Erkenn (2008) 68:21–39
DOI 10.1007/s10670-007-9054-1

ORIGINAL ARTICLE

Reflections on the deBroglie–Bohm Quantum Potential

Peter J. Riggs

Received: 25 September 2005 / Accepted: 7 May 2007 / Published online: 11 August 2007

Abstract The deBroglie–Bohm quantum potential is the potential energy function of the wave field. The quantum potential facilitates the transference of energy from wave field to particle and back again which accounts for energy conservation in isolated quantum systems. Factors affecting energy exchanges and the form of the quantum potential are discussed together with the related issues of the absence of a source term for the wave field and the lack of a classical back reaction.

Keywords DeBrogle–Bohm theory · Quantum potential · Wave field · Potential energy

1 Introduction

There has been a great deal of ill-informed commentary about quantum mechanics since its beginning around 1924. Much of this has been speculation based upon improper renderings of the uncertainty relations, identifying probabilistic outcomes with an absence of causality, and unconditionally accepting the quantum ‘no-go’ theorems as final proof that a more complete description of phenomena cannot be given. These speculations together with sentiments expressed by some of the founders of quantum mechanics about the impossibility of depicting a quantum ontology led to the abandonment of a set of concepts and principles that were strongly held prior to 1924 (Woit 2006, 147). Examples of such abandoned principles at the micro-level include: realism; causality; continuity of process; and (occasionally) energy conservation (Cushing 1998, 284). The concept of realism is a case in point. Realism asserts that there exists an objective physical realm independent of any sentient beings to observe it. One of the best known founders of

P. J. Riggs (✉)
Research School of Social Sciences, Australian National University, Canberra, ACT 0200, Australia
e-mail: peter.riggs@anu.edu.au

quantum mechanics, Niels Bohr, explicitly denied the independent existence of an objective quantum realm:

> ... There is no quantum world. There is only an abstract quantum physical description. It is wrong to think that the task of physics is to find out how nature is. ... (Bohr quoted in Petersen 1963, 8).

The dominant paradigm of quantum physics is known as the Copenhagen Interpretation of Quantum Mechanics, also called Orthodox Quantum Theory (Jammer 1966, 361; Baggott 1992, 82; Cushing 1998, 289). Orthodox Quantum Theory came about (in part) from the abandonment of those previously held physical concepts and principles mentioned above. The state of a quantum system is represented in Orthodox Quantum Theory by a (state) vector in an abstract, mathematical space. The state vector is assumed to contain all possible information about the system. Consequently, any questions relating to a quantum system that go beyond what can be found from the state vector are considered *meaningless* (Bohm 1957, 92; Jammer 1966, 330; Holland 1993, 9). Orthodox Quantum Theory is primarily an algorithm for obtaining statistical predictions for the results of experiments and a prescription for avoiding fundamental questions. In other words, it is essentially an instrumental theory (Smart 1968, 159) and has nothing to tell us about quantum ontology.

There are certainly many, new features to be learnt about the microworld and which quantum mechanics can inform about. This does not necessarily require abandoning physical principles and ontological concepts that have been previously developed and have served physics successfully. Such well-established physical concepts and principles should not be given away until such time as they are clearly shown to be inappropriate, not applicable, or simply false. Rather than renouncing physical principles and ontological concepts, it should be more a case of modifying these principles and concepts as required to suit new knowledge. This is a better way to proceed if we are to gain a fuller understanding of the foundations of quantum mechanics. It is accepted by most philosophers of science that *one* aim of science is to provide explanations of physical phenomena. The way to accomplish this in the context of the quantum realm is to specify both the ontology and the laws that govern the realm in addition to those rules by which we predict the outcome of experiments. This is achieved in the deBroglie–Bohm Causal Theory of Quantum Mechanics which is an empirically adequate, realist description of quantum phenomena with more than 70 years of progress (e.g., de Broglie 1924; Bohm 1952; Holland 1993; Cushing et al. 1996). A main attraction of deBroglie–Bohm Theory is its ontology—quantum entities (i.e., particles embedded in a quantum field) are prescribed to have a real existence in space and time. The deBroglie–Bohm Theory, therefore, offers a better possibility for making sense of physical reality than does Orthodox Quantum Theory and for assisting in gaining a more coherent understanding in areas such as relativistic quantum field theory and quantum gravity.

The conceptual structure of deBroglie–Bohm Theory is quite distinct from Orthodox Quantum Theory. In the one-particle case, deBroglie–Bohm Theory asserts that a quantum particle has an accompanying field that together form a single physical system. The quantum field is commonly called the 'wave field' for historical reasons. It is a physical process that propagates in three-dimensional Galilean space over time. A quantum particle is a point-like object localised in three-dimensional Galilean space with an inertial mass and a well-defined position at all times. This does not conflict with the uncertainty relations which are interpreted as expressing the statistical scatter of measured values of complementary variables in an ensemble of systems (Holland 1993, 360). Although particle and wave field are physically inseparable parts of a single entity, these two aspects can be given limited, individual descriptions.

