PHYSICS REPORTS (Review Section of Physics Letters) 144, No. 6 (1987) 321-375. North-Holland, Amsterdam

AN ONTOLOGICAL BASIS FOR THE QUANTUM THEORY

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Received July 1986

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0 370-1573/$19.25 © Elsevier Science Publishers B.V. (North-Holland Physics Publishing Division)

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I. Non-relativistic Particle Systems

D. Bohm and B.J. Hiley

Abstract:

In this paper we systematically develop an ontology that is consistent with the quantum theory. We start with the causal interpretation of the quantum theory, which assumes that the electron is a particle always accompanied by a wave satisfying Schrödinger's equation. This wave determines a quantum potential, which has several qualitatively new features, that account for the difference between classical theory and quantum theory.

Firstly, it depends only on the form of the wave function and not on its amplitude, so that its effect does not necessarily fall off with the distance. From this, it follows that a system may not be separable from distant features of its environment, and may be non-locally connected to other systems that are quite far away from it.

Secondly, in a many-body system, the quantum potential depends on the overall quantum state in a way that cannot be expressed as a preassigned interaction among the particles. These two features of the quantum potential together imply a certain new quality of quantum wholeness which is brought out in some detail in this article.

Thirdly, the quantum potential can develop unstable bifurcation points, which separate classes of particle trajectories according to the "channels" into which they eventually enter and within which they stay. This explains how measurement is possible without "collapse" of the wave function, and how all sorts of quantum processes, such as transitions between states, fusion of two systems into one and fission of one system into two, are able to take place without the need for a human observer. Finally, we show how the classical limit is approached in a simple way, whenever the quantum potential is small compared with the contributions to the energy that would be present classically. This completes our demonstration that an objective quantum ontology is possible, in which the existence of the universe can be discussed, without the need for observers or for collapse of the wave function.

1. Introduction

Quantum theory has been presented almost universally as a theory giving nothing but statistical predictions of the results of measurements. The fact that the predictions are statistical is, of course, not in itself very novel, but the basically new feature on which most interpretations agree is that the theory can be formulated only in terms of the results of a *measurement process*, and that there is no way even to conceive of the individual actual system, except insofar as it manifests itself through the phenomenona that are to be observed in such a process of measurement.*

In contrast, previous theories (e.g., Newtonian mechanics or classical statistical mechanics) started with assumptions about what each individual system *is* and discussed both measurement and statistics as having a secondary kind of significance that is ultimately based on assumptions as to what is, with regard to individual systems. That is to say, previous theories dealt basically with the ontology of individual actual systems while the quantum theory appears to have a fundamental and irreducible epistomological and statistical component. Indeed, without discussing the statistical results of measurements there would be nothing to talk about in the theory at all except for pure mathematics with no physical interpretation.

In 1952 one of us (DB) [3] proposed a causal interpretation of the quantum theory based on the assumption that an individual electron, for example, is constituted of a particle satisfying certain

* Ballantine [1] has proposed what he regards as a statistical interpretation which is consistent with the assumption that the concept of an individual electron in its actual movement is meaningful. However, he discusses only ensembles, and never makes any statement as to what the individual electron actually *is*. The same is true of Lande's [2] theory.

equations of motion *and* a wave satisfying Schrödinger's equation. Both particle and wave are taken to be objectively real whether they are observed or not. This has raised the possibility that quantum mechanics can be understood essentially in terms of ontological assumptions concerning the nature of individual systems so that the epistemological and statistical content would then take on a secondary role as in Newtonian mechanics.

In support of this view, we have analysed the measurement process in great detail in a further paper using the causal interpretation [4], and we have shown that the objective reality of the individual measurement process can be maintained consistently, without the need either for "collapse of the wave function" or for consciousness to play a fundamental role at this level. However, Shimony [5] has pointed out that in the very act of discussing in this way in terms of a *measurement* process, we are still, in effect, giving epistomology a basic role, at least tacitly. Perhaps one could say that Shimony is asking us *explicitly* to formulate a theory solely in terms of what Bell [6, 7] has called "beables", without bringing in "observables" except as a special case of what is happening among the beables.

To meet this challenge we shall in this paper further discuss the ontological significance of the causal interpretation of the quantum theory, in terms of a few illustrative examples, that serve to bring out what is implied in this approach. These will include the processes of quantum transition, of "fusion" of two particles into a single combined system, and of "fission" of such a combined system into two particles. (The latter cases have already been discussed to some extent in an earlier paper [8].)

We feel that these examples are sufficiently broad in their implications to indicate how the ontology works out more generally, and to enable us to understand individual quantum processes without having to bring in measurement. Indeed, we shall see that what is now called a measurement is itself only a special case of the above-mentioned processes of "fusion" and "fission", in which the thing measured and the measuring apparatus may now be seen to constitute an *indivisible* whole. And as Bell [7] has suggested, the term measurement is therefore somewhat inappropriate, since what is "measured" is, to a considerable extent, formed within the activity of the process itself.

Although we do thus interpret the quantum theory in terms of a particular kind of ontology, this requires us to introduce several fundamentally new notions as to the nature of being (or of "beables") that may be admitted into physical theories. However, as these notions are of a quite different kind to those that have generally been accepted in physics thus far, it will not be useful to summarize them at this point. Rather, it will be best to let them come out in full detail at appropriate stages of the discussion. All that we wish to emphasize here is that the quantum theory can be understood intuitively on an ontological basis only if we are ready to consider the assumption of fundamentally new kinds of qualities and properties for matter rather than merely to continue the development of past lines of thought.

Finally, we show that the causal interpretation leads to a simple approach to the classical limit in which, on the basis of a single notion of reality that is valid at all levels, one can see that classical behaviour results when a certain typically quantum mechanical contribution to this reality (the quantum potential) can be neglected.

2. New ontological implications of the causal interpretation. The single-particle system

We shall now develop in some detail the main new implications of the causal interpretation of quantum theory. Firstly, as has indeed been suggested in the Introduction, we suppose that the electron, for example, actually *is* a certain kind of particle following a continuous and causally

determined trajectory. As we have also suggested, however, this particle is never separated from a new type of quantum wave field that belongs to it and that fundamentally affects it. This quantum field, $\psi(\boldsymbol{x}, t)$, satisfies Schrödinger's equation, just as the electromagnetic field satisfies Maxwell's equations. It too is therefore causally determined.

In classical physics, a particle moves according to Newton's laws of motion, and, as is well known, the forces that enter into these laws can be derived from the classical potential, V. The basic proposal in the causal interpretation is that the quantum theory can be understood in a relatively simple way by assuming that the particle is also acted on by an additional quantum potential, Q, given by

$$Q = \frac{-\hbar^2}{2m}\frac{\nabla^2 R}{R} \qquad \text{where } R = |\psi|^2 \tag{1}$$

and $\hbar$ is Planck's constant, while m is the mass of the particle. Evidently, the quantum potential is determined by the quantum wave field, ψ.

To justify this proposal, we begin by considering Schrödinger's equation for a single particle

$$\mathrm{i}\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\psi + V\psi\,. \tag{2}$$

We write

$$\psi = R\,\mathrm{e}^{\mathrm{i}S/\hbar}$$

and obtain

$$\frac{\partial S}{\partial t} + \frac{(\nabla S)^2}{2m} + V + Q = 0 \tag{3}$$

where Q is given in eq. (1) and

$$\frac{\partial P}{\partial t} + \nabla\cdot\frac{(P\nabla S)}{m} = 0\,, \qquad \text{with } P = R^2\,. \tag{4}$$

Clearly eq. (3) resembles the Hamilton–Jacobi equation except for an additional term, Q. This suggests that we may regard the electron as a particle with momentum $p = \nabla S$ subject not only to the classical potential V but also to the quantum potential Q. Indeed the action of the quantum potential will then be the major source of the difference between classical and quantum theories. This quantum potential depends on the Schrödinger field ψ and is determined by the actual solution of the Schrödinger equation in any particular case.

Given that the electron is always accompanied by its Schrödinger field, we may then say that the whole system is causally determined; hence the name "causal interpretation".

Equation (4) can evidently be regarded as a continuity equation with $P = R^2$ being a probability density, as Born suggested. The function P has, however, two interpretations, one through the quantum potential and the other through the probability density. It is our proposal that the fundamental meaning of R (and therefore indirectly of P) is that it determines the quantum potential. A secondary meaning is that it gives the probability density for the particle to *be* at a certain position. Here we differ from Born who supposed that it was the probability of *finding* the particle there in a suitable measurement.

Indeed, as has been pointed out in the Introduction, in the causal interpretation, the measurement process itself has to be interpreted as a particular application of the theory, which is formulated basically in terms of "beables" rather than of "observables" [7] (while the observables are treated as statistical functions of the beables).

Equation (4) implies that it is consistent to interpret P as a probability density in a statistical ensemble of well-defined trajectories, each following the causal laws described above. For if the density holds initially, then this equation guarantees that it will hold for all time. We shall discuss the question of why there is such a statistical ensemble further on, as well as why its probability density will approach $|\psi|^2$, no matter what the initial form of this density may have been.

As the theory develops, we shall find that the electron is by no means a structureless particle. Rather, what is suggested by its behaviour is that it is a highly complex entity that is deeply affected by its quantum field in an extremely subtle and dynamic way. Moreover, this entity is not to be regarded (as is done in the usual interpretations) as somehow directly possessing both particle-like and wave-like properties. Rather, the observed wave-like properties will follow, as we shall see, from the general effect of the quantum wave field on the complex structure of the particle.

At first sight, it may seem that to consider the electrons as some kind of particle that is affected by the quantum field, ψ, is a return to older classical ideas. Such a notion is generally felt to have long since been proved to be inadequate for the understanding of quantum processes. However, closer inspection shows that this is not actually a return to ideas of this sort. For the quantum potential has a number of strikingly novel features, which do not cohere with what is generally accepted as the essential structure of classical physics. As we shall see, these are just such as to imply the qualitatively new properties of matter that are revealed by the quantum theory.

The first of these new properties can be seen by noting that the quantum potential is not changed when we multiply the field intensity ψ by an arbitrary constant. (This is because ψ appears both in the numerator and the denominator of Q.) This means that the effect of the quantum potential is independent of the strength (i.e., the intensity) of the quantum field but depends only on its *form*. By contrast, classical waves, which act mechanically (i.e., to transfer energy and momentum, for example, to push a floating object) always produce effects that are more or less proportional to the strength of the wave.

To give an analogy, we may consider a ship on automatic pilot being guided by radio waves. Here too, the effect of the radio waves is independent of their intensity and depends only on their form. The essential point is that the ship is moving with its own energy, and that the *information* in the radio waves is taken up to direct the much greater energy of the ship. We may therefore propose that an electron too moves under its own energy, and that the information in the *form* of the quantum wave directs the energy of the electron.

This introduces several new features into the movement. First of all, it means that particles moving in empty space under the action of no classical forces still need not travel uniformly in straight lines. This is a radical departure from classical Newtonian theory. Moreover, since the effect of the wave does not necessarily fall off with the intensity, even distant features of the environment can profoundly affect the movement. As an example, let us consider the interference experiment [9]. This involves a system of two slits. A particle is incident on this system, along with its quantum wave. While the particle can only go through one slit or the other, the wave goes through both. On the outgoing side of the slit system, the waves interfere to produce a complex quantum potential which does not in general fall off with the distance from the slits. This potential is shown in fig. 1. Note the deep "valleys" and broad "plateaux". In the regions where the quantum potential changes rapidly there is a strong force on the

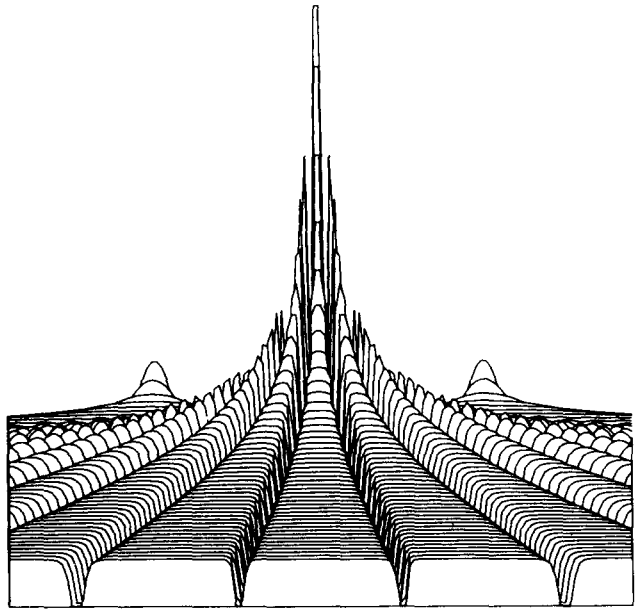

Fig. 1. The quantum potential for the two-slit experiment.

particle. The particle is thus deflected, even though no ordinary type of force is acting. The movement of the particle is therefore modified as shown in fig. 2 (which contains an ensemble of possible trajectories).

In this explanation of the quantum properties of the electron, the notion of information plays a key role. Indeed it is helpful to extend this notation of information and introduce what could be called *active* information. The basic idea of active information is that a *form* having very little energy enters into and directs a much greater energy. The activity of the latter is in this way given a form similar to that of the smaller energy. It is therefore clear that the original energy-form will "inform" (i.e. put form into) the activity of the larger energy.

As an example, consider a radio wave whose form carries a signal. The energy of the sound that we hear from our radio comes, however, not from this wave but from its power plug or batteries. This is essentially an "unformed" energy, that takes up the form or information carried by the radio wave. The information in the radio wave is, in fact, *potentially* active everywhere, but it is *actually* active, only where and when its form enters into the electrical energy within the radio. A more developed example of active information is obtained by considering the computer. The information content in a silicon chip can similarly determine a whole range of potential activities which may be actualized through having the form of this information enter the electrical energy coming from a power source. Which of these possibilities will in any given case be actualized depends on a wider context that includes the software programmes and the responses of the computer operator at any given moment.

