate standard QM in a covariant way!

$\Rightarrow$ Treat time on an equal footing with space:

1. space probability density $\longrightarrow$ spacetime probability density
2. single-time wave function $\longrightarrow$ many-time wave function

- To make particle BM compatible with QFT and particle/destruction:
  1. Represent QFT states with wave functions (depending on an infinite number of coordinates).
  2. Use quantum theory of measurements $\Rightarrow$ effective collapse into states of definite number of particles.

Thank You!