The wave field is described by a wavefunction Ψ which evolves in time according to the Schrödinger equation:

$$i\hbar\frac{\partial\Psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\Psi + \mathrm{V}\Psi \tag{1}$$

where $\nabla^2$is the Laplacian differential operator, V is an external (classical) potential, $\hbar$ is Planck's Constant divided by 2π, m is the particle's inertial mass, and $i = \sqrt{-1}$. The motion of the particle is causally governed by its wave field. If the wavefunction is expressed in polar form $\Psi = \mathrm{R}e^{iS/\hbar}$(where R, S are real functions of the space and time coordinates, with $\mathrm{R} \geq 0$) and substituted into Eq. 1, then two differential equations result. One of these is called the Quantum Hamilton-Jacobi equation (Bohm and Hiley 1993, 29):

$$-\frac{\partial S}{\partial t} = \frac{(\nabla S)^2}{2m} + V - \frac{\hbar^2}{2m}\left(\frac{\nabla^2 R}{R}\right) \tag{2}$$

(The other differential equation will not be of concern here). The last term in Eq. 2, i.e., $(-\hbar^2/2m)(\nabla^2\mathrm{R}/\mathrm{R})$, is the quantum potential (denoted Q) and does not appear in classical mechanics. The presence of the quantum potential accounts for most of the differences between classical and quantum physics. It is important to explore the nature and role of the quantum potential for this reason. Additionally, in deBroglie–Bohm Theory, the momentum **p** of a quantum particle is given by:

$$\mathbf{p} = (\nabla S) \tag{3}$$

and the particle's total rate of change of momentum with respect to time is:

$$(\mathrm{d}\mathbf{p}/\mathrm{d}t) = -\nabla(V + Q) \tag{4}$$

It follows from Eq. 3 that the term $[(\nabla \mathrm{S})^2/2m]$ in Eq. 2 is the particle's kinetic energy.

An N-particle quantum system ($\mathrm{N} > 1$) is a generalisation from the one-particle case. There is only a single guiding wave field represented by the wavefunction Ψ, but now Ψ is defined on a 3N-dimensional configuration space. The quantities and equations defined for a one-particle system have their many-particle analogues. In particular, we note that the many particle quantum potential Q = Q(x_1, x_2, x_3, ..., x_N, t) is given by:

$$Q = \sum_{i=1}^{N} Q_i = -\sum_{i=1}^{N} \left(\frac{\hbar^2}{2m_i R}\right) \nabla_i^2 R \tag{5}$$

where ∇_i^2 is the Laplacian evaluated at the position of the *i*-th particle and m_i is the mass of the *i*-th particle. A many-particle quantum system exhibits non-local effects as its quantum potential allows for a direct interconnection between particles which depends on the state of the whole system (Holland 1993, 282). Although there is an instantaneous connection between the particles, there is no instantaneous energy transfer and consequently Special Relativity is not violated.

An examination of the existence and role of the quantum potential in deBroglie–Bohm Theory will allow quantum phenomena to be explained utilising those physical principles and ontological concepts that are abandoned in Orthodox Quantum Theory. This is what we shall turn to now.

2 The ‘Active Information’ Hypothesis

We begin this section by noting that different forms of energy (e.g., kinetic, gravitational, heat, etc.) are defined in each relevant domain of physics. Although no quantitative definition of energy which covers all its aspects is currently known, there is a well-known generalised concept of energy that typically draws on particular examples in order to illustrate itself. In what follows, it will be assumed that this generalised concept of energy and the principles of the conservation of total energy and of momentum, are sufficiently familiar that they do not require elaboration.

The Active Information Hypothesis due to David Bohm and Basil Hiley is an explanation of the nature and role of the quantum potential that differs from Bohm’s ideas of the 1950s and reflects the view Bohm held in later life. (For a discussion of why Bohm altered his opinion, see Guarini 2003.) Bohm and Hiley pointed out that the value of the quantum potential Q is unchanged when the wave field’s amplitude R is multiplied by a constant. Therefore, the effect of the wave field (via the quantum potential) does not depend on the wave field’s intensity, where intensity is proportional to the square of the amplitude of the wave field. On this basis, Bohm and Hiley made the conclusion that the wave field does not transfer energy and momentum, unlike a classical wave. Their reasoning is summarised in the following comparison between classical and quantum waves:

> ... the effect of the quantum potential is independent of the strength (i.e., the intensity) of the quantum field but depends only on its *form*. By contrast, classical waves, which act mechanically (i.e., to transfer energy and momentum, for example, to push a floating object) always produce effects that are more or less proportional to the strength of the wave (Bohm and Hiley 1987, 326).

Instead of taking the role of the quantum potential Q as being similar to classical potentials, Bohm and Hiley interpreted Q as representing information that encodes details relevant to the whole of a given experimental arrangement or environment. They called this 'active information' as it is postulated that the information becomes 'active' upon entering an entity that can process the information. The basis of their hypothesis is that information carried by something with only a small amount of energy can direct something else with much greater energy (Bohm and Hiley 1993, 35).

In the context of quantum physics, Bohm and Hiley postulated that 'active information' (which is carried by the wave field and represented by the quantum potential) determines a quantum particle's path and its velocity by using the particle's own energy. They illustrated this idea with an analogy concerning a ship being automatically guided by a radio signal. Obviously the radio signal does not push the ship, nor does the effect of the signal depend on its intensity, for a weak signal will do just as well as a strong signal if properly received. What is important is the form of the signal for this carries information which, when processed by the ship's autopilot, determines how the ship's own energy will be used. They wrote:

> ... the effect of the radio waves is independent of their intensity and depends only on their form. The essential point is that the ship is moving with its own energy, and that the *form* of the radio waves is taken up to direct the much greater energy of the ship (1993, 31–32).

Bohm and Hiley argued that the quantum potential works in a similar manner—by 'informing' a quantum particle about how it will move under its own energy. The other illustrations provided by Bohm and Hiley are not especially helpful and will not be discussed here.