Although the computer does thus indicate a kind of objective significance for information, nevertheless like the radio set it depends on a structure set up through the thought of human beings and so it still retains a trace of subjectivity. An example that does not involve structures set up by human beings is the function of the DNA molecule. The DNA is said by biologists to constitute a code, that is to say, a language. The DNA molecule is considered as information content for this code, while the "meaning" is expressed in terms of various processes; e.g., those involving DNA molecules, which "read" the

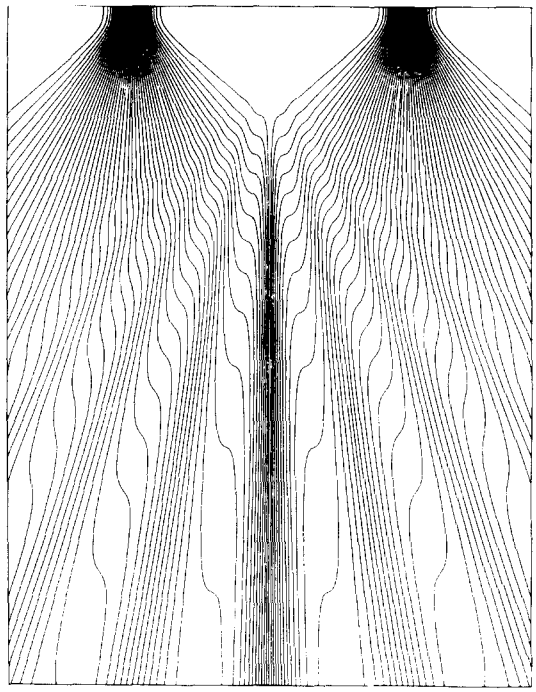

Fig. 2. Trajectories for the two-slit experiment.

DNA code, and carry out the activities that are implied by particular sections of the DNA molecule. The comparison to our notion of objective and active information is very close. Thus, in the process of cell growth it is only the *form* of the DNA molecule that counts, while the energy is supplied by the rest of the cell (and indeed ultimately by the environment as a whole). Moreover, at any moment, only a part of the DNA molecule is being "read" and giving rise to activity. The rest is potentially active and may become actually active according to the total situation in which the cell finds itself.

The notion of active information is not only relevant in objective contexts as those described above, but it evidently also applies to subjective human experience. For example, in reading a map we apprehend the information context through our own mental energy. And by a whole set of virtual or potential activities in the imagination, we can see the possible significance of this map. Thus the information is immediately active in arousing the imagination but this activity is still evidently inward within the brain and nervous system. If we are actually travelling in the territory itself then, at any moment, some particular aspect may be further actualized through our physical energies, acting in that territory (according to a broader context including what the human being knows and what he is perceiving at that moment).

It is clear that the notion of active information already has a widespread application in many areas of human experience and activity, as well as beyond these (in the case of DNA). Our proposal is then to extend the notion of the possibility of the objective applicability of the notion of active information to the quantum level. That is to say the information in the quantum potential is *potentially* active everywhere, but *actually* active only where this information enters into the activity of the particle.

This implies, however, that as we have already suggested, an electron, or any other elementary particle, has a complex and subtle inner structure (e.g., at least comparable to that of a radio). This

notion goes against the whole tradition of modern physics which assumes that as we analyse matter into smaller and smaller parts its behaviour grows more and more elementary. But our interpretation of the quantum theory indicates that nature is far more subtle and strange than previously thought. However, this inner complexity is perhaps not as implausible as may appear at first sight. For example, a large crowd of people can be treated by simple statistical laws, whereas individually their behaviour is immensely more subtle and complex. Similarly, large masses of matter reduce approximately to a simple Newtonian behaviour, whereas the molecules and atoms out of which matter is built have a more complex inner structure.

To make this suggestion yet more plausible, we note that between the shortest distances now measurable in physics (of the order of 10^{-16} cm) and the shortest distances in which current notion of space time probably has meaning, of the order of 10^{-33} cm, there is a vast range of scale in which an immense amount of yet undiscovered structure could be contained. Indeed, this range of scale is comparable to that which exists between our own size and that of the elementary particle.

So far, the double slit experiment has been discussed in terms of a single particle. But a typical experiment, in fact, involves a statistical ensemble of particles all having the same quantum field, ψ. The experiment can be so arranged that each particle, along with its quantum field, goes through the slit system separately and independently. Each of these electrons start from a different initial position, and so go through the slit system in a different way, following one of the ensemble of trajectories shown in fig. 2. As a result, it will arrive at a different point on the screen. After many such particles have gone through, one then obtains a statistical ensemble of spots on the detection screen, which add up to the well-known interference pattern.

Once the quantum field, ψ, is specified, the results of each experiment are in principle determinate and depend only on the initial conditions of the particle. The latter will however fluctuate from one case to the next, because the particles emerge from a source (e.g., a hot filament), in which they are subject to irregular thermal perturbations. Within this source, the actual trajectory of a particle is extremely sensitive to the complex details of the quantum potential, which is in turn sensitive to the small thermal disturbances from the other particles in the thermodynamic system. The motion is therefore unstable and chaotic in a sense, that has been defined, for example, by Kadonoff et al. [10], and that we shall discuss later in section 4. This means that complete predictability and controllability of the initial conditions is, in practice, essentially impossible. And so, it will also not be possible to predict just where each electron will arrive on the detection screen. In this fashion, statistical notions of the *predictions* of the quantum theory can be explained while causality is still basic to the theory itself.

As to why the probability density in a thermal source should approach $|\psi|^2$, there are several possible explanations. First of all, one can give reasons why it is plausible that this will be the equilibrium distribution resulting from the effects of chaotic thermal disturbances on the quantum potential of any particle that emerges [11]. And secondly, it is possible to assume a further stochastic process of random fluctuations of the momentum around on average, $\bar{\bar{\boldsymbol{p}}} = \nabla S$, representing, for example, the action of a subquantum level that would lead to $P = R^2$ in the long run [12, 13].

The fact that each particle responds to information from the entire environment gives a simple and tangible account of Bohr's notion of the undivided wholeness of the experimental conditions and the experimental results [14, 15]. Thus, each particle goes through only one of the slits, and yet its motion is fundamentally affected by information coming from both slits. One has therefore to consider the whole of the relevant experimental solution, to understand what happens in each case. However, this sort of wholeness implied by the causal interpretation differs in an important way from that of Bohr. For in the causal interpretation, while the entire experiment has to be treated as an undivided whole, this whole is

analysable in thought (e.g., through the movement of the particle acted on by the quantum potential). However, in Bohr's approach, the entire experimental situation is an *unanalysable* whole, about which nothing more can be said at all.

3. Extension to the many-body system

We shall now go on to consider the many-body system in which we shall see, in several striking ways, a further development of this difference between classical and quantum ontologies.

We shall begin by considering the two-body system. The wave function $\psi(\boldsymbol{r}_1, \boldsymbol{r}_2, t)$ satisfies the Schrödinger equation

$$i\hbar \frac{\partial \psi}{\partial t} = \left[-\frac{\hbar^2}{2m} (\nabla_1^2 + \nabla_2^2) + V \right] \psi \tag{5}$$

where ∇_1 and ∇_2 refer to particles 1 and 2 respectively. Writing $\psi = R\, e^{iS/\hbar}$ and defining $P = R^2 = \psi^* \psi$, we obtain

$$\frac{\partial S}{\partial t} + \frac{(\nabla_1 S)^2}{2m} + \frac{(\nabla_2 S)^2}{2m} + V + Q = 0 \tag{6}$$

where

$$Q = -\frac{\hbar}{2m} \frac{(\nabla_1^2 + \nabla_2^2) R}{R} \tag{7}$$

and

$$\partial P/\partial t + \mathrm{div}_1 (P \nabla_1 S/m) + \mathrm{div}_2 (P \nabla_2 S/m) = 0 \,. \tag{8}$$

As in the case of the one-body system, eq. (6) can be interpreted as the Hamilton–Jacobi equation with the momenta of the two particles being respectively

$$\boldsymbol{p}_1 = \boldsymbol{\nabla}_1 S \qquad \text{and} \qquad \boldsymbol{p}_2 = \boldsymbol{\nabla}_2 S \,.$$

But this time it is the Hamilton–Jacobi equation for a system of two particles responding not only to the classical potential, V, but also the quantum potential, Q. This latter now depends on the position of both particles in a way that does not necessarily fall off with the distance. We thus obtain the possibility of a *non-local interaction* between the two particles.

Going on to consider the N-body system we will have

$$Q = Q(\boldsymbol{r}_1, \boldsymbol{r}_2, \ldots, \boldsymbol{r}_N, t)$$

so that the behaviour of each particle may depend non-locally on all the others, no matter how far away they may be.

If we take $P = R^2$ as the probability density in the configuration space of the two particles, then

clearly eq. (8) is the equation for the conservation of this probability. With this definition of probability, it follows, as has been shown, by means of a detailed and extensive treatment [3, 4], that all the statistical results of measurements are (as in the one-body case) the same in the causal interpretation as they are in the usual formulation of the theory.

For several centuries, there has been a strong feeling that non-local theories are not acceptable in physics. It is well known, for example, that Newton felt uneasy about action-at-a-distance [16] and that Einstein regarded this action as "spooky" [17]. One can understand this feeling, but if one reflects deeply and seriously on this subject one can see nothing basically irrational about such an idea. Rather it seems to be most reasonable to keep an open mind on the subject and therefore to allow oneself to explore this possibility. If the price of avoiding non-locality is to make an intuitive explanation impossible, one has to ask whether the cost is not too great.

The only serious objection we can see to non-locality is that at first sight it does not seem to be compatible with relativity because non-local connections in general would allow a transmission of signals faster than the speed of light. However, we have extended the causal interpretation to a relativistic quantum field theory. Although this interpretation now implies that events outside each other's light cones can be connected but the additional new features of the quantum potential do not permit a signal to be transmitted faster than light [4]. It follows then that the causal interpretation is compatible with the essential implications of the theory of relativity.

While non-locality as described above as an important new feature of the quantum theory, there is yet another new feature that implies an even more radical departure from the classical ontology, to which little attention has been paid thus far. This is that the quantum potential, Q, depends on the "quantum state" of the whole system in a way that cannot be defined simply as a pre-assigned interaction between all the particles.

To illustrate what this means in more detail, we may consider the example of the hydrogen atom, whose wave function is a product of function $f(\boldsymbol{x})$, where $\boldsymbol{x}$ is the centre-of-mass coordinate, and $g(\boldsymbol{r})$ where $\boldsymbol{r}$ is the relative coordinate,

$$\Psi = f(\boldsymbol{x})\ g(\boldsymbol{r})\ .$$

The quantum potential will contain a term representing the interaction of electron and proton

$$Q_1 = -\frac{\hbar^2}{2\mu}\frac{\nabla^2 g(\boldsymbol{r})}{g(\boldsymbol{r})}\ .$$

In the s state, Q_1 is a function only of r itself, while in the p state, it depends on the relative angles, θ and ϕ, as well. Evidently, it is impossible to find a single pre-assigned function of $\boldsymbol{r}$, which would simultaneously represent the interaction of electron and proton in both s and p states. And, of course, the problem would be still more sharply expressed if we brought in all the other states (d, f, etc.).

It is clear from the above that the wholeness of the entire system has a meaning going beyond anything that can be specified solely in terms of the actual spatial relationships between particles. This is indeed the feature which makes quantum theory go beyond mechanism of any kind. For the essence of a mechanical behaviour is that the parts interact in some pre-assigned way to make up the whole. Even if the interaction is non-local, the system is still mechanical as long as the interaction potential is a fixed and pre-assigned function of the particle variables. But in the causal interpretation of the quantum theory, this "interaction" depends upon the wave function of the entire system, which is not only

contingent on the state of the whole but also evolves with time according to Schrödinger's equation. Something with this sort of independent dynamical significance that refers to the whole system and that is not reducible to a property of the parts and their inter-relationships is thus playing a key role in the theory. As we have stated above *this is the most fundamentally new ontological feature* implied by the quantum theory.

The above-described feature should, in principle, apply to the entire universe. However, as has been shown elsewhere [18], when the wave function of a system factorises into two parts, the corresponding subsystems will behave independently. In this case, each subsystem can be treated on its own. It is this feature which provides the ground for the possibility of understanding how, in spite of quantum wholeness, the world still behaves in the classical limit as a set of relatively independent parts that interact mechanically.

Such considerations are crucially important to understand the application of quantum mechanics to the many-body system. For example, a chemical bond in a molecule in a certain quantum state can be seen to be a consequence of the quantum potential for the whole molecule, which is such that in this state it tends to hold the electrons in places where they contribute to the bonding. On the other hand, in a different quantum state, the quantum potential will be different, so that the molecule may not be stable.

Similarly, in the superconducting state of a many-electron system, there is a stable overall organized behaviour, in which the movements are coordinated by the quantum potential so that the individual electrons are not scattered by obstacles. One can say indeed that in such a state, the quantum potential brings about a coordinated movement which can be thought of as resembling a "ballet dance".

As the temperature goes up, the property of superconductivity disappears. In terms of our approach, this is because the wave function breaks up into a set of independent factors which can be thought of as representing independent pools of information, so that the electrons will cease to be guided by a common pool of information and will instead respond to independent pools of this kind. Therefore, the electrons begin to behave like an unorganized crowd of people who are all acting more or less independently and thus begin to jostle each other, so that the property of superconductivity disappears.

This suggests that the quantum potential arising under certain conditions has the novel quality of being able to organize the activity of an entire set of particles in a way that depends directly on the state of the whole. Evidently, such an organization can be carried to higher and higher levels and eventually may become relevant to living beings [19].

The possibility that information can be in one common pool or divide into independent pools in the way described above arises basically from the multi-dimensional nature of the wave function, which constitutes one pool when it is not factorizable and many when it is.

This is what enables us to answer one of the principle objections that has been made to causal interpretation, i.e., that the wave function, being in a configuration space, cannot be understood in a field in a 3 + 1-dimensional space time. For we are now regarding the wave function as belonging to an information structure that can quite naturally be considered to be multi-dimensional (organized into as many sets of dimensions as may be needed), rather than as a simple source of a mechanical force.

4. A brief resumé of the theory of measurements in the causal interpretation

We shall now give a brief resumé of our treatment of the quantum measurement process [4], as this treatment introduces a number of key new concepts that will be carried over into our discussion of the quantum potential approach.

Our first step is, for the sake of convenience, to divide the overall process of measurement into two stages. In the first stage, the "observed system" interacts with the "measuring apparatus" in such a way that the wave function of the combined system breaks into a sum of non-overlapping packets, each corresponding to a possible distinct result of the measurement. Let the initial wave function of the observed system be

$$\Psi_0(\boldsymbol{x}) = \sum_n C_n \psi_n(\boldsymbol{x}) ,$$

where the $\psi_n(\boldsymbol{x})$ are the possible eigenfunctions of the operators that are being "measured". The initial wave function of the combined system is then

$$\psi_0 = \phi_0(y) \sum_n C_n \psi_n(\boldsymbol{x})$$

where $\phi_0(y)$ is the initial wave function of the measuring apparatus. After the two systems have interacted Ψ_0 goes over into

$$\Psi_f = \sum_n n\psi_n(\boldsymbol{x})\, \phi_n(y)$$

where the $\phi_n(y)$ are the different wave packets of the relevant parameters of the apparatus that correspond to the possible results of the measurement. For a proper measurement to be made, the packets $\phi_n(y)$, must, of course, be distinct and non-overlapping.

As we show in Bohm and Hiley [4], during the period of interaction, the wave functions, and therefore the quantum potential, become very complex and rapidly fluctuating functions of the time. When the packets have separated, the "apparatus particles" must have entered one of them (say m) and will have zero probability of leaving (because there is no probability of entering the spaces in between packets). From then on, the quantum potential acting on the particles will be determined only by the packet $\psi_m(\boldsymbol{x})\, \phi_m(y)$, because all the other packets (which do not overlap this one) will not contribute to it. So, at least as far as the particles are concerned, we may ignore all the other packets, and regard them as, in the sense discussed in section 2, constituting inactive or physically ineffective information. Here, it must be emphasized that this will still happen even when there is some spatial overlap between $\psi_m(\boldsymbol{x})$ and the remaining packets, $\psi_n(\boldsymbol{x})$. This is because of the multi-dimensional nature of the many-body wave function, which implies that the packet, $\psi_m(\boldsymbol{x})\phi_m(y)$, and any other packet, say $\psi_v(\boldsymbol{x})\phi_v(y)$, will fail to overlap as long as *one* of its factors fails to overlap, even though the other factor would still have some overlap.

One may describe what happens in another way by saying that each packet, $\phi_n(y)$ forms a kind of "channel". During the period of interaction, the quantum potential develops a structure of bifurcation points, such that trajectories of the apparatus particles on one side of one of these points enter a particular channel (say $\phi_m(y)$), while the others do not. Eventually, each particle enters one of the channels to the exclusion of all the others and thereafter stays in this channel. When this has happened, the "observed" particle will behave from then on, as if its wave function were just $\psi_n(\boldsymbol{x})$, even if $\psi_m(\boldsymbol{x})$ and the rest of the $\psi_n(\boldsymbol{x})$ should still overlap. The fact that the "apparatus particles" must enter one of their possible channels and stay there is thus what is behind the possibility of a set of clearly distinct results of a quantum measurement.

At this point, however, one may ask what is the role of the "inactive" packets, not containing the particles. Can we be sure that they must necessarily remain permanently inactive? The answer is that in

principle, it is in fact still possible to bring about activity of such packets. For example, one may apply an interaction Hamiltonian to one of these inactive packets, say $\psi_r(\boldsymbol{x})$, such that it comes to coincide once again with $\psi_m(\boldsymbol{x})$, while leaving $\phi_m(y)$ unchanged. The two packets together will then give us $\phi_m(y)(\psi_m(\boldsymbol{x}) + \psi_r(\boldsymbol{x}))$. If $\psi_m(\boldsymbol{x})$ and $\psi_r(\boldsymbol{x})$ overlap, there will be interference between them, and this will give rise to a new quantum state, in which the previously inactive packet, $\psi_r(\boldsymbol{x})$, will now affect the quantum potential, so that it will once again be active.

But here, it is necessary to note that thus far, the "measuring apparatus" and the "observed system" have been treated on an essentially symmetrical footing. We have not, as yet, brought into the theoretical description anything that would assign a special role to the state of the measuring apparatus as something that was actually capable of being known by a human being. It was here that we introduced our second stage of the measurement process, which contained a detection or registration device capable of amplifying the distinctions in the states of the "apparatus particles" to a large scale level that is easily observable by ordinary means. Such a registration device will contain a very large (macroscopic) number, N, of particles. When this device interacts with the "apparatus particles", y, its wave function $\lambda(Z_1, \ldots Z_N)$ will have to be brought into the discussion. To each distinct state, n, of the "apparatus particles", there will be a corresponding state, $\lambda_n(Z_1, \ldots Z_N)$ of the registration devices. The wave function of the relevant system will then be

$$\Psi = \sum_n C\, \phi_n(y)\, \psi_n(\boldsymbol{x})\, \lambda_n(Z_1, \ldots Z_N)\,.$$

Each of the λ_n will also not overlap with the others, so that even if the $\phi_n(y)$ should later come to overlap, this would still not affect the quantum potential, as the particles of the registration device will now be in distinct channels.

Could the channels of the registration device in turn be made to overlap again? In Bohm and Hiley [4] it was emphasized that this would have essentially zero probability, because in registration, there has occurred a thermodynamically irreversible process. (So that, for example, to have overlap here would be as improbable as for a kettle of water placed on ice to boil.)

It follows then that once registration has occurred, the packets (or channels) not containing all the relevant particles (including those constituting the registration device) will indeed be permanently inactive or physically ineffective. From this, we can see that there is no need to introduce a "collapse" of the wave function in a measurement. For, because of the very behaviour of all the relevant particles and of the wave functions, the "observed system" will from now on act entirely as if it were in the quantum state, $\psi_n(\boldsymbol{x})$. This state will correspond to the channels determined by the wave function of all the constituents of the registration device, that are actually occupied by the particles constituting this device. So it follows that everything will from this point on take place as if the wave function had "collapsed" to the actual result, without the need for any such collapse ever to occur. And this comes about without the need to bring in the consciousness of an observer, in arguments such as have been proposed by Wigner [20]. As in the case of superconductivity discussed in section 3, we can consider this process in terms of the "dance" of the particles, as guided by "pools" of information, represented by the wave function. The two systems begin by executing independent "dances", because the wave function then factorises, and corresponds to separate "pools" of information. During the period of interaction, both systems move according to a single and more complex pool of information, so that they are carrying out a common "dance". It then follows that what happens in the over-all process described above can better be regarded as a mutual transformation of observed system and observing apparatus, rather than as a measurement in which the two systems are governed by separate and

independent pools of information. Thus during the period of interaction this single more complex pool of information determines (among other things), a set of bifurcation points. Finally, as the particles enter one of the channels, as determined by the bifurcation points, the two systems are moving once again according to independent pools of information, and therefore executing separate "dances". But their movements are correlated, in the sense that each state of movement of the observed system goes together with a corresponding state of the apparatus. In this process of mutual transformation, one can indeed hardly say that anything has been measured for what are called the results of a measurement are not present to begin with, but come about in the process of interaction itself.

While it is clear that what is automatically called the measurement process has thus been given an overall causal and objective description, one may nevertheless ask what is the meaning of all the "empty" wave packets (i.e., those not containing particles). Those still satisfy Schrödinger's equation, but are nevertheless permanently inactive, in the sense that they never manifest themselves in the movements of the particles at all. Such packets seem to be floating, almost like wraiths in a strange multi-dimensional world. One can see, for example, in the "many worlds" interpretation of Everett [21], the problem is dealt with in a certain way, as each packet would correspond to a different universe, with its different measuring instrument (along with its different human observer as well). What do all these "empty" packets signify in the causal interpretation?

To help make clear what the permanent loss of potential for activity of such packets means, we may consider as an analogy a device that was used to illustrate what has been called the implicate order [22]. In this device, a droplet of insoluble ink was inserted into a viscous fluid such as glycerine, which was then subject to a shearing rotation in a controlled way. The ink droplet gradually became invisible as it was drawn out into a fine thread. When the rotation was reversed, this time the thread was gradually drawn together, until the ink droplet suddenly emerged again into visibility. While it was drawn out and invisible, the distribution of ink particles appeared to be "at random" with no order. Yet it had a hidden order, which was revealed when the droplet came together again. This order was described as implicate or enfolded, while the ink droplet itself was explicate or unfolded.

Let us now imagine printing a message in ink droplets suspended in glycerine. As these are enfolded, the information in the message becomes inactive, within the field of anything that is sensitive only to a concentration of ink beyond a certain minimum threshold. But it is still potentially active there, as can be seen by the fact that it can be unfolded again into its original form. If, however, the fluid had not been viscous, there would have been an irreversible diffusion. After such a process, the information in the message would be permanently inactive in the field in question, as the original form could never be reconstituted again. Nevertheless, in a certain sense, some highly enfolded transformation of the original structure of the printed message is still in principle present, regardless of how much diffusion has taken place.

The analogy to the quantum theory is clear. As long as the measuring apparatus interacts reversibly with the classical system, channels that are inactive with regard to the particles are still potentially active. But as soon as the irreversible interaction with the registration device takes place, the channels not containing particles are permanently inactive. In the usual language, we would say that the information has been "lost", but as with the diffusion of ink particles, it has merely ceased to be capable of acting in the manifest domain.

From the above, we see that the information content in the wave function is quite generally in a non-manifest (implicate) order (generally multi-dimensional), while the particles are in the ordinary (explicate) order of space time. Indeed, it can be shown [22] that the Green's function, which describes the movement of this wave function, corresponds to just a process of enfoldment and unfoldment. The

movements of the particles then express the meaning of the information content in a manifest (explicate) order.

In the above example, the information was ultimately "carried" or "held" by ink droplets. In other applications of information theory, there has always been some material system or field that carries the information. This implies that such information has a certain usually small energy, but that the energy in the activity which is its "meaning" is much larger and has an independent source. Can we look back at the information in the quantum field in this way?

As yet, we have no theory as to what is the origin of the quantum potential. However, we may suppose that the information it represents is carried in some much more subtle level of matter and energy, which has not yet manifested in physical research. In this connection, we may recall that in section 2, we proposed that the behaviour of matter does not always become simpler as we go to smaller low dimensions, and that a particle may have a structure (somewhere between 10^{-16} cm and 10^{-33} cm) which is complex and subtle enough to respond to information in ways that might even resemble, for example, the activity of a ship guided by radar waves. It is in this structure (which could include generalized fields of a very subtle nature as well as particles) that we may look for the basis of this information in physical structure.

Another analogy to the process in which information becomes inactive can be obtained by thinking of what happens when we make a decision from a number of distinct possibilities. Before the decision is made, each of these possibilities constitutes a kind of information. This may be displayed *virtually* in imagination as the sort of activities that would follow if we decided on one of these possibilities. Immediately after we make such a decision, there is still the possibility of altering it. However, as we engage in more and more activities that are consequent on this decision, we will find it harder and harder to change it. For we are increasingly caught up in its irreversible consequences and sooner or later we would have to say that the decision can no longer be altered. Until that moment, the information in the other possibilities was still potentially active, but from that point on such information is permanently inactive. The analogy to the quantum situation is clear for the information in the unoccupied wave packet becomes more and more inactive as more and more irreversible processes are set in train by the channel that is actually active.

In the case of our own experience of choice, the inactive possibilities may still have a kind of "ghostly existence" in the activity of the imagination, but eventually this too will die away. Similarly, according to our proposal, the inactive information in the quantum potential exists at a very subtle level of the implicate order. We may propose, however, that perhaps this too will eventually die away because of as yet unknown features of the laws of physics going beyond those of quantum theory.

The above discussion should help to make it clear how it is possible to understand quantum processes without bringing the collapse of the wave function and without bringing in the consciousness of an observer. D'Espagnat [23] has however suggested in a critical vein that our ability to do without the collapse of the wave function does in fact depend ultimately on consciousness. For in his view, we have "assumed that we perceive only the supplementary (particle) variables", and for him this implies that all the unoccupied packets are dropped from our account, simply because they are not perceived and therefore do not enter our consciousness. In our approach, however, our basic assumption has actually nothing to do with consciousness. Rather, it is that the particles are the direct manifest reality, while the wave function can be "seen" only through its manifestations in the motions of the particles. This is similar to what happens in ordinary field theories (e.g., the electromagnetic), on which the fields likewise can manifest themselves only through the forces that they exert on the particles.* Moreover the

* The main difference is that the particles can be sources of fields, whereas, in the quantum theory, particles do not serve as sources of the wave function. But evidently this does not affect the question of how the fields are manifest themselves.

conclusion that after an irreversible detection process has taken place, the unoccupied packets will never manifest themselves in the behaviour of the particles follows, as we have seen, from the theory itself and has nothing to do with our not being conscious of these packets.

We can further justify our assumption that the immediately manifest level is that of the particles by noting that (as will be shown in more detail in section 7) in the classical limit, from which the content of our sensory data comes, the quantum potential can be neglected. This means that all that we learn about the world through the senses has to pass through a level in which the wave function plays no essential role. That is to say, as we have indeed already indicated, empirical knowledge referring to the wave function has ultimately to come from inferences drawn from observations of the behaviour of structures of particles, as manifested at the classical level. This conclusion is however a consequence of our basic assumptions concerning the nature of reality as a whole and does not depend on any further hypothesis concerning the particular content of our consciousness.

5. Quantum transitions discussed independently of measurement

We are now ready to extend the notions developed here to show how specific but typical processes can be discussed apart from the context of measurement, and indeed apart from the need to bring in the activity of any human beings at all (e.g., no one is needed to prepare a quantum state).

We begin this section with a discussion of a transition process. If an atom, for example, is to jump from one stationary state to another, it is necessary that there be an additional system available which can take up the energy. This is usually the electromagnetic field within which, for example, a photon can be created. However, to avoid the complexities arising in the causal interpretation to the electromagnetic field [3, 4], we shall suppose that the energy is taken up in an Auger-like effect by an additional particle in the neighbourhood, that was originally in a bound state near the atom in question. Clearly the principles involved will be the same no matter what are the details of the system that carries away the energy.

Let us consider an atom containing an electron with coordinates x and with an initial wave function $\psi_0(x)\exp(-iE_0t)$ corresponding to a stationary state with energy E_0. (We shall write $\hbar = 1$ in the rest of the paper.) The additional particles will have coordinates y and an initial wave function $\phi_0(y, t)$. In general $\phi_0(y, t)$ will represent a state in which the y particle is bound near a centre. The combined system will then have an initial wave function,

$$\Psi_0 = \psi_0(x)\exp(-iE_0t)\,\phi_0(y, t)\,.$$

Through interaction between the atomic electron and the additional particle the wave function will begin to include other stationary states of the electron. To simplify the discussion, we will suppose that only one of these, $\psi_f(x)\exp(-iE_ft)$, contributes significantly. The corresponding wave function of the additional particle will be $\phi_f(y, t)$, which represents a particle that is no longer bound. (We suppose, of course, that the energy, $E_0 - E_f$, given off by the electron is considerably greater than the binding energy of the y particle.) As a function of time the wave function of the combined system will then be

$$\Psi_f = \psi_0(x)\exp(-iE_0t)\,\phi_0(y, t) + \int_0^t \alpha(t', t)\,\psi_f(x)\exp(-iE_ft)\,\phi_f(y, t - t')\,dt' \tag{9}$$

where $\alpha(t', t)$ can be calculated using ordinary time-dependent perturbation theory [26].

What the above formula means is that during a small interval of time dt' a contribution to the wave function will be produced which is described by the integrand. That is to say, there will be a term $\psi_f(x)\exp(-iE_f t)$ corresponding to the final state of the electron which is multiplied by $\phi_f(y, t-t')\,dt'$. This latter function corresponds to a y particle that during the time interval $(t-t')$ moves away from the atom very rapidly because it has absorbed the energy difference $E_f - E_0$. As a result there will be a negligible overlap with $\phi_0(y, t)$. As time goes on the total contribution to $\psi_f(x)\exp(-iE_f t)$ will accumulate in a way that is proportional to the integral in the above equation. This integral represents a sum of a set of contributions $\phi_f(y, t-t')\,dt'$, each of which has moved a different distance from the atom, but practically all of which will have a negligible overlap with $\phi_0(y, t)$.