The Active Information Hypothesis opens up a whole host of questions and issues that are extremely problematic. Consider first the difficulties encountered with particle structure. Quantum particles would require complex internal structures with which the 'active information' is processed in order that the particle be directed through space. Bohm and Hiley readily acknowledge this:

> The fact that the particle is moving under its own energy, but being guided by the information in the quantum field suggests that an electron or other elementary particle has a complex and subtle inner structure (e.g., perhaps even comparable to that of a radio) (1993, 37).

It has not been specified what these complex structures consist of or how they might be organised within elementary particles. Nor has it been suggested how the actual processing of the 'active information' could occur. Bohm and Hiley's account is presented solely by way of alluding to a number of indirect analogies (e.g., portable radios, computers, DNA) and not by detailed and specific arguments. What's more, it seems likely that at least some elementary particles do not have the kind of structure that would be necessary. Electrons, which are a prime example for Bohm and Hiley, do not seem to have any constituent parts (Veltman 2003, 54–55; Close 2004, 40) and therefore cannot have a complex internal structure.

Second, consider the difficulties with satisfying physical laws. If information carried by something with only a small amount of energy is to direct something else with much greater energy, where does this greater energy come from in the case of quantum particles? Marcello Guarini has also expressed this question, stating:

> Radios have batteries or some other power source to draw on. Metaphorically speaking, where are the electron's batteries? (2003, 82).

!Energy conservation necessitates that either the quantum particle would have to have an internal energy content to draw on or that energy be transferred to the particle from a source external to itself. Further, in the case of a particle that is increasing its speed, it would need a continuous supply of energy during periods of (positive) acceleration. After a large number of such speed increases (which might be interspersed with periods of deceleration), any internal energy content would become depleted. The particle would not be able to 'speed up' thereafter. If the particle's energy comes from an external source, what is it? Other than a mere conjecture about vacuum fluctuations as a possible reservoir of energy (Bohm and Hiley 1993, 48), there is no explanation provided of where the required energy might originate from or how such energy might be 'tapped into'.

The Law of Inertia requires that there be some change made to a body's momentum for its path to be altered. If the Active Information Hypothesis is correct, then a quantum particle would have to be deviated from its initial trajectory (i.e., its momentum changed) as a result of the internal processing of the 'active information'. If we relate this to the ship analogy, a ship can have the highest quality radio receiver, a state-of-the-art autopilot, a large reserve of fuel (i.e., energy content), but if it has no engines then none of these other components will affect any change in the ship's momentum. We might ask, metaphorically, what constitutes the electron's engines? Bohm and Hiley give no indication as to how the interior make-up of a quantum particle can possibly affect its momentum.

Another aim of philosophy of science is to highlight the shortcomings of scientific theories. In keeping with this aim, substantial flaws in the Active Information Hypothesis have been exposed. In particular, the Active Information Hypothesis:

- leaves too many questions unanswered about its operation;
- cannot be applied to some elementary particles;
- would seem to require violations of the Law of Inertia; and
- does not provide a proper account of energy conservation.

Philosophy of science also has a role in proposing new avenues of research. The problems of the Active Information Hypothesis are sufficiently severe that they warrant its abandonment in favour of a more promising line of development. This will be presented below in terms of the physical characteristics of the wave field.

3 The Physical Nature of Potential Energy

Before proceeding to an alternative explanation for the quantum potential, it will be necessary to deal with the issue of the physical underpinnings of potential energy, as this will be crucial to subsequent discussion. In undergraduate studies of (classical and quantum) mechanical systems, the energy of a system is divided into kinetic and potential quantities. Potential energy is introduced to account for the ability of a physical entity to perform work on its surroundings (where work is given by the product of force and displacement) and for the purposes of energy conservation. The formal potential energy term is (explicitly or implicitly) defined as the potential energy of a particle or object. In electrostatics for example, a (point) particle with an electric charge q_1 at a distance r from another particle with charge q_2 is defined to have a potential energy V given by:

$$V = \frac{1}{4\pi\varepsilon}\frac{q_1 q_2}{r} \tag{6}$$

where ε is the electric permittivity constant. Such definitions of potential energy are drummed into students to the extent that it is accepted that potential energy is a particle characteristic which depends on its position. This is, in a strict sense, *incorrect*. The labelling of potential energy as a particle attribute is only a convenient description which is a statement of convention and not a matter of physical reality.

The familiar potential energy term is a potential energy function which gives the amount of the field's potential energy that is available to a particle situated within the field. In other words, potential energy is energy properly associated with fields, not particles. A field, for example, may be present in a spatial region which is totally devoid of any particles because fields can propagate enormous distances into otherwise empty regions of space, regions which might be many cubic light-years in size. Yet, despite the absence of particles, such a spatial region possesses a (potential) energy density due to the presence of a field (Jackson 1975, 46). Potential energy being a field attribute is only occasionally stated in the physics literature and indeed, assigning potential energy to particles is a standard and almost unquestioned practice. This is because, in the majority of physical contexts (such as particle mechanics), it makes no difference to the final result by assigning potential energy to a particle. Further, regarding potential energy as a particle property is easier to use and simpler for students to assimilate for this treatment acts as a kind of 'shortcut' to the actual location of potential energy in a field. This 'shortcut', however, is not possible in all physical situations, in particular those involving non-linear interactions (Freistadt 1957, 17). A notable exception to labelling potential energy as a particle attribute appears in the works of the respected physicist Wolfgang Rindler. He writes:

> In classical mechanics, a particle moving in an electromagnetic (or gravitational) field is often said to possess potential energy, so that the sum of its kinetic and potential energies remains constant. This is a useful "book-

> keeping“ device, but energy conservation can also be satisfied by debiting the *field* with an energy loss equal to the kinetic energy gained by the particle (1977, 83).

and again in a later text:

> ... [particle] potential energy, which is really nothing but a useful ‘book-keeping’ device. But physically it is more satisfactory to credit the field *itself* with whatever momentum or energy is required to ‘balance the books’ (1982, 132).

This is surely the critical point, i.e., a physically satisfactory account of the nature of potential energy (in both linear and non-linear interactions) requires that fields, not particles, possess potential energy.

In order to illustrate this, consider the following two examples. The first example concerns the everyday supply of household electricity. In most industrialised countries, electricity is supplied by power generating stations through heavy duty metallic cables using a form of alternating current, i.e., current that changes direction over a short time interval (typically with a frequency of 50 Hz). The regular change in the polarity of the electricity requires the electrons in the cables to oscillate back and forth about equilibrium positions. Consequently, there is no net electron flow along the cables from an electricity power station to a consumer. The electrons cannot, therefore, transport the electrical energy since they do not travel from source to user. Instead, the energy is transferred as potential energy in the generated electric field. It is the field and not the particles that possess potential energy. Ian Sefton is physics educator who has strongly argued against the mistaken view that electrons possess potential energy. He writes:

> ... [There is a] basic misconception that is often implied and sometimes explicitly stated in texts. The mistake is to speak of the electric potential energy (PE) of an electron as though the electron owns all the PE—it doesn’t. This error seems to be a reflection of a similar sloppy way of talking about gravitational PE. When you lift a brick do you increase its PE? No, ... PE is not stored in either Earth or the brick ... electrons don’t have PE of their own ... and in a circuit they generally don’t go anywhere much but energy is transferred very rapidly ... (2002, 2).

A second example may bring this into sharper focus. Consider an electrically charged particle placed in an external electric field. Such an external field may be produced by applying an electrical potential difference to two (usually parallel) metal plates. If the charged particle is released at rest between the plates before they become charged, the particle remains at rest. However, if the particle is released at rest between the plates when they are charged, the particle will immediately accelerate. The electric field between the charged plates imparts energy to the particle as it had no kinetic energy initially. This energy is gained at the expense of some (but not all) of the potential energy stored in the field between the charged plates, i.e., by a small fraction of the potential energy contained within the external electric field. Moreover, if we were to ‘shoot’ the charged particle in a direction towards the plate of similar charge to itself, the particle would decelerate and then come to a (momentary) stop. The particle's kinetic energy would then be instantaneously zero. If at the instant when the particle stops, we arrange for the electric field between the plates to be zero, then the value of the potential energy would also be zero. If potential energy is taken to be a particle property, then all the particle's energy (i.e., kinetic and potential) would have just disappeared from existence! This situation is inexplicable. The loss of potential energy when the external field is turned off can only be accounted for in a manner that is physically reasonable if potential energy is contained in the field.

It is also important to distinguish between the potential energy available to a particle situated in a field and the total energy stored in the field. In the electric field example, if the plates are the same size and shape, are parallel, and the particle is a perpendicular distance y from the plate of opposite charge, then the former energy is given by (qEy), where q is the particle's electric charge and E is the strength of the electric field. The total energy stored in the field is given by ($1/2\varepsilon AdE^2$) where A is the surface area of one plate, and d is the separation of the plates (Johnk 1975, 210). The amount of potential energy available to the particle depends on a number of factors such as the particle's location in the field. In this respect, the physicist and mathematician Hermann Weyl wrote:

> Not only the field as a whole, but every portion of the field has a definite amount of potential energy ... (1952, 70)

4 What is the Quantum Potential?

It was concluded in Sect. 2 that the Active Information Hypothesis cannot account for quantum phenomena and fails to accurately describe the quantum potential. What then, is the quantum potential Q? The mathematical expression of Q is very different from that found for potentials in classical physics. In Bohm's original papers, the form of the quantum potential was merely accepted as given by the mathematics (i.e., as derived from the Schrödinger equation). Others have seen a need to specify an origin or a source. A recent physics e-print by Parmenter and DiRienzo asks the same question. They write:

> There are, however, weaknesses in the original [deBroglie–Bohm] theory. One of the most obvious of these relates to the quantum potential Q: What is its source? Typically in physics a force, and its associated potential, have a source. However, nowhere in the literature is this fundamental question addressed in a physically reasonable way (2004, 2).

Parmenter and DiRienzo provide their own solution. They assume an isolated, many-body quantum system in which the wave field is *not* the origin of the quantum force (i.e., the particle's rate of change of momentum with respect to time) that is associated with the quantum potential. Their explanation for the origin of the quantum force is gained by concluding that each particle exerts a force on all other particles in the quantum system. The exact nature of this force is not specified. The source of the net quantum force on any one particle in the system is then just all the other particles (Parmenter and DiRienzo 2004, 7). Even if it were to be accepted that the quantum force does not originate from the wave field, the solution of Parmenter and DiRienzo is still untenable for it cannot explain the motion of a single quantum particle, such as occurs in the Double Slit experiment when only one particle is present between the slits and the screen at any particular time. If only a single particle is between slits and screen, then there are no other particles to exert a force to change the trajectory to one consistent with the resultant Double Slit interference pattern. We have to look elsewhere to account for the quantum potential.