As brought out in section 4, this feature of negligible overlap implies the establishment of a separate "channel" along with bifurcation points that divide all trajectories into two classes, i.e., those that enter the channel and those that do not. For small values of t, the number of trajectories entering the channel is proportional to t, and as a simple calculation shows, the proportionality factor yields the correct probability. This means that while interaction is taking place, the y particle and the electron will (as in the case of superconductivity) be executing a common "dance" but that afterwards their "dances" will be independent. However, they will be correlated, in the sense that for all initial conditions in which the electron has undergone a transition to a new "dance", the y particle will be moving freely and far away from the atom, while all other initial conditions will remain bound.

In the way discussed in section 4, the information in whichever is the unoccupied channel has become inactive. Of course, at this stage, it is still potentially active. But, as happened in the case of the measuring apparatus there is a second stage. In this stage, the y particle interacts irreversibly with its macroscopic environment. The same arguments used in section 4 will show that the unoccupied channels will then have lost the potential for activity, and so may from this point on, be dropped from the physical account.

Clearly, the entire process has, in this way, taken place with an individual system, and there is moreover no need to discuss a collapse of the wave function to that of the bound state. There is also no need to talk about the preparation of the initial state by a human being, for the x and y particles could fall into stationary states on their own account by giving up energy (e.g., to the electromagnetic field). A similar analysis would show how any suitable initial state (e.g., of free particles) could give rise, in a transition to the state from which we initially started. This means that we can regard the whole system as existing in its own right whether there are human beings to prepare states and observe them or not.

A very important feature of our interpretation is that it makes possible a simple and precise definition of what is to be meant by the time at which a transition takes place. In terms of the wave function, all that we can talk about is the mean life time. For example, the mean life time of a uranium atom is of the order of 2×10^9 years. Nevertheless, in any specimen, some uranium atoms are observed to decay in a very short time, which can indeed in principle be measured very precisely (e.g., within 10^{-10} s). There is no clear way to discuss this in terms of the wave function alone [24]. However, in the causal interpretation, the wave function does not exhaust the whole of reality. There is also the particle, which may be bound in a stationary state for a long time until by chance it comes near a bifurcation point [25] in which case it very rapidly moves out into an unbound state, thus implying a relatively well-defined time of transition. This is a definite advantage of the causal interpretation, not only over the usual interpretation, but also over the many-world's theory, which (as we shall discuss in more detail in section 8) also has no clearly defined concept of the time of transition.

Finally, we can now clear up a question that has long been puzzling in terms of the usual interpretation, i.e., that of the "watched dog effect [24]" (or Zeno's paradox). If one supposes that an

electron is continually "watched" by a piece of apparatus, the probability of transition has been shown to go to zero. It seems that the electron can undergo transition only if it is not "watched". This appears to be paradoxical in the usual interpretation which can only discuss the results of "watching" and has no room for any notion of the electron existing while it is not being "watched". But in the causal interpretation with its objective ontology, this puzzle does not arise because the system is evolving whether it is watched or not. Indeed, as our theory of measurement shows, the "watched" system is profoundly affected by its interaction with the measuring apparatus [4] and, so we can understand why, if it is "watched" too closely, it will be unable to evolve at all.

6. The quantum processes of "fusion" and "fission"

We now go on to extend the results of the previous section to the examples of "fusion" and "fission". We begin with a discussion of "fusion". We will consider the case of an atom containing an electron with coordinate $\boldsymbol{x}$. This atom is able to capture an incident particle with coordinate $\boldsymbol{Z}$ to form a new combined system, which is a negative ion. As happened in the case of a transition, the excess energy will be carried away by an additional particle with coordinate $\boldsymbol{y}$ bound to a centre in the neighbourhood. This will represent the fusion of two independent systems to form a new whole.

The initial state of the electron in the atom will be denoted by $\psi_0(\boldsymbol{x}) \exp(-\mathrm{i}E_0 t)$ and the initial state of the incident particle by $\chi_0(\boldsymbol{Z}, t)$. The initial state of the additional particle will be $\phi_0(\boldsymbol{y}, t)$, as in the previous section. The combined initial wave function for the entire system will then be

$$\Psi_0(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{Z}, t) = \chi_0(\boldsymbol{Z}, t)\, \phi_0(\boldsymbol{y}, t)\, \psi_0(\boldsymbol{x}) \exp(-\mathrm{i}E_0 t)\,. \tag{10}$$

Because of the interaction between the three particles with coordinates $\boldsymbol{x}$, $\boldsymbol{y}$ and $\boldsymbol{Z}$, this combined wave function will change. We shall assume that the main possibility for change is the introduction of a bound state for particle $\boldsymbol{Z}$ which will correspond to a wave function $\lambda(\boldsymbol{x}, \boldsymbol{Z}) \exp(-\mathrm{i}E_\mathrm{f} t)$ while the additional particle that takes up the energy released has a wave function $\phi_\mathrm{f}(\boldsymbol{y}, t)$, representing a state that is no longer bound. Using time-dependent perturbation theory in the way discussed in the previous section, it can be shown that the final wave function becomes

$$\Psi(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{Z}, t) = \Psi_0(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{Z}, t) + \int_0^t \alpha(t', t)\, \phi_\mathrm{f}(\boldsymbol{y}, t - t')\, \lambda(\boldsymbol{x}, \boldsymbol{Z}) \exp(-\mathrm{i}E_\mathrm{f} t)\, \mathrm{d}t'\,. \tag{11}$$

The final wave function contains a "channel" corresponding to a free y particle and a state in which the x and Z particles have "fused" into a single whole. While the particles are all interacting, they are executing a common "dance", in which there are critical (bifurcation) points, such that those on one side enter the channel corresponding to fusion while those on the other do not. Finally, the $\boldsymbol{y}$ particle executes a "dance" independent of the other two, but correlated to their state. We emphasize once again that all this happens without the presence of any apparatus either to prepare or measure the system, and without any collapse of the wave function.

We now come to the fission process which is essentially the inverse of the fusion process. It is only necessary for us to give a brief sketch of how this takes place as the treatment is similar in both cases.

The main difference is that the y particle must now *supply* the energy needed for fission, whereas with fusion, it had to *absorb* the energy released. We therefore suppose that the y particle is incident with a high energy, in a state represented by $\phi_0(y, t)$, and that after this particle interacts with the atom, it is scattered into one of a set of states of lower energy, represented by $\phi_n(y, t)$. The initial state of the combined system can be represented by the wave function

$$\Psi_0(x, y, Z\, t) = \phi_0(y, t)\, \lambda(x, Z) \exp(-iE_i t)\,.$$

Through interaction this becomes

$$\Psi_f(x, y, Z, t) = \Psi_0(x, y, Z, t) + \int_0^t \alpha(t', t)\, \psi_f(x) \exp(-iE_f t) \sum_n \chi_n(Z, t - t')\, \phi_n(y, t - t')\, dt'\,. \qquad (12)$$

The sum over states appears above because the outgoing particle can move in many directions and the additional particle will move in a correlated way. However, in time, the various directions become separated in space and the functions $\phi_n(y, t - t')$ will cease to overlap. The y particle will enter one of the channels corresponding to a particular $\phi_n(y, t - t')$ and will remain in it. All the other channels will be empty and can be left out of the discussion from this point on.

The probabilities for transition, whether for fusion or fission, can be obtained by integrating all the particle coordinates over the relevant packets and as has been shown elsewhere [3] this will come out the same as for the usual interpretation.

It can be seen from this discussion that the measurement process treated in section 4 is actually a combination of fusion and fission. At first, the measuring apparatus undergoes fusion with the observed system, and this is followed by fission. This means that what has commonly been called "measurement" is actually a transformation of the entire system. In this transformation the apparatus and the "observed" system enter into a common "dance" for a short time and then separate to give rise to independent movements in which the "observed" system is executing a new "dance" corresponding to the wave function $\psi_n(x)$.

We therefore emphasise, once again, Bell's suggestion that the process described above should not even be called measurement, except perhaps in the classical limit where the effect of these transformations is negligible. In every such transformation, that which is "measuring" and that which is "measured" are both altered in a certain irreducible way. Of course, in a rough treatment that is adequate for the large scale level the alterations may generally be neglected, but any suitable refined treatment must take such alterations into consideration.

7. Quantum wholeness and the approach to the classical limit

We have seen thus far that the quantum behaviour of matter shows a certain kind of wholeness, brought about by the quantum potential. This latter functions as active information that may reflect distant features of the environment and may give rise to a non-local connection between particles that depends on the "quantum state" of the whole, in a way that is not expressible in terms of the

relationships of the particles alone. How then do we account for the classical limit in which matter generally does not show such wholeness, but behaves as if it were constituted of independent parts, which do not depend significantly on distant features of the environment?

We shall begin by discussing how quantum mechanics approaches the classical limit. There already exists a well-known way of treating this question mathematically; i.e., the W.K.B. approximation. This is indeed obtained just by neglecting the quantum potential, which leaves us with the Hamilton–Jacobi equation. Such neglect will be justified when the quantum potential term is actually small compared with the other terms in the Hamilton–Jacobi equation. Roughly speaking, then, classical behaviour is approached, when the classical potential dominates over the quantum potential; and when this is not so, we obtain typical quantum behaviour. The approach to the classical limit has therefore nothing to do with setting $\hbar = 0$, as Planck's constant is always actually the same, while the relative importance of the terms in the equations is what may change with conditions.

The W.K.B. approximation is obtained simply by solving for S, with Q set equal to zero. In the one-dimensional case, we obtain, for a stationary state of energy, E,

$$S = \int_0^x \frac{\mathrm{d}x}{\sqrt{2m(E - V(x))}} \,. \tag{13}$$

From the conservation equation, we obtain $P \sim (E - V(x))^{-1/2}$ so that $R \sim (E - V(x))^{-1/4}$. The approximate wave function is then

$$\psi = A(E - V(x))^{-1/4} \exp\left[2 \int_0^x \mathrm{d}x \, (2m(E - V(x)))^{-1/2}\right].$$

The test for the validity of this approximation is simply to evaluate the quantum potential with the appropriate wave function, and to show that it is small compared with $(E - V(x))$. This leads to the criterion

$$-\frac{\hbar^2}{2m} \frac{1}{(E - V(x))^{-1/4}} \frac{\partial^2}{\partial x^2} (E - V(x))^{-1/4} \ll 1 \,. \tag{14}$$

In simple physical terms, this criterion will be satisfied when the classical potential, $V(x)$, does not change much in relation to $E - V(x)$, within a quantum wave length [26]. This criterion tends to be satisfied in most large-scale situations where the quantum number is large, and the wave length is therefore short enough so that $V(x)$ does not change appreciably within it. Nevertheless, wherever there are turning points, at which $E - V(x) = 0$, it will necessarily break down. Because the approximation breaks down in this way, a special treatment is needed there, which shows that in general a wave will be reflected from such points, with a phase change, the details of which depend somewhat on the precise shape of the potential [26]. Thus, for a stationary state, we must take a linear combination of waves running in opposite directions,

$$\psi \sim (E - V(x))^{-1/4} \left\{ \exp\left[\mathrm{i} \int_a^x \mathrm{d}x \, (2m(E - V(x)))^{-1/2}\right] + \exp\left[-\mathrm{i} \int_a^x \mathrm{d}x \, (2m(E - V(x)))^{-1/2} + \alpha\right]\right\}$$

where α is a suitable phase factor, determined in the way indicated above. The wave function is now a combination of sines and cosines, so that the amplitude $R(x)$ oscillates in each wave length. Since, in the limit of high quantum numbers, the wave length is very short, the quantum potential will still be large. Indeed, this is just an example of the possibility of the strong dependence of the quantum potential on distant features of the environment. (In this case, the region with $E - V(x) = 0$ where the wave is reflected.) This means that the criterion (14) for the validity of the WKB approximation does not always guarantee that the quantum potential is small, even in states with high quantum numbers, and that therefore a *system does not always approach the classical limit for high quantum states.*

The situation described above, in which the wave function is effectively real, implies that $\nabla S = 0$. In the causal interpretation, this means that the particles are at rest. Such a conclusion violated Einstein's physical intuition, which was such that he felt that, at least in states with high quantum numbers, the particles ought to be moving back and forth with equal probabilities for each direction [27]. However, it must be admitted that whether we use the usual interpretation or the causal interpretation or indeed any other interpretation, the quantum theory clearly predicts for this case that there will be a trigonometrically varying probability density for the particles, with zeros at the nodes of the wave function. If the particles are moving back and forth, how can they possibly cross these modes, at which they can never be present? (Unless their speeds are infinite, which would violate classical intuition even more than would the notion that their speed is zero.)

One of us [28] answered Einstein, but we would like here to present such an answer in more detail. Firstly, we emphasize again that $p = 0$ is, for this case, a reasonable result, consistent with the existence of nodes in the wave function, whereas the intuitive notion of equiprobability of opposite velocities is not. What was behind Einstein's objection was the commonly accepted notion we have referred to before, namely, that high quantum numbers must *always* imply classical behaviour. But if we reflect on this point further, we can see that Einstein's objection implied that the quantum mechanics must be wrong for this example, in the sense that prediction of nodes in states of high quantum numbers for the probability density cannot be right. Therefore, Einstein's criticism against the causal interpretations is misdirected – it should have been directed against the quantum theory itself, because it implies that this theory must fail for high quantum numbers.

We wish however to argue here that the quantum theory is in fact valid for high quantum numbers. To show this, consider replacing the impenetrable walls (i.e. the nodes of the wave function) with high but penetrable barriers, so that this experiment would become essentially an interference experiment. Indeed, it would just be the equivalent of the Fabry–Perot interferometer for neutrons or electrons. Considering the many kinds of macroscopic scale quantum interference experiments with electrons and neutrons, one can therefore see no reason to doubt that nodes would be discovered in this Fabry–Perot case, even for high quantum number, if the experiments were ever done.

In this connection there is a common and indeed very natural tendency to form the notion of some kind of averaging process that would wipe out such nodes. For most large-scale systems this would, in fact, be correct. For example, there might be averaging over random thermal disturbances due to the environment or, in a simpler case of the isolated system, one may take a linear combination of solutions with a small range of energies to form a local wave packet. In such situations the amplitude of the wave function would change slowly so that the quantum potential would be negligible and the classical limit would therefore be approached for high quantum numbers. However, if the system is isolated and its energy is well-defined so that only one energy eigenfunction will be present in the wave function, then the nodes predicted by the quantum mechanics will necessarily have to be present. However, such a state of well-defined energy requires a very special kind of experimental situation to bring it about and

maintain it, because it is highly unstable to small perturbations (as will indeed be brought out in more detail later on in this section).

We therefore emphasise again that although high quantum numbers generally lead to classical behaviour, there are special conditions in which they do not do so.

It is an advantage of the causal interpretation that it gives a simple and correct criterion for the classical limit, i.e., that the quantum potential be negligible. We are thus able to treat reality on all levels in the same way. It would seem that in any investigation into macroscopic quantum phenomenon, or into the relationship between classical and quantum domains, clarity will be enhanced, if we can thus treat the process without a break or "cut" between these levels, such as seems to be required in other interpretations.

The above discussion of the approach to a classical limit has been in terms of a one-particle system. To extend this to the many-particle system, we have also to take into account the non-local and state-dependent connection between particles, which is brought about by the many-body quantum potential. In particular, this sort of connection will arise when the wave function is not factorisable (recall that when the wave function is factorisable the particles behave independently). Such a non-factorised wave function will generally be present in a bound state which is stabilised by attractive classical potentials. Of course, for such bound states, quantum wholeness plays an essential part. However, bound states can be broken up (i.e., they can undergo fission) if they are bombarded with particles, including photons and phonons, having sufficient energy. Our question is then: what happens to quantum wholeness when this break-up occurs?

We have already discussed fission as brought about by interaction with an external particle that supplies the necessary energy. In this case, as shown in eq. (12), the external particle with coordinate y eventually goes into one of its possible sets of wave packets. Thereafter the other packets have no effect on the quantum potential so that the effective wave function is then represented by a set of factors corresponding to independent behaviour of the various particles (whose coordinates are x, y and $\mathbf{Z}$). This means that distant systems originating in fission of a single larger system will in general be in states corresponding to independent behaviour and so quantum non-local connection will not be present.

However, it is possible to have fission under special conditions in which such non-local connection remains. This indeed is what happens in the experiment of Einstein, Podolsky and Rosen. A typical way of bringing this about is through the *spontaneous* disintegration of a system into two parts. Thus let an unstable molecule be represented by the wave function

$$\Psi_{\mathrm{i}}(\boldsymbol{x}, \mathbf{Z}) = \sum_n C_n \, \psi_n(\boldsymbol{x}) \, \chi_n(\mathbf{Z}) \, . \tag{15}$$

After disintegration the wave function will be

$$\Psi_{\mathrm{f}}(\boldsymbol{x}, \mathbf{Z}) = \sum_n C_n \, \psi_n(\boldsymbol{x} - a_n) \, \chi_n(Z - b_n) \tag{16}$$

where a_n and b_n represent the respective distances that the two particles have moved. This corresponds to a function in which there are still correlations even though the particles have separated by macroscopic distances and in which the quantum potential corresponds to a non-local interaction.

As we have shown in our measurement paper [4], any measurement of the properties of these particles will bring about factorisation without the need for a collapse of the wave function. However, since the purpose of this paper is to discuss the ontology independently of the question of measurement

we shall now show how factorised wave functions are brought about by almost any kind of small interaction or perturbation that may be present naturally or fortuitously. For example, suppose that a particle with coordinate y interacts with the particle x. Let the initial wave function of the y particle be $\phi_0(y)$. The effect of the interaction is to change terms such as $\phi_0(y)\ \psi_n(x - a_n)$ into linear combinations $\Sigma_i\ \alpha_{in}\phi_i(y)\ \lambda_i(x)$ where λ_i form a complete set. The total wave function will become

$$\Psi_f(x, y, Z) = \sum_n \sum_i C_n\ \alpha_{in}\ \phi_i(y)\ \lambda_i(x)\ \chi_n(Z - b_n)\ . \tag{17}$$

The y particle eventually goes into one of the separate packets into which Ψ_f finally develops. Let this be denoted by s. The other packets will then not contribute to the quantum potential and the effective wave function will become

$$\Psi_e(x, y, Z) = \phi_s(y)\ \lambda_s(x) \sum_n C_n\ \alpha_{sn}\ \chi_n(Z - b_n)\ . \tag{18}$$

Clearly this reduces to a set of factors for all of the particles and so the quantum non-local connections will be destroyed.

What the above shows is that quantum non-local connection is fragile and easily broken by almost any disturbance or perturbation. (Zeh [29] has come to a similar conclusion, but on a different basis.) This kind of connection is stable only when a system is held together by classical potentials and even then only when the system is not disturbed by interactions with enough energy to disintegrate it. It is clear then that such non-local connections may be expected to arise mainly at very low temperatures for which the random thermal notions do not have enough energy to break up the system. It will also arise under special conditions established in the laboratory in which a system breaks up without any external disturbances (as in the experiment of EPR). But as a rule we may expect that non-local connection will not normally be encountered under ordinary conditions, in which every system is bathed in electromagnetic radiation and is subjected to external perturbations of all kinds as well as random thermal energies (usually in the form of phonons).

Moreover, because typical systems are constituted of particles with classical potentials $V(r)$ that fall off with the distance, they tend at appreciable temperatures to form relatively independent blocks with correspondingly weak interaction between the blocks, along the lines suggested by Kadanoff et al. [10]. The various disturbances mentioned above, will then ensure that the blocks will have product wave functions, so that the various parts of a system will be executing independent "dances". This completes the demonstration that the relative independence characteristic of classical mechanics will generally be maintained at the large-scale level while on a sufficiently small scale, the size of which is dependent on temperature and other conditions, there will generally be non-local connections and other forms of quantum wholeness.

The above-described fragility of quantum behaviour at a large-scale level holds also for the one-particle system. We have indeed already indicated, for example, that special experimental conditions are needed to establish well-defined quantum states with very high quantum numbers. Consider, for example, a particle with coordinates x, in such a stationary state that interacted with a free particle with coordinate y, through a classical interaction potential $V(x - y)$ of rather short range. A treatment similar to that used for transition, fission and fusion would then show that a set of separate channels would ultimately be created, in which the x particle will be in localized packets, with a spread corresponding roughly to the range of the potential. Since these packets are too narrow to be significantly restricted by the containing potential barriers, the parts corresponding to movements in

opposite directions will form two separating packets. The particle will enter one of these, and from then on, the interference between different directions of motion of the wave function that is responsible for the nodes in the wave function will vanish. The behaviour will then be classical. The essential point is that almost any random disturbance would produce a similar result. And this explains why we would not expect to find stationary states of high quantum number under conditions that arise naturally (except possibly at very low temperatures), and why we need special experimental conditions to bring them about.

Finally, such considerations may also be applied to discuss the possibility, already mentioned in section 3, of using the non-local connection supplied by the quantum potential as a means of transmitting signals faster than light. The state dependence and fragility of this connection is just what makes the transmission of such a signal impossible. Thus, one way of trying to convey a signal would be to have a particle A initially in a definitely known state, and to have this state changed instantaneously by a measurement made on a distant particle B. But if the state of particle A were measured before that of B, the wave function of the whole system would be altered into a product of factors, in which all long-range connection would have vanished. A subsequent measurement of B would then not affect A, so that no signal would be possible.

Another way of carrying a signal would be to try to "modulate" the quantum potential as one would a radio wave into a signal having a definite form and order, with a corresponding meaning. But once again, any interaction aimed at doing this would result in factorised states, so that long-range connections needed to carry such a signal would vanish. More generally, one can show that although the total system is, in principle, deterministic, nevertheless, the actual behaviour of long-range quantum connections is too fragile to be controllable in ways required for transmitting a signal, as well as too full of unstable bifurcation points to make the behaviour of the system predictable enough for this purpose.

8. Summary and conclusions

Throughout this article we have developed an ontology that enables us consistently to understand what may be the individual nature of the reality that underlies the quantum behaviour of matter. The main advantages of this ontology are the following:

(1) It permits an objective description of quantum processes, in which neither measurement nor preparation of a state plays a fundamental part. This enables us to understand intuitively what the quantum theory means.

(2) It avoids the need for introducing a "collapse" of the wave function that would violate the basic laws of physics, such as Schrödinger's equation.

(3) It gives a clear discussion of the time of transition, and of the "watched dog" effect.

(4) It gives a clear notion of the meaning of the classical limit, as arising in a very simple way whenever the quantum potential is negligible compared with the other terms in the Hamilton–Jacobi equation. It also makes it clear that the classical limit is not *always* valid on a macroscopic scale, because, under certain special conditions, the quantum potential can still be large in such systems.

(5) A more detailed analysis [3] shows that it covers the whole range of the quantum theory. In this connection, it is important to note that one can extend the causal interpretation to quantum fields [3], and thus deal with all bosonic systems. We shall in fact give a more detailed treatment of this question in the next paper [30], where we shall also discuss how fermions are to be incorporated into the causal interpretation.

(6) It has important advantages for cosmology, especially because it can discuss the universe apart from the questions of the "collapse of the wave function" and of the absolute need for the presence of observers, which are characteristic of other interpretations. Thus, in the commonly accepted interpretation, one has necessarily to invoke a measuring apparatus, which ultimately cannot be treated within the framework of the theory. In effect, this apparatus is regarded as being outside the universe, and thus, a consistent cosmology cannot be obtained.

Current theories involve what is called the wave function of the universe, which is especially important to consider, at or near the moment of its supposed origin. But how could this wave function have been observed or measured, unless there was an apparatus present at that time, as current interpretations indeed demand? Evidently, under conditions that prevailed during this period (in which not even atoms and molecules could exist), this would have been impossible. We recall that Wigner [20] has attempted to avoid the whole dilemma of the unexplained whole "collapse" of the wave function by an appeal to the mind of an observer, whose consciousness is supposed to cause this collapse. At or near the point of origin of the universe, this observer would have to be a pure disembodied spirit, who could constantly be observing the wave function of the universe, and thus bringing about a sequence of collapses that corresponded to the actual evolution of the universe. Everett, however, proposes a non-countable infinity of universes, along with their non-countable infinity of observers, that are constantly proliferating by dividing and subdividing. But here, the question is once again: How could these observers have been present, in conditions near the original "big bang"?

It seems to be much simpler, as is done in the causal interpretation, to assume an objective universe, with its particles and with its wave function, which is not dependent on observers, though it may contain them.

At this point, it seems to be worthwhile to compare our approach in more detail to that of the many-worlds interpretations, which is the only other one that treats the universe as objective, and yet avoids bringing in the collapse of the wave function. Evidently, the two interpretations are similar, in that, in a certain sense, the "many worlds" appear in both. The difference is that in the Everett view, all these worlds actually exist in manifest form, while in our approach, they are present only in the implicate order as non-manifest and inactive information. However, in the causal interpretation, only one of these worlds is fully actualized and manifest. (Rather, as in the case of a dancer, many possibilities are present, but only one is actualised.)

We would like to draw attention to the fact that there are several important difficulties in the many-worlds view that do not arise in our approach. Thus, one of the main problems of the many-worlds interpretation is that like the usual interpretation, it has no concepts in terms of which it could discuss the time at which a transition actually takes place. (Indeed, we have been able to do this only by enriching the conceptual structure to include a set of particles, as well as the wave function or the quantum state.) In the many-worlds view, it is thus not clear how one would even confront the question of discussing *just when* a new universe is suddenly created, in which there is a new quantum state, along with a new state of the observing apparatus. Recall here that the many-worlds' interpretation takes the overall wave function as a complete description of reality and expressly avoids bringing in probability in a fundamental way, so that it cannot just assume a statistical distribution of times at which new universes are supposed to be created.

To try to avoid this difficulty, would one say perhaps that a new universe is formed at the moment when the apparatus wave packets cease to overlap? But then, as we have seen in section 4, interference is still possible between these packets, so that we would have to say that later, these universes could be withdrawn or destroyed. Would the proponents of this view be ready to contemplate both constant

creation and constant destruction of universes? Or would they want to say that a new universe is created only when the registration device has functioned irreversibly in the way described in section 4? If so, at just what point in the functioning would a new universe come into being? Or would the creation (and destruction) of universes be a gradual process? (Rather as happened with the gradual disappearance of the Cheshire cat.) And even if all these ambiguities are somehow dealt with, there is still the fact that a *complete* transition may actually take place in a time much shorter than that in which the wave function changes appreciably. There do not seem to be any concepts within the many-worlds' point of view which allow for this to be described consistently.

This sort of difficulty also comes out in another way, which shows itself when the results of different measurements do not have equal coefficients in the wave function of the whole system (so that in the usual interpretation, one would say that they do not have equal probabilities). One seems to need here a further concept corresponding to a variable "intensity" with which each universe could exist. Perhaps one could modify the basic idea by proposing that, corresponding to each possible result of a measurement, there are many universes, all of which are in the same quantum state. The number of such universes would then be proportional to the square of the corresponding coefficient that appears in the wave function. But then, what can it mean to have a vast set of universes that are all different, and yet in identical quantum states? In what way are they actually different?

It should be clear that none of the difficulties described above arises in our approach. And as we have seen, our approach also avoids the basic difficulties of other interpretations. We feel therefore that in spite of the novel and somewhat strange features that we have ascribed to the quantum potential, it is, on balance, the best interpretation that is available.

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PHYSICS REPORTS (Review Section of Physics Letters) 144, No. 6 (1987) 349–375. North-Holland, Amsterdam

II. A Causal Interpretation of Quantum Fields

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Abstract:

We extend the approach to the causal interpretation given in the previous paper [1] to quantum mechanical field theories. We begin by presenting a systematic causal interpretation of the real scalar field. We analyse the field into normal modes and discuss their meaning for excited states of the field, including both Fock states and coherent states. We then go on to discuss the absorption of a quantum of energy by an atom. In this process the field is initially continuously distributed. However, because of the non-linearity and non-locality of the super-quantum potential of the total field, energy is "swept in" from the entire wave packet, so that it concentrates on the atom as a single quantum of energy $h\nu$. We then see that while the field is continuously distributed, it manifests itself in a discrete, particle-like way, and this explains the wave–particle duality. We extend this approach to explain interference phenomena, and the Pfleegor–Mandel experiment involving interference of two separated lasers. Finally, we discuss briefly the extension of the causal interpretation to a relativistic assemblage of fermions.

1. Introduction

In the previous paper [1] we have given a systematic account of how the quantum theory can be reformulated so that it refers to individual quantum systems without the need for an observer. Statistics then plays only a secondary role. However, this treatment was restricted to systems of particles only. Mandel [2] has questioned whether it is possible to discuss individual events in quantum field theory. In an earlier paper an extension of the causal interpretation has been made to the case of quantum fields although in a somewhat condensed way [3]. In the present paper, we shall develop this treatment in considerably more detail with special emphasis on making clear how individual physical processes actually take place. Moreover, we shall show that such processes are independent of acts of preparation and observation.