Given the explanation of the nature of potential energy in Sect. 3, a (partial) answer to the question of what constitutes the quantum potential may be fleshed out in terms of Q being the potential energy function of the wave field. The quantum potential has some features in common with classical potentials for this reason, such as the relationship expressed by Eq. 4. This relationship shows that classical and quantum potentials are on an 'equal footing' in regard to affecting the motion of a quantum particle (Holland 1993, 74). However, Q cannot be completely equivalent to an external classical potential for such potentials are due to fields which do not, in general, travel along with the particle. Nor is a classical field that is externally imposed on a particle intrinsic to the physical system so created in the way that the wave field is intrinsic to a quantum system. The wave field acts on the quantum particle through the quantum potential and, as such, it is the wave field that is the origin of the quantum force. Where then does the energy that is necessary for the wave field to act upon the particle come from? An isolated, one-particle quantum system provides the answer for, in such a system, the only possible repositories of energy are the wave field and its accompanying particle. In this case, the wave field may gain energy at the expense of the particle's kinetic energy or may lose energy to the particle, as may be seen from the following example.

Consider the simple case of a cubical box with no classical potential inside and infinite potential outside. In other words, the particle cannot escape while the box remains intact. If an initially free (spinless) particle is trapped in such a box of side length L, then its wavefunction would take on the stationary waveform:

$$\psi = (2\mathbf{L})^{3/2}|\sin(\mathrm{n}_1\pi\mathrm{x}/\mathbf{L})\sin(\mathrm{n}_2\pi\mathrm{y}/\mathbf{L})\sin(\mathrm{n}_3\pi\mathrm{z}/\mathbf{L})|\,\mathrm{e}^{-i\mathrm{E_n t}/\hbar} = \mathrm{R}\,\mathrm{e}^{i\mathrm{S}/\hbar}$$

with total energy $\mathrm{E} = (\mathrm{n}_1^2 + \mathrm{n}_2^2 + \mathrm{n}_3^2)(\pi^2\hbar^2/2m\mathrm{L}^2)$, where n_1, n_2, n_3 are positive integers. In Orthodox Quantum Theory, the particle is assumed to have kinetic energy only and to be bouncing back and forth between the walls of the box. In deBroglie–Bohm Theory, the value of the quantum potential is given by: $\mathrm{Q} = -(\hbar^2/2m)(\nabla^2\mathrm{R})/\mathrm{R} = (\mathrm{n}_1^2 + \mathrm{n}_2^2 + \mathrm{n}_3^2)(\pi^2\hbar^2/2m\mathrm{L}^2)$. This is the same magnitude as the particle's kinetic energy is assumed to have in Orthodox Quantum Theory. However, since $\mathrm{S} = -\mathrm{Et}$, $\nabla\mathrm{S} = 0$, i.e., the particle has zero momentum and therefore zero kinetic energy. All the energy of the system is potential with the kinetic energy of the quantum particle having become stored in the wave field (Bohm 1952, 184; Riggs 1999, 3072). What's more, this energy will be returned to the particle if the wave field's stationary state is disturbed, e.g., if any side of the box is removed. Surprisingly, this explanation was originally suggested by David Bohm when he wrote:

> ... the kinetic energy of the particle will come from the ψ field, which *is able to store up even macroscopic orders of energy when its wave-length is small* (1953, 14, italics mine).

Bohm failed to take this idea to its logical conclusion opting in later years for his Active Information Hypothesis. Although this shows that the wave field may gain or lose energy to the quantum particle, it does not provide the exact mechanism for these energy transfers. However, this is also the case in classical theory, e.g. Newtonian gravitation does not give a mechanism for energy transfers between a massive particle and a gravitational field (Doughty 1990, 123). The explication of a mechanism for energy transfer in quantum systems will require a relativistic approach.

A complete explanatory account of physical phenomena requires the explication of the dynamics of the system. In deBroglie–Bohm Theory, this is provided by the existence and role of the quantum potential. In particular, the quantum potential is essential to account for energy conservation (as will be shown explicitly in Sect. 5). The conservation of energy is, of course, a principal reason why the concept of potential energy was introduced into physics. Indeed, problems in theoretical chemistry and solid state physics within the context of the deBroglie–Bohm Theory have shown that they require an application of the quantum potential approach for their solution (e.g., Garashchuk and Rassolov 2004, 1181–1190; 2003, 358–363; Grubin et al. 1994, 855–858).

5 Energy in a Quantum System and the Role of the Quantum Potential

The storage of energy may now be seen as a physical characteristic of the wave field. In what follows we shall restrict attention to a classically-free (i.e., V = 0), isolated, one-particle system for this brings out the essential features under examination. Analogous to the electric field case, the value of the quantum potential Q does not give (in a non-stationary quantum state) the total field energy but represents an amount of energy in the wave field that is available to the particle at its specific position in the field. Let's consider the storage of energy in the wave field that comes from its accompanying quantum particle. Let E = (T + Q) be the energy available to a quantum particle, where T is the particle's kinetic energy. Then the total rate of change with respect to time of E and Q are given by:

$$\frac{\mathrm{dE}}{\mathrm{d}t} = \frac{\mathrm{dT}}{\mathrm{d}t} + \frac{\mathrm{dQ}}{\mathrm{d}t} = \left(\frac{1}{2m}\right)\frac{\mathrm{d}}{\mathrm{d}t}(\nabla \mathrm{S})^2 + \frac{\mathrm{dQ}}{\mathrm{d}t} = (-\nabla \mathrm{Q})\cdot(\nabla \mathrm{S}/m) + \frac{\mathrm{dQ}}{\mathrm{d}t} \tag{7}$$

and

$$\frac{dQ}{dt} = \sum_{i=1}^{3} \frac{\partial Q}{\partial x^i}\frac{dx^i}{dt} + \frac{\partial Q}{\partial t} = (\nabla Q)\cdot(\nabla S/m) + \frac{\partial Q}{\partial t} \tag{8}$$