2. The causal interpretation of the scalar field

In the earlier paper, to which we have referred above [3], the causal interpretation was applied to the electromagnetic field. But this field introduces formal complications that tend to hide the physical meaning of the theory. We shall therefore simplify the problem by discussing a scalar massless bosonic field for which the principles are essentially the same as those of quantum electrodynamics.

We begin with the real scalar field $\phi(\boldsymbol{x})$ and its canonical momentum $\pi(\boldsymbol{x})$. In units for which $c = 1$ and $\hbar = 1$, the Lagrangian is

$$\mathscr{L} = \tfrac{1}{2}(\partial\phi/\partial t)^2 - \tfrac{1}{2}(\nabla\phi)^2 \tag{1}$$

so that

$$\pi(\boldsymbol{x}) = \partial\phi/\partial t\,. \tag{2}$$

Classically this field satisfies the wave equation

$$\partial^2\phi/\partial t^2 = \nabla^2\phi \,. \tag{3}$$

To quantize this field we may introduce a super-wave function $\psi(\ldots\phi(\boldsymbol{x}, t), \ldots)$ which depends on the values of ϕ at every point in the universe. $|\psi|^2$ then gives the probability of any given total field configuration. $\pi(\boldsymbol{x})$ is now replaced by an operator which can be symbolised as

$$\pi_{\mathrm{op}}(\boldsymbol{x}) = -\mathrm{i}\,\delta/\delta\phi(\boldsymbol{x}) \tag{4}$$

where $\delta/\delta\phi(x)$ is the usual functional derivative of quantum field theory. This operator acts on the super-wave function $\psi(\ldots\phi(\boldsymbol{x}, t), \ldots)$.

It is clear that quantum field theory has consequences similar to those of quantum particle theory. The main difference is that the basic degrees of freedom are the field variables rather than the particle variables. This can be made even clearer by considering the Fourier analysis of the field in a box with periodic boundary conditions. The non-denumerable infinity of field variables is replaced by a denumerable infinity ϕ_k with k_x, k_y, k_z restricted to integral multiples of $2\pi/L$ where L is the size of the box. The functional derivative becomes an ordinary derivative and the similarity to ordinary quantum mechanics becomes very obvious.

It should be added, of course, that if we start from quantum field theory, the particles come out as quantized excitations of the entire field. Therefore the particle is no longer a fundamental concept in quantum field theory but rather it is derived as a structural feature of the field. It will be important to keep this in mind throughout our discussion, as we shall emphasise that in the causal interpretation the particle is just a dynamical structure within the field.

To obtain the causal interpretation of the field theory, we do just what was done in the particle theory, that is, we assume the basic dynamical variables, $\phi(\boldsymbol{x}, t)$, have well-defined values that change continuously. To see how they change we must begin with the super-Schrödinger equation, which is,

$$\mathrm{i}\partial\psi/\partial t = H\psi \tag{5}$$

where H is the Hamiltonian given by

$$H = \tfrac{1}{2}\int_{\substack{\text{All}\\ \text{space}}} \left[-\frac{\delta^2}{(\delta\phi(\boldsymbol{x}, t))^2} + (\nabla\phi(\boldsymbol{x}, t))^2\right] \mathrm{d}V \,. \tag{6}$$

We then write

$$\psi = R(\ldots\phi(\boldsymbol{x}, t), \ldots, t)\exp[\mathrm{i}S(\ldots\phi(\boldsymbol{x}, t), \ldots, t)]$$

and obtain

$$\partial S/\partial t + \tfrac{1}{2}\int[(\delta S/\delta\phi)^2 + (\nabla\phi)^2]\,\mathrm{d}V + Q = 0 \tag{7}$$

and

$$\frac{\partial P}{\partial t} + \int \frac{\delta}{\delta \phi}\left[P \frac{\delta S}{\delta \phi}\right] \mathrm{d}V = 0 \tag{8}$$

where $P = |\psi|^2$ and Q is the super-quantum potential which we shall define presently.

Equation (7) is evidently the extended Hamilton–Jacobi equation for the field which contains the super-quantum potential, Q, in addition to the other terms that would be present classically.

We can write

$$\pi(x, t) = \delta S(\ldots \phi(x, t), \ldots, t)/\delta \phi(x, t) \tag{9}$$

which is comparable in the particle case to

$$p = \nabla S\,. \tag{10}$$

The super-quantum potential will be

$$Q = -\frac{1}{2}\int \frac{(\delta^2/(\delta\phi(x, t))^2) R(\ldots \phi(x, t), \ldots, t)\, \mathrm{d}V}{R(\ldots \phi(x, t), \ldots, t)} \tag{11}$$

and this is comparable in the many-particle system to

$$Q_{\mathrm{MP}} = -\frac{1}{2m}\sum_i \frac{\nabla_i^2 R(\ldots x_i, \ldots, t)}{R(\ldots x_i, \ldots, t)}\,. \tag{12}$$

From the extended Hamilton–Jacobi equation (7) we obtain the "field velocity"

$$\partial \phi/\partial t = \delta S/\delta \phi. \tag{13}$$

Then eq. (8) clearly represents the conservation of probability. The remaining equation of motion is obtained from (7) as

$$\partial \pi/\partial t = -\delta H/\delta \phi = \nabla^2 \phi - \delta Q/\delta \phi\,. \tag{14}$$

Putting (7) into the above we obtain

$$\partial^2 \phi/\partial t^2 = \nabla^2 \phi - \delta Q/\delta \phi\,. \tag{15}$$

As shown in the previous paper [1], the classical limit is approached whenever Q becomes negligible. In this case, eq. (15) reduces to the ordinary wave equation. However, in the quantum domain there is an extra term $-\delta Q/\delta \phi$. In general, this is a non-linear and non-local function of ϕ. As we shall see, it profoundly alters the meaning of the equation and leads to radically new implications. We emphasize again, however, that in the classical limit none of these will be significant.

3. Analysis into normal modes and the ground state of the field

In order to discuss the ground state and excited states of the field, it will be convenient to go to a representation in terms of normal modes. To simplify the problem we may, as we have already suggested, consider a cubic box of side L and assume periodic boundary conditions. We take L to be finite but much larger than any dimension of physical interest, i.e., in principle, as large as we please. The normal modes will then be trigonometric waves and an arbitrary field can be expressed as

$$\phi(\boldsymbol{x}, t) = \sqrt{\frac{2}{V}} \sum_{\boldsymbol{k}}{}' \left[a_{\boldsymbol{k}}(t) \operatorname{Cos}(\boldsymbol{k} \cdot \boldsymbol{x}) + b_{\boldsymbol{k}}(t) \operatorname{Sin}(\boldsymbol{k} \cdot \boldsymbol{x})\right] \tag{16}$$

where

$$\boldsymbol{k} = k_x, k_y, k_z \quad \text{with } k_x = 2\pi n_x/L \; ; \; k_y = 2\pi n_y/L \; ; \; k_z = 2\pi n_z/L$$

and where n_x, n_y, n_z are integers and V is the volume of the box. Because $-\boldsymbol{k}$ and $\boldsymbol{k}$ cover the same functions we should sum only over half the possible values of $\boldsymbol{k}$, e.g., n_x and n_y cover all the integers positive, negative and zero, while n_z is restricted to values greater than or equal to zero. Whenever the sum is taken with the above restrictions, we shall write it as above $\Sigma'_{\boldsymbol{k}}$.

We also introduce $\pi(\boldsymbol{x}, t)$, the momentum canonically conjugate to $\phi(\boldsymbol{x}, t)$, which satisfies the Poisson bracket relations

$$\{\phi(\boldsymbol{x}, t), \pi(\boldsymbol{x}', t)\} = \delta(\boldsymbol{x} - \boldsymbol{x}') \, . \tag{17}$$

We Fourier analyse $\pi(\boldsymbol{x}, t)$ writing

$$\pi(\boldsymbol{x}, t) = \sqrt{\frac{2}{V}} \sum_{\boldsymbol{k}}{}' \left[\alpha_{\boldsymbol{k}}(t) \operatorname{Cos}(\boldsymbol{k} \cdot \boldsymbol{x}) + \beta_{\boldsymbol{k}}(t) \operatorname{Sin}(\boldsymbol{k} \cdot \boldsymbol{x})\right] . \tag{18}$$

With the aid of (17) we then obtain the Poisson bracket relations

$$\{a_{\boldsymbol{k}}, \alpha'_{\boldsymbol{k}}\} = \{b_{\boldsymbol{k}}, \beta'_{\boldsymbol{k}}\} = \delta'_{\boldsymbol{k}\boldsymbol{k}}$$

with all other Poisson brackets equal to zero. This means that $a_{\boldsymbol{k}}$, $\alpha_{\boldsymbol{k}}$ and $b_{\boldsymbol{k}}$, $\beta_{\boldsymbol{k}}$ form a set of canonical variables.

From (6) we obtain the Hamiltonian

$$H = \tfrac{1}{2} \int \left[\pi^2(\boldsymbol{x}) + (\nabla\phi(\boldsymbol{x}))^2\right] \mathrm{d}V \tag{19}$$

which in terms of Fourier analysis becomes

$$H = \tfrac{1}{2} \sum_{\boldsymbol{k}}{}' \left[(\alpha_{\boldsymbol{k}})^2 + k^2(a_{\boldsymbol{k}})^2 + (\beta_{\boldsymbol{k}})^2 + k^2(b_{\boldsymbol{k}})^2\right] . \tag{20}$$

It is convenient to go to the complex representation of the normal modes. Writing

$$q_k = \frac{1}{\sqrt{2}}(a_k - \mathrm{i}b_k), \qquad \pi_k = \frac{1}{\sqrt{2}}(\alpha_k - \mathrm{i}\beta_k), \tag{21}$$

we obtain

$$\phi(\boldsymbol{x}, t) = \frac{1}{\sqrt{V}} \sum_k q_k \exp(\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{x}) \tag{22}$$

$$\pi(\boldsymbol{x}, t) = \frac{1}{\sqrt{V}} \sum_k \pi_k \exp(\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{x}). \tag{23}$$

From the above we see that the reality of ϕ implies that $q_k = q^*_{-k}$ and $\pi_k = \pi^*_{-k}$.
We then obtain the Poisson bracket relations

$$\{q_k, \pi^*_{k'}\} = \{q^*_k, \pi_{k'}\} = \delta_{kk'}$$

with all other Poisson brackets being zero. And the Hamiltonian becomes

$$H = \sum_k{}' [\pi^*_k \pi_k + k^2 q^*_k q_k]. \tag{24}$$

We verify the classical equations of motion:

$$\dot{q}_k = \partial H/\partial \pi^*_k = \pi_k, \qquad \dot{\pi}_k = -\partial H/\partial q^*_k = -k^2 q_k$$

$$\therefore \ddot{q}_k + k^2 q_k = 0$$

which are the correct equations of motion.

We now go to the quantum theory. We represent the super-wave function in terms of the q_k and $q^*_k = q_{-k}$. We are restricting ourselves to half the values of $\boldsymbol{k}$ as before. We then write

$$\Psi = \Psi(\ldots q_k \ldots; q^*_k \ldots; t).$$

Because the system reduces to a set of independent harmonic oscillators, the wave function can be expressed as a linear combination of the product functions,

$$\Psi_0 = \psi(q_{k1}, q^*_{k1})\psi(q_{k2}, q^*_{k2}) \cdots \psi(q_{ki}, q^*_{ki}) \ldots. \tag{25}$$

Each $\psi(q_k, q^*_k)$ satisfies an independent Schrödinger equation representing a harmonic oscillator,

$$\mathrm{i}\frac{\partial \psi}{\partial t}(q_k, q^*_k) = \left[\frac{-\partial^2}{\partial q_k \partial q^*_k} + k^2 q^*_k q_k\right] \psi(q_k, q^*_k). \tag{26}$$

Writing $\psi(q_k, q^*_k) = R_k \exp(\mathrm{i}S_k)$ we obtain the equivalent Hamiltonian–Jacobi equation

$$\frac{\partial S_k}{\partial t} + \left(\frac{\partial S_k}{\partial q_k}\right)\left(\frac{\partial S_k}{\partial q^*_k}\right) + k^2 q^*_k q_k - \frac{1}{R_k}\frac{\partial^2 R_k}{\partial q_k \partial q^*_k} = 0. \tag{27}$$

The above result is similar to that obtained in the causal interpretation of a particle undergoing harmonic oscillation. The ground state of eq. (26) then represents the zero point energy. Its wave function is

$$\psi_0(q_k, q_k^*) = \exp[-kq_k^* q_k] . \tag{28}$$

The ground state of the whole set of oscillators is then

$$\Psi_0 = \exp\left[- \sum_k{}' \, kq_k^* q_k \right] . \tag{29}$$

The quantum potential for this state is

$$Q_0 = \sum_k{}' \, (-k^2 q_k^* q_k + k) . \tag{30}$$

Writing

$$\partial S/\partial t = -E$$

we get

$$E = \sum_k{}' \left[\left(\frac{\partial S}{\partial q_k} \right) \left(\frac{\partial S}{\partial q_k^*} \right) + k \right] . \tag{31}$$

Since the wave function is real $\partial S/\partial q_k = \partial S/\partial q_k^* = 0$ and we obtain

$$E = \sum_k{}' \, k . \tag{32}$$

This is, evidently, the usual expression for the zero point energy (note that each $\boldsymbol{k}$ corresponds to *two* real oscillators a_k and b_k so that the usual factor 1/2 is cancelled).

We shall now discuss the meaning of the wave function for the ground state, eq. (28). Firstly, let us note that although the quantum potential will, in general, be non-local, in the special case of the ground state it reduces to a *local* function. In fact, it cancels the term $\Sigma_k' k^2 q_k^* q_k$, which, in the space representation, corresponds to $\nabla\phi^* \nabla\phi$. This is evidently a local function (as we shall see later it is only in the excited states that non-locality appears, although in the classical limit it disappears again).