where we have used Eqs. 3 and 4. The term $[(\nabla Q)\cdot(\nabla S/m)]$ is equal to minus the rate of change of the particle's kinetic energy with respect to time, i.e., $(-dT/dt)$, as can be seen from Eq. 7. Thus:

$$\frac{dQ}{dt} = -\frac{dT}{dt} + \frac{\partial Q}{\partial t} \tag{9}$$

If Eq. 8 is substituted into Eq. 7, then the expression for (dE/dt) becomes:

$$\frac{dE}{dt} = \frac{\partial Q}{\partial t} \tag{10}$$

which shows that E is not constant if $(\partial Q/\partial t) \neq 0$, i.e. the energy available to the particle will change over time if the value of Q changes with time. The quantum potential Q gives the potential energy available to the particle at its specific position in the field but Q does not (in a non-stationary state) coincide with the total field energy.

In order to see this more concretely, consider the Hamiltonian density $\mathcal{H}$ associated with the time-dependent Schrödinger equation. The Hamiltonian density is the total density of mass-energy in an observer's frame of reference (Misner et al.1973, 137). In the non-relativistic context, this equates to the system's total energy density. Let the quantity H be defined by:

$$H = \int\int\int_{-\infty}^{\infty} \mathcal{H} d^3\mathbf{x} \tag{11}$$

The integration in Eq. 11 shows that the value of H is constant in the classically-free case (Holland 1993, 116). The quantity H may then be interpreted as the total energy of an isolated, classically-free quantum system (i.e., particle and wave field). H is the system's total energy and not just the energy of the wave field for the following reasons:

- particle and wave field are intrinsic parts of a single quantum system (i.e., the particle is not an 'add-on' to, or an enlargement of, the system);
- in any isolated system, total energy is a conserved quantity;
- the quantum particle receives energy from the wave field (Holland 1993, 120);
- the quantum potential represents part of the wave field's energy; and
- there are isolated, classically-free quantum systems where the field energy decreases and so cannot be constant (Riggs 1999, 3073–74).

Let the energy content of the wave field (in a non-stationary state) other than that given by the quantum potential be U, then

$$U = \mathrm{H} - \mathrm{E} = \mathrm{H} - (\mathrm{T} + \mathrm{Q}) \tag{12}$$

Since H is constant in the classically-free case, the total time rate of change of the energy content U is found by differentiating Eq. 12:

$$\frac{\mathrm{dU}}{\mathrm{d}t} = \frac{\mathrm{dH}}{\mathrm{d}t} - \left(\frac{\mathrm{dT}}{\mathrm{d}t} + \frac{\mathrm{dQ}}{\mathrm{d}t}\right) = -\frac{\mathrm{dE}}{\mathrm{d}t} = -\frac{\partial \mathrm{Q}}{\mathrm{d}t} \tag{13}$$

using Eq. 10. In other words, changes in the energy content of the wave field appear as an opposite change in the value of the quantum potential. Equation 13 shows that (∂Q/∂t) gives the change of the quantum potential due to changes in U, i.e., the time rate of change of Q due to changes in the amount of energy stored in the wave field other than at the particle's position (Riggs 1999, 3071). The particle's kinetic energy will then increase (decrease) with decreases (increases) in the amount of energy stored in the wave field. Any change in the particle's kinetic energy is explained by an energy conversion process, the concept of which is common to all branches of physics. Energy transfers, therefore, occur through a process whereby T⇆Q⇆U, with the direction of the arrows depending on whether the particle is losing or gaining kinetic energy. The quantum potential is the physical interface between particle and wave field and its role is to channel energy from wave field to particle and back again. These conversions need to be registered when accounting for the total energy of an isolated, classically-free quantum system. A suitable summary of such energy exchange was provided by Hermann Weyl (albeit from another field context):

> The total energy ... remains unchanged: they merely stream from one part of the field to another, and become transformed from field energy ... into kinetic-energy ... and *vice versa* (1952, 168).

This account of energy conservation and energy transfer provides an explanation of quantum mechanical tunnelling of a many-particle system from a potential well. (There have been a number of papers devoted to aspects of tunnelling within deBroglie–Bohm Theory, e.g., Dewdney and Hiley 1982, 27–48). Consider a situation where quantum particles are trapped in a potential well (such as can be produced by an electric field) with insufficient kinetic energy to escape. Despite this, quantum mechanics predicts that there is a small probability that some particles can be found outside the well. In Orthodox Quantum Theory, tunnelling arises as a consequence of the mathematics (i.e., by the constraints of continuity for the wavefunction and its first derivative at boundaries) and is experimentally confirmed. If we have an N-particle system ($N > 1$) with the mass of the i-th particle being m_i then classically one would expect the particles to be held in a well with a (finite) potential V if

$$[(\nabla_i \mathrm{S})^2/2m_i] < |\mathrm{V}|$$

for all i, $1 \leq i \leq N$. (It is usual for a well exerting an attractive force to have its potential energy defined to be zero at the 'top' of the well which then requires V to be negative inside the well.)

How can any particles become free if bound by an attractive force field? The solution becomes evident when the role of the quantum potential is recognised as facilitating the exchange of energy between the wave field and particles. In the many particle case, individual particles can gain energy from, or lose energy to, the wave field through their associated quantum potential (the Q_i as defined by Eq. 5) depending on their position in the wave field. The condition for the i-th particle to escape from the well is:

$$\left(\frac{1}{2m_i}\right)(\nabla_i S)^2 > \left| V - \left(\frac{\hbar^2}{2m_i R}\right)\nabla_i^2 R \right| \tag{14}$$

This condition can be satisfied in two ways depending on the nature of the potential well, the form that the wave field takes within the well, and the positions of the particles. First, an individual particle might gain sufficient energy from the wave field so that the particle's kinetic energy becomes large enough to satisfy the inequality (14). Second, a small part of the wave field might increase its energy content (and thereby increase the magnitude of quantum potentials Q_i associated with individual particles situated in that part of the wave field) so that the absolute value of the net potential energy in this region of the wave field (i.e., $|Q_i + V|$) is less than an individual particle's kinetic energy. The additional energy in both cases is gained at the expense of a portion of the kinetic energies of other (non-tunnelling) particles in the system. Either way, this would allow a small fraction of the total number of particles to break free of the binding force of the well.

Unlike a classical field, the form of the wave field has greater physical significance than its amplitude. The form of the wave field may be described with reference to its wavefronts. A wavefront is defined as a surface over which the phase of the wave ($S/\hbar$) is constant. Common examples of wavefronts include spherical waves which have expanding spheres as their wavefronts and plane waves whose wavefronts are flat planes perpendicular to the direction of wave propagation. The shape of a wavefront depends (in part) on what the wave field encounters, i.e., whether its initial shape has been altered by passing over or through an obstruction. It is generally the case that when a wave changes its shape there will be a change in its amplitude. Returning to a one-particle system, the total rate of change of the amplitude R with respect to time is:

$$\frac{dR}{dt} = \sum_{i=1}^{3} \frac{\partial R}{\partial x_i}\frac{dx_i}{dt} + \frac{\partial R}{\partial t} = (\nabla R).\left(\frac{\nabla S}{m}\right) + \frac{\partial R}{\partial t}$$

The term $[(\nabla R) \cdot (\nabla S/m)]$ gives the rate of change in the value of R due to changes in the size of the wave field (e.g., due to the wave expanding). The term $(\partial R/\partial t)$ shows the rate of change of R explicitly due to changes over time in the shape of its wavefronts. The dependence of $(\partial Q/\partial t)$ on $(\partial R/\partial t)$ is given by:

$$\frac{\partial Q}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial}{\partial t}\left(\frac{\nabla^2 R}{R}\right) = -\frac{\hbar^2}{2mR}\nabla^2\left(\frac{\partial R}{\partial t}\right) - \frac{Q}{R}\left(\frac{\partial R}{\partial t}\right) \tag{15}$$

which clearly shows that, in general, $(\partial Q/\partial t) \neq 0$ over the time interval of a change in the wave field's shape. The more pronounced the change in shape is, the greater will be the amount of energy exchanged between particle and wave field. Since $(dU/dt) = -(\partial Q/\partial t)$ from Eq. 13, it can be seen from Eq. 15 that the condition for energy exchange between wave field and particle (and vice-versa) is $(\partial R/\partial t) \neq 0$. This energy exchange process is not what would be expected classically.

A classically-free quantum particle that is initially not subject to any barriers or external fields moves with constant velocity and its wave field is represented by a plane wave with constant amplitude. The value of Q for such a plane wave is zero. If this wave field is distorted by an encounter with an object or other obstruction then the wave field's shape will change and the wavefront will no longer be a plane wave of constant amplitude. The value of Q will then be different from zero since $(\partial Q/\partial t) \neq 0$ whilst $(\partial R/\partial t) \neq 0$, as can be seen from Eq. 15. In the case where the particle passes through a narrow slit, energy will be transferred from the particle to the wave field as the field's shape is altered during its passage through the slit. Both the kinetic energy of the particle and the amount of energy stored in the wave field will be time-dependent but the total energy of the system will remain constant. Once the particle has passed through the slit and thereafter the quantum system is unaffected by external influences, the wave field will expand and tend to the form of a plane wave. The field energy will decrease as the system's energy becomes kinetic, i.e., energy will transfer back from the wave field to the particle which will accelerate until the value of the quantum potential drops effectively to zero (Riggs 1999, 3072). It is interesting to note that although Bohm and Hiley denied any explanation in which the wave field itself transferred energy, a reading of the following description by them of the spread of a wave field packet leads to questioning their consistency on this issue:

> It is clear then that the particles are accelerated ... This acceleration is evidently a result of the quantum potential ... the quantum potential decreases as the wave packet spreads, falling eventually to zero.
>
> The picture is then that as the wave packet spreads, the particle gains kinetic energy ... the energy represented by the quantum potential was turned into kinetic energy (1993, 47).

6 The Nature of the Quantum Potential and Related Issues

We have seen that the quantum potential Q is the potential energy function of the wave field and represents a portion of the wave field's energy. This is not an entirely satisfactory response to the question of the quantum potential's origin and is, by no means, a complete explanation of its nature. We have not, for example, explained why Q takes the form $(-\hbar^2/2m)(\nabla^2 R/R)$rather than one more akin to the familiar classical potential functions (such as Eq. 6) which essentially have a (1/r) dependence, where r is the distance from the relevant particle. Nor have we completely explained why the effect of the wave field is independent of its intensity. These two issues are not mutually exclusive and are deserving of more attention than can be given in the current paper. It is however, fitting to make a few relevant remarks which make use the results reached earlier. Dealing first with the latter issue—the effect of the wave field (and therefore its energy content) cannot depend on its intensity (i.e., its amplitude squared) for if it did, the wave field could not have an amplitude large enough to store energy up to macroscopic orders of magnitude (cf. Bohm's comment quoted in Sect. 4). We also saw in Sect. 5, that the change of shape of the wave field is an important ingredient in determining energy transfer and storage. These factors strongly imply that the mechanism here is completely different to classical cases and cannot be amplitude dependent.