Let us now consider the significance of the probability distribution. (Wheeler [4] has considered this problem for the electromagnetic field and has obtained results similar to those that will be given here. His analysis is, however, developed in less detail and, of course, does not consider the role of the quantum potential.) We begin by expressing the q_k in terms of $\phi(x)$ i.e.,

$$q_k = \frac{1}{\sqrt{V}} \int \exp[-i\boldsymbol{k} \cdot \boldsymbol{x}] \, \phi(\boldsymbol{x}) \, dV . \tag{33}$$

[We have suppressed the coordinate t which will not be relevant in this context.] This yields

$$\Psi_0 = \exp\left[- \int\int \phi(x)\, \phi(x')\, f(x' - x)\, \mathrm{d}V\, \mathrm{d}V' \right] \tag{34}$$

where

$$f(x' - x) = \frac{1}{V} \sum_k{}' \, k \exp[\mathrm{i}k \cdot (x' - x)] \, .$$

To turn this sum into an integral, we introduce the density of k-vectors which is $V/(2\pi)^3$ and we obtain

$$f(x' - x) = \frac{1}{(2\pi)^3} \int k \exp[\mathrm{i}k \cdot (x' - x)] \, \mathrm{d}k \, . \tag{35}$$

This is not a properly defined function but we can evaluate it by taking a limit in which there is a cut-off in k-space. We can represent such a cut-off by multiplying the integrand by a factor $\mathrm{e}^{-\lambda k}$ and allowing λ to become as small as we please. These integrations are then straightforward and the result is

$$f(r) = \frac{1}{\pi^2} \frac{r^2 - 3\lambda^2}{(\lambda^2 + r^2)^3} \tag{36}$$

where $r = |x' - x|$.

The probability function is then

$$P = |\Psi_0|^2 = \exp\left[- \int\int \frac{\phi(x)\, \phi(x')}{\pi^2} \frac{r^2 - 3\lambda^2}{(\lambda^2 + r^2)^3} \, \mathrm{d}V\, \mathrm{d}V' \right] . \tag{37}$$

This probability is largest when $\phi(x)$ and $\phi(x')$ have different signs. Therefore it is quite clear that for a given value of $\phi(x)$ it is highly improbable that a neighbouring field $\phi(x')$ will have the same sign within a distance of the order of λ. (Recall that λ is as small as we please.) This point can be brought out more clearly by going to spherical polar coordinates where $r = |x' - x|$. The contribution to the probability coming from a field $\phi(x)$ which is on a shell of radius r will be

$$\exp\left[- \frac{\phi(x)\, \phi(x + r)}{\pi^2} \frac{r^2 - 3\lambda^2}{(r^2 + \lambda^2)^2} \, r^2 \, \mathrm{d}r \, \mathrm{d}\Omega \right] .$$

Clearly this probability can become very small for small λ and small r, when $\phi(x)$ and $\phi(x + r)$ have the same sign. It is therefore most probable that the field $\phi(x)$ is highly discontinuous, the sharpness of the discontinuity being dependent on the cut-off radius λ. Such a discontinuity implies that the most probably spatial form of the field will be chaotic, i.e., in the sense of modern chaos theory [5]. However, this chaotic variation will be limited, because the variation of the q_k will be the order of $1/\sqrt{k}$ as can be seen from eq. (29).

Because $\partial S/\partial q_k = \pi_k = \dot{q}_k = 0$ it follows that the field is static. This result is surprising as one generally thinks that the zero point fluctuations of the vacuum correspond to some kind of chaotic *dynamical* behaviour. We can, indeed, obtain such a dynamical behaviour by introducing the further assumption that the field is taking part in a random-type of fluctuation in which the mean value of $\dot{\pi}_k = 0$. This would be along the lines suggested by Bohm and Vigier [6] and later developed in a more

systematic way by Nelson [7] and de la Pena [8]. However, for the purposes of the present article, we shall confine ourselves to the initial deterministic model, which we emphasise is completely self-consistent.

The ground state energy which we ordinarily ascribe to the dynamic behaviour will now be attributed to the super-quantum potential. In the analogy with the particle case in a stationary state, we have $p = 0$ and $E = V + Q$, or alternatively $Q = E - V$. In the classical case we have $T = E - V$. So Q is in some sense playing a role analogous to T. For example, in an excited state, the energy of the quantum potential enters into the quantum transitions in the same way that the kinetic energy would in the classical limit. And if, as we have suggested above, the energy of the quantum potential is accounted for by further random motions, then the analogy to our ordinary way of thinking will be very close.

Whether we take the static or dynamic model, the ground state is not covariant because it defines a favoured frame in which either $\dot{q}_k = 0$ or its mean value is zero. Will this be consistent with relativity?

First of all, as shown in our previous paper [1], the quantum potential is too unstable and fragile to carry a signal. Therefore, at least in this respect relativity will not be violated.

Secondly, as shown originally by one of us [3], the statistical results of this theory are the same as those obtained in the usual interpretation. But this latter is known to be covariant and therefore no experimental results can be obtained that violate relativity.

Finally, in the classical limit, the quantum potential becomes negligible and, as we have seen, we obtain the usual covariant field equation in this limit.

It follows that the non-covariant ground state in our interpretation will not be accessible to observation as long as the quantum theory in its current form is valid. And so no violations of relativity will be obtained. However, it is possible, as pointed out in the earlier paper [3], that quantum theory will fail to hold in some, as yet, unexplored domain. For example, if we extend our theory to include stochastic processes in the manner suggested earlier, there will be some relaxation time, τ, for the probability function to approach the usual one, $|\psi|^2$. Measurement in times shorter than τ might show this discrepancy and these results would in general not be covariant. If this should happen then relativity would evidently hold only as a statistical approximation valid for distributions close to equilibrium in the stochastic process underlying the quantum mechanics.

4. The excited state of the field

There is a considerable range of possible forms for the excited state of the field. One of these is the well-known set of Fock states in which the number of quanta and the energy are well defined. Another is the set of coherent states [9] which are especially suited to the discussion of the time dependence of the field and its approach to the classical limit. For the case where the mean quantum number is low, both kinds of states lead to the same results for most purposes. However, as we shall see for states with high quantum numbers there may be significant differences between the two types of state.

Let us begin with a treatment of the Fock state. A typical wave function for the excitation of a normal mode q_k with a definite k-vector is

$$\Psi = q_k \exp\left[- \sum k q_k^* q_k \right] = q_k \Psi_0 . \tag{38}$$

The probability function is

$$P = |q_k^* q_k|\,|\Psi_0|^2 . \tag{39}$$

What this means is that the actual field in the excited state will in general not look very different from that of the ground state. As a function of x we will have

$$P = \int\int \phi(x)\,\phi(x')\exp[\mathrm{i}k\cdot(x'-x)]\,\mathrm{d}V\,\mathrm{d}V'\,|\Psi_0|^2 . \tag{40}$$

The factor $|\Psi_0|^2$ will still represent a generally chaotic variation. However, the integral in front implies a tendency for waves with vector k to be emphasised relative to the original chaotic distribution. From the actual value of the field one could therefore not say for certain whether the state is excited or not because the same field configuration could exist in either state. Indeed, the degree of excitation is determined only with the aid of the super-quantum potential which involves the state of the whole as discussed in our previous article in the treatment of the many-body system.

The state having only a single normal mode excited is an extreme abstraction. A more realistic representation of the state would involve something like a wave packet. To represent such a packet we write

$$\Psi = \sum_k f_k q_k \Psi_0 . \tag{41}$$

In the above equation, the sum is over all k and we do not make the restriction that $f_{-k} = f_k^*$ because the super-wave function is, in general, complex even though $\phi(x)$ is real. Indeed considering that $q_{-k} = q_k^*$, we can also write

$$\Psi = \sum_k{}' [f_k q_k + f_{-k} q_k^*]\Psi_0 \tag{42}$$

where the Σ_k' indicates summing a suitable half of the total set of values of k as discussed before. For the applications made in this paper we may assume that $f_0 = 0$ i.e. the space average of the field is zero.

To obtain the time dependence of Ψ, we note that the state $q_k \Psi_0$ and $q_k^* \Psi_0$ both have the same energy k, so that they oscillate as $\mathrm{e}^{-\mathrm{i}kt}$. We obtain

$$\Psi(t) = \sum_k{}' [f_k q_k\,\mathrm{e}^{-\mathrm{i}kt} + f_{-k} q_k^*\,\mathrm{e}^{-\mathrm{i}kt}]\Psi_0 .$$

The q_k terms correspond to running waves in the direction $+z$, while the q_k^* terms correspond to running waves in the $-z$ direction.

To simplify the discussion let us form a wave packet running only in the $+z$ direction. This will be sufficient to illustrate the general meaning of the super-quantum potential for these wave packets. Writing

$$q_k = \frac{1}{\sqrt{V}} \int \exp[-\mathrm{i}k\cdot x]\,\phi(x)\,\mathrm{d}V$$

we obtain

$$\Psi(t) = \int F(\boldsymbol{x}, t)\, \phi(\boldsymbol{x})\, \mathrm{d}V\, \Psi_0 \tag{43}$$

where

$$F(\boldsymbol{x}, t) = \frac{1}{\sqrt{V}} \sum_{k}{}' f_k \exp[-\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{x} - \mathrm{i}kt]\,. \tag{44}$$

To interpret this, it is convenient to write in accordance with (16)

$$\phi(\boldsymbol{x}, t) = \sqrt{\frac{2}{V}} \sum_{k}{}' [a_k(t)\,\mathrm{Cos}(\boldsymbol{k}\cdot\boldsymbol{x}) + b_k(t)\,\mathrm{Sin}(\boldsymbol{k}\cdot\boldsymbol{x})]$$

and define

$$\phi_{\mathrm{c}} = \sqrt{\frac{2}{V}} \sum{}' a_k(t)\,\mathrm{Cos}(\boldsymbol{k}\cdot\boldsymbol{x})$$

$$\phi_{\mathrm{s}} = \sqrt{\frac{2}{V}} \sum{}' b_k(t)\,\mathrm{Sin}(\boldsymbol{k}\cdot\boldsymbol{x})$$

with

$$\phi = \phi_{\mathrm{c}} + \phi_{\mathrm{s}}\,.$$

We note that ϕ_{c} and ϕ_{s} are orthogonal. Similarly, we split $F(\boldsymbol{x})$ into two parts F_{c} and F_{s} with

$$F_{\mathrm{c}} = \frac{1}{\sqrt{V}} \sum{}' f_k \exp[-\mathrm{i}kt]\,\mathrm{Cos}(\boldsymbol{k}\cdot\boldsymbol{x})$$

and

$$F_{\mathrm{s}} = -\frac{\mathrm{i}}{\sqrt{V}} \sum{}' f_k \exp[-\mathrm{i}kt]\,\mathrm{Sin}(\boldsymbol{k}\cdot\boldsymbol{x})\,.$$

We further note that F_{s} is orthogonal to ϕ_{c} and F_{c} to ϕ_{s}. Using these orthogonality relations we then obtain

$$\Psi(t) = \int [F_{\mathrm{c}}(\boldsymbol{x}, t)\, \phi_{\mathrm{c}}(\boldsymbol{x}) + F_{\mathrm{s}}(\boldsymbol{x}, t)\, \phi_{\mathrm{s}}(\boldsymbol{x})]\, \mathrm{d}V\, \Psi_0\,. \tag{45}$$

This wave function will be large in absolute magnitude for fields in which $\phi_{\mathrm{c}} = \alpha_{\mathrm{c}} F_{\mathrm{c}}$ and $\phi_{\mathrm{s}} = \alpha_{\mathrm{s}} F_{\mathrm{s}}$ where α_{c} and α_{s} are real proportionality factors. This follows because the total integrand is then a non-negative function of both integrands. Fields that are not proportional in this way will tend to produce a smaller integrand because of cancellation. Indeed fields that are orthogonal to those described above will give no contribution at all to the integrals.

Introducing $\Gamma(\boldsymbol{x})$ to represent the part of the field that is orthogonal to $F_{\mathrm{c}}(\boldsymbol{x}, t)$ and $F_{\mathrm{s}}(\boldsymbol{x}, t)$ we can write

$$\phi(\boldsymbol{x}, t) = \alpha_{\mathrm{c}}\, F_{\mathrm{c}}(\boldsymbol{x}, t) + \alpha_{\mathrm{s}}\, F_{\mathrm{s}}(\boldsymbol{x}, t) + \Gamma(\boldsymbol{x}, t)\,. \tag{46}$$

The most probable state will then be one in which α_c and α_s are as large as possible within the limitation of the Gaussian function Ψ_0 which tends to bound the variation of the overall field.

It is therefore most likely that the field $\phi(x, t)$ will have the form of a wave packet corresponding to $\alpha_c F_c(x, t) + \alpha_s F_s(x, t)$. The functions $\Gamma(x, t)$ which are orthogonal to $F_c(x, t)$ and $F_s(x, t)$ will then not affect the factor in front of Ψ_0. This means that their variation will be the same as it is in the ground state. Therefore the generally chaotic variation of the field as a whole will be modified by a statistical tendency to vary around an average which has the form of a wave packet as described above.

We shall now show that inside this wave packet the super-quantum potential introduces non-local connections between fields at different points separated by a finite distance (unlike what happens in the ground state). To obtain the quantum potential we must first write down the absolute value of the wave function in the q_k representation.

Recalling that we are choosing $f_{-k} = 0$ in eq. (42) we write

$$\Psi = \sum_k{}' f_k q_k \exp[-ikt]\Psi_0 .$$

Then writing

$$g = \sum_k{}' f_k q_k \exp[-ikt]$$

we have

$$R = \sqrt{\Psi^*\Psi} = \sqrt{gg^*}\Psi_0 . \tag{47}$$

The quantum potential is

$$Q = -\sum_k{}' \frac{\partial^2 R}{\partial q_k^* \partial q_k} \Big/ R .$$

Let us now evaluate the change of quantum potential, ΔQ, from the ground state. This is

$$\Delta Q = -\frac{1}{4}\sum_k{}' \frac{f_k f_k^*}{g^* g} + \frac{1}{2}\sum_k{}' \frac{k f_k q_k \exp[-ikt]}{g} + \text{c.c.} \tag{48}$$

In a wave packet which has only a small range of k, the second term on the right-hand side reduces to k which is just the extra energy of the whole field above the ground state (recalling that only one quantum has been excited). More generally, this term varies with the time in a rather complex way but it will be sufficient for our purposes to consider only those wave packets in which the variation of k makes a negligible contribution to this term.

What will be of interest to us is the remaining term. When the q_k is expressed in terms of $\phi(x)$ through eq. (33) the quantum potential then reduces to

$$\Delta Q = -\frac{1}{4}\sum_k{}' \frac{f_k f_k^*}{\sqrt{\int F(x, t)\,\phi(x)\,\mathrm{d}V}\,\sqrt{\int F^*(x', t)\,\phi(x')\,\mathrm{d}V'}} . \tag{49}$$

Clearly this term implies a non-local interaction between $\phi(\boldsymbol{x})$ at one point and the $\phi(\boldsymbol{x}')$ at other points for which the integrand is appreciable. Writing

$$Q = \Delta Q + Q_0$$

where Q_0 is the quantum potential of the ground state given in eq. (30) we can write the field equation (15) as

$$\partial^2\phi/\partial t^2 = \nabla^2\phi - \delta\,\Delta Q/\delta\phi - \delta Q_0/\delta\phi\ . \tag{50}$$

Using eq. (30) and expressing Q in terms of $\phi(\boldsymbol{x})$ by Fourier analysis we obtain

$$\frac{\partial^2\phi}{\partial t^2} = \frac{1}{8}\left(\sum_k{}' f_k f_k^*\right)\frac{F(\boldsymbol{x},t)}{\left(\int F(\boldsymbol{x},t)\,\phi(\boldsymbol{x})\,\mathrm{d}V\right)^{3/2}\left(\int F^*(\boldsymbol{x}',t)\,\phi(\boldsymbol{x}')\,\mathrm{d}V'\right)^{1/2}} + \text{c.c.} \tag{51}$$

Recall that in the ground state the field was static because the effect of the quantum potential cancels out $\nabla^2\phi$ in the field equation. But now in the excited state there is a further term which causes the average wave packet to move. As happens with the quantum potential itself, the field equation is non-local as well as non-linear. This non-locality represents instantaneous connection of field at different points in space. However, it is significant only over the extent of the wave packet. In the usual interpretation the spread of the wave packet corresponds to a region within which, according to the uncertainty principle, nothing at all can be said as to what is happening. Therefore the causal interpretation attributes non-locality only to those situations in which, in the usual interpretation, no well defined properties at all can be attributed.

It is important to note that a state of the form

$$\Psi = q_k \exp[-\mathrm{i}kt]\Psi_0$$

does not correspond to the usual picture of an oscillation. This can be seen from eq. (51) because the term $\nabla^2\phi$ is absent. This result follows essentially because, as explained in our previous paper [1], stationary wave functions generally correspond to a static state in contradiction to our intuitive expectations of a dynamic state of movement. In order to obtain wave functions that give results that are closer to our intuitive notions we have to use coherent states [9].

To illustrate what these are, let us consider a particle of coordinate $\boldsymbol{x}$ undergoing harmonic oscillation. Its Schrödinger equation is

$$\mathrm{i}\hbar\,\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\,\frac{\partial^2\psi}{\partial x^2} + \frac{k}{2}\,x^2\psi\ .$$

We introduce the variable $\xi = \sqrt{(m\omega/\hbar)}x$ where $\omega = \sqrt{k/m}$. We obtain

$$\mathrm{i}\hbar\,\frac{\partial\psi}{\partial t} = \frac{\hbar\omega}{2}\left[-\frac{\partial^2}{\partial\xi^2} + \xi^2\right]\psi\ .$$

The ground state wave function is

$$\psi_0 \propto \exp[-\xi^2/2] .$$

One can show by direct differentiation that a solution for Schrödinger's equation is obtained by writing

$$\psi \propto \exp[-\tfrac{1}{2}(\xi - \gamma\, e^{-i\omega t})^2] \tag{52}$$

where γ is a complex number which we can write as

$$\gamma = |\gamma|\, e^{-i\theta} .$$

We can rewrite the above as

$$\psi \propto \exp[-\tfrac{1}{2}(\xi - |\gamma| \cos(\omega t + \theta))^2] \exp[i|\gamma| \sin(\omega t + \theta)\xi]$$
$$\times \exp[-i|\gamma|^2 \sin(\omega t + \theta)\cos(\omega t + \theta) + \tfrac{1}{2}|\gamma|^2 \sin^2(\omega t + \theta)] . \tag{53}$$

The latter exponential is a factor that can be absorbed into the normalising coefficient. This solution, therefore, represents a Gaussian wave packet whose centre oscillates harmonically with amplitude $|\gamma|$ and initial phase θ. The mean momentum (in these units) at any time, t, is $|\gamma| \sin(\omega t + \theta)$.

Evidently the packet oscillates as a unit. States of this kind which are, of course, not stationary are called coherent states. They represent the nearest approximation to the classical behaviour of an oscillator.

If a^+ represents the creation operator for a quantum (so that $a^+ a = N$ is the number operator) then one can show [9] that

$$\psi = \exp[-\tfrac{1}{2}|\gamma|^2] \sum_{n=0}^{\infty} \frac{a^{+n}\gamma^n \psi_0}{n!} . \tag{54}$$

The probability for the nth state of excitation is

$$P_n = \exp[-|\gamma|^2]|\gamma|^{2n}/n! .$$

This is a Poisson distribution and the most probable value of n is

$$n = |\gamma|^2 .$$

The root-mean-deviation is

$$\delta_n \simeq \sqrt{n}$$

or

$$\delta_n / n \simeq 1/\sqrt{n} .$$

For large values of $|\gamma|$ the number of quanta becomes indefinite but the fractional deviation of this number from the mean approaches zero. This behaviour further shows the classical significance of coherent states of high $|\gamma|$ because their energy becomes nearly determinate in the sense that the fractional deviation is small, while the absolute magnitude of the indeterminacy in energy becomes large enough so that one quantum more or less makes no significant difference. Therefore in interactions with other systems the oscillator in a coherent state of high mean energy behaves nearly like a classically well-defined object.

To further show up the classical significance of the coherent state, we evaluate the quantum potential and compare it with the classical potential. The quantum potential (in these units) will be

$$Q = -\tfrac{1}{2}[\xi - |\gamma| \cos(\omega t + \theta)]^2 + \tfrac{1}{2}\,, \tag{55}$$

but because of the Gaussian probability function, the range of fluctuation of particle position and therefore of the quantum potential (which is essentially proportional to the argument of the Gaussian) will be of the order of 1 (in these units). Evidently, as the absolute value of $|\gamma|$ becomes large, the total energy (which is kinetic + potential + quantum potential) will be correspondingly large and therefore much greater than the quantum potential. Since the quantum potential can be neglected, the dynamics will then be essentially that of classical mechanics.

Of course, the coherent state represents classical motion under conditions in which, in the usual interpretation, we would say that x and p are defined as accurately as possible within the limits of the uncertainty principle. More general wave functions are clearly possible which approach the classical limit in the sense that the quantum potential can be neglected but which correspond to much greater uncertainty in x and p.

It is evident that we can discuss coherent states of the radiation oscillators in a straightforward way. For low degrees of excitation the coherent state of an oscillator (54) can be approached as

$$\Psi \propto (1 + |\gamma| a^{*})\Psi_0\,. \tag{56}$$

This represents a time-dependent linear combination of the ground state and the first excited state. For most purposes, such as calculating probability of transitions, the results obtained from such a state are essentially the same as those obtained from the corresponding Fock state which is not time-dependent and in which the number of quanta is well defined. The only difference is that the mean field will oscillate harmonically in the coherent state.

The most interesting case to consider is that of a wave packet in which the mean value of ϕ actually moves with the form of this packet even as we approach the classical limit. To obtain this we go back to eq. (52) writing for the wave function of an oscillator

$$\psi \propto \exp[-\xi^2/2 + \xi\gamma\, \mathrm{e}^{-\mathrm{i}\omega t}]\,.$$

It will be convenient here to return to expressing the field ϕ in terms of a_k and b_k, the real and imaginary parts of q_k. This field is

$$\phi(\boldsymbol{x}, t) = \sqrt{\frac{2}{V}} \sum_{k}{}' \,[a_k(t) \operatorname{Cos}(\boldsymbol{k}\cdot\boldsymbol{x}) + b_k(t) \operatorname{Sin}(\boldsymbol{k}\cdot\boldsymbol{x})]\,.$$

We want a packet in this field whose average behaviour is given by

$$a_k = |r_k| \cos[\omega_k t + \delta_k]$$
$$b_k = |s_k| \cos[\omega_k t + \varepsilon_k] .$$

The appropriate overall wave packet will have to be made up of a product of states

$$\Psi \propto \exp\left[- \sum k \frac{(a_k^2 + b_k^2)}{2} \right] \exp[r_k \exp(\mathrm{i}(\delta_k + \omega_k t))\, a_k] \exp[s_k \exp(\mathrm{i}(\varepsilon_k + \omega_k t))\, b_k] . \tag{57}$$

For large r_k and s_k this will represent a very nearly well-defined state of movement as a wave packet (corresponding, for example, to the oscillations of an electromagnetic field in a resonant cavity). In this state, the average field will have a packet form, and there will be random Gaussian deviations from this packet which become negligible when the mean amplitude is great enough.

5. The concept of a photon

5.1. The absorption of a quantum of energy

Thus far in discussing wave packets we have not encountered any of the discrete quantised properties of the electromagnetic field. We shall now show how these come in by discussing the process of absorption of energy from the field by an atom which goes from the ground state to an ionized state. If the field is in an excited state with a single quantum of energy then it will interact with the atom. During this interaction, a very complex super-wave function of the field together with the atom will result. The resulting super-quantum potential can change dramatically to bring in bifurcation points. When the process is complete the whole system of field plus particle will then either remain in the initial state or will enter into a new state in which a quantum of energy has gone into the atom while the field is left in the ground state. The super-quantum potential, being non-local, in this way sweeps up energy from the whole wave packet and brings it to the atom. The discrete quantized properties of the field are thus explained.

The treatment is very similar to that of the Auger-like effect which we discussed in our previous paper [1]. We begin with the initial state of the field, as given by eq. (41) which represents a packet of Fock states with one quantum of excitation,

$$\Psi_0^{\mathrm{f}} = \sum_k f_k q_k \exp\left[- \sum_k{}' k' \, q_{k'}^* q_{k'} \right] .$$

The initial state of the electron in an atom is $\psi_0(\boldsymbol{\xi} - \boldsymbol{\xi}_0)$ where $\boldsymbol{\xi}_0$ represents the centre of the atom. The combined initial state is

$$\Psi_0 = \sum_k f_k q_k \, \psi_0(\boldsymbol{\xi} - \boldsymbol{\xi}_0) \exp\left[- \sum_k{}' \boldsymbol{k}' q_{k'}^* q_{k'} \right] .$$

We assume an interaction Hamiltonian

$$H_1 = \lambda \int \phi(x)\, \delta(\boldsymbol{x} - \boldsymbol{\xi})\, \mathrm{d}^3\boldsymbol{x} = \lambda\, \phi(\xi) = \lambda \sum_k q_k \exp[\mathrm{i}\boldsymbol{k} \cdot \boldsymbol{\xi}]\,. \tag{58}$$

What this means is that the particle interacts locally, i.e., only at the central point of the atom.

As happened in the Auger-like effect, the interaction Hamiltonian will introduce further terms into the overall wave function. The coefficient q_k is the sum of a creation operator and an annihilation operator. In this case, only the annihilation operator will be significant and it will produce the ground state of the field. The quantum of energy will then go into the particle $\boldsymbol{\xi}$. We assume that this energy is considerably more than enough to ionize the atom. The final wave function of the electron can be written as

$$\psi_{\mathrm{f}} = \sum_n C_n\, \psi_n(\boldsymbol{\xi}, t)$$

where the $\psi_n(\boldsymbol{\xi}, t)$ represents packets going in various directions. Perturbation theory then enables us to compute C_n as function of the time. The final wave function for the whole system is

$$\Psi = \Psi_0^{\mathrm{f}}\, \psi_0(\boldsymbol{\xi} - \boldsymbol{\xi}_0) + \sum_n \int_0^t \alpha_n(t', t)\, \psi_n(\boldsymbol{\xi}, t - t')\, \mathrm{d}t'\, \Psi_0^{\mathrm{f}}\,, \tag{59}$$

where $\alpha_n(t', t)$ can be calculated using ordinary time-dependent perturbation theory.

The coefficient $\alpha_n(t', t)$ is proportional to V_{n0} which is the matrix element between the initial state ψ_0 and one of the final states ψ_n of the electron. In more detail this is

$$V_{n0} = \lambda \int \psi_0(\boldsymbol{\xi}') \sum_k f_k \exp[\mathrm{i}\boldsymbol{k} \cdot (\boldsymbol{\xi}' + \boldsymbol{\xi}_0)]\, \psi_n(\boldsymbol{\xi}')\, \mathrm{d}V' \tag{60}$$

with $\boldsymbol{\xi}' = \boldsymbol{\xi} - \boldsymbol{\xi}_0$. (The term $\Sigma_k f_k \exp[\mathrm{i}\boldsymbol{k} \cdot (\boldsymbol{\xi}' + \boldsymbol{\xi}_0)]$ has arisen by taking the matrix element of the field between its initial excited state and final ground state.)

What (59) means is that during a small time interval $\mathrm{d}t'$, a contribution to the wave function will be produced which will be described by the integrand. This contribution is multiplied by the ground state of the field.

The packet functions $\psi_n(\boldsymbol{\xi}, t - t')$ correspond to particles that move in a direction represented by n and are set in motion at $t = t'$. These packets will all separate so that they cease to overlap. And in addition they will eventually not overlap the initial wave function ψ_0.

Returning to the causal interpretation as applied to the electron, we see that the corresponding particle will eventually either enter one of the outgoing packets or it will end up in the initial state still bound to the atom. After it enters one of these channels, it will remain there indefinitely. Moreover, because the packets do not overlap with each other or with the initial state, the quantum potential in each channel will be the same as if the other channels did not exist. In other words, as explained in our previous paper [1], all the effects of the collapse of a wave function will be present without any actual collapse having occurred. Furthermore, as also explained in our previous paper, when the electron goes on to interact with the thermal environment, the probability that these packets could ever come to interfere will become negligible.

The effective wave function of the combined system then reduces to

$$\Psi_{\text{eff}} = \psi_{n_f}(\boldsymbol{\xi}, t - t')\, \Psi_0^{\text{f}} \tag{61}$$

where n_f signifies the packet actually occupied by the particle. This represents a situation in which the particle now carries away a discrete quantum of energy, while the field is left in its ground state.

As we have indeed already indicated, during the period of transition the quantum potential will be very complex and will contain many bifurcation points, implying a highly unstable movement of the whole system. According to the initial conditions of the particle and of the total field, the whole system will in some cases enter channels corresponding to a complete transition in which a whole quantum of energy has been absorbed by the particle. In other cases, however, the whole system will remain unaltered. When the probability of transition is worked out using the assumptions that the initial conditions have a distribution of field and particle variables corresponding to $P = |\psi|^2$, then the usual probability of transition will be obtained.

This probability is proportional to $|V_{n0}|^2$ where V_{n0} is the matrix element given in (60). It will be useful to transform (60) by first introducing the function

$$K(\boldsymbol{\xi}' + \boldsymbol{\xi}_0) = \sum_{\boldsymbol{k}} f_{\boldsymbol{k}} \exp[\mathrm{i}\boldsymbol{k} \cdot (\boldsymbol{\xi}' + \boldsymbol{\xi}_0)] .$$

This is essentially the spatial form of the mean wave packet as determined by the Fourier coefficients $f_{\boldsymbol{k}}$ which appeared in the original wave function of the field (41). Therefore the wave function of the field determines a function $K(\boldsymbol{\xi})$ which is the nearest one can come to what the classical field would be in the corresponding classical situation. We may