In relation to the former issue of the non-classical form of Q, we have seen that the quantum potential is structured in such a manner so as to facilitate energy storage in the wave field and energy exchange with quantum particles. In a many-particle quantum system this is manifested as a non-local effect which is orchestrated by the quantum potential and does not necessarily fall off with increasing distance. A potential energy function that can perform these roles could not be of the same form as classical potential functions, i.e., since the effect may not fall off with distance, the quantum potential cannot be proportional to (1/r). Here again this implies that the mechanism must be different to the classical case.

Although it is acknowledged in some of the literature on the deBroglie–Bohm Theory that the wave field provides energy-momentum to the particle, it is also claimed that energy is not conserved for the quantum system as a whole (Holland 1993, 120). Energy non-conservation is claimed because Newton's Third Law (Action-Reaction) is not obeyed, i.e. the wave field acts on the quantum particle but the particle does not react back on the wave field. This is correct in the sense that the shape or size of the wave field is not directly affected by the particle. The issue of the back reaction is really a more general question of the status of the Third Law and its applicability outside classical physics. Yet, even in classical physics, it is possible to find examples where there is an action without a corresponding *equal and opposite* reaction (Goldstein 1980, 7–8; Fowles 1977, 44). Although this is a topic that goes beyond the scope of the current paper, it is appropriate to make some comment on the applicability of the Third Law to quantum systems.

In the earlier example of a charged particle accelerated by an external electric field between charged plates, there is an obvious action of the external field on the particle but what is the reaction and how is it mediated? Before answering this question, consider the following description of fields by Noel Doughty in his text *Lagrangian Interaction*:

> Fields are thus of two forms, those like gravity or electromagnetism which are generated by a source (for example mass or electric charge), and those which are not and represent the sources themselves, such as the non-relativistic Schrödinger wave function ... The field equations of a sourced, or mediated

field, can be recognised by the presence in them of a term, the *source* term, which does not contain the field itself (1990, 139).

A charged particle is surrounded by its own very small electric field which is independent from any external field. Both the particle's field and the external field (each with its own source) are distorted in shape by their mutual interaction. The answer to the above question is, of course, that the particle's electric field exerts a force on the plates equal and opposite to that which it experiences allowing the Third Law to hold. Classical action-reaction holds in cases of contact phenomena and of most mediated field interactions. However, as Doughty rightly points out, the Schrödinger wave field is not a mediated field and therefore there is no familiar means to carry a classical reaction from the quantum particle to the wave field. The lack of a classical reaction on the wave field should not be viewed as a flaw in deBroglie–Bohm Theory as has been suggested by some commentators (e.g., Anandan and Brown 1995, 359). It has also been mooted that a source term should be added to the Schrödinger equation in order to rectify the 'problem' (Squires 1994, 129; Abolhasani and Golshani 1999, 304). This, however, would lead to a non-linear wave equation which would produce predictions in conflict with well-established empirical results.

Instead of viewing the absence of a classical reaction as a defect, it should be seen as a new insight into the quantum domain. Indeed, the late James Cushing argued that our intuitions about classical action-reaction might not be reliable in the quantum realm (Cushing 1994, 45–46). This suggestion is very plausible in light of the following factors. First, a better understanding of action-reaction in deBroglie–Bohm Theory might be arrived at by beginning with a many-body system and deducing the dynamics of its sub-systems. In this way, the influences of the parts of a system on each other should become apparent (ibid.). Second, in an isolated, classically-free quantum system, it is still consistent to claim that the total energy is conserved, despite the absence of a classical reaction, because it can be shown that the field gains energy from the particle with energy transformations facilitated through the quantum potential.

It would seem suitable at this juncture to consider whether we might be looking at this problem from the wrong perspective. The discussion so far has been treating wave field and particle as separate but interacting entities for the purposes of the Third Law (like a charged particle in an external electric field). They are not, of course, separate entities and this needs to be taken into account. Equations 13 and 15 together indicate that the energy exchanges between particle and wave field are related to changes in the shape of the wave field. One part of a quantum system merely responds to changes in another part of the system without this being of a classically expected kind. It must be remembered that what is occurring in the quantum case are changes in a *single entity*.

The conclusions of this section imply that the level of analysis which is appropriate to the issue of origin of the quantum potential is where the nature of the wave field itself is the subject. An in-depth ontological account of the wave field would provide a more substantial explanation of the origin and nature of the quantum potential than has been possible here. In this regard the following questions should be addressed:

- What fundamental properties of the wave field can be identified?
- Why does the wave field propagate according to the Schrödinger equation?
- What is the exact process by which energy transfers are achieved between particle and wave field?
- What further insights into the wave field would studies of many-particle quantum systems disclose?
- Is there a preferred frame of reference that would reveal more about the wave field?
- What other aspects of the wave field might be found by examination of relativistic deBroglie–Bohm Theory?

7 Conclusions

The quantum potential is the potential energy function of the wave field. It gives the amount of the wave field's potential energy that is available to quantum particles. The well-established principle of energy conservation holds in classically-free quantum systems. This is achieved by energy exchanges between the quantum particles and wave field. The quantum potential facilitates these exchanges and provides an explanation of quantum phenomena such as tunnelling from a potential well. The quantum potential has novel features that would be unexpected on the basis of our understanding of classical potentials. The question of the origin of the quantum potential ultimately has to be a question about the nature of the wave field.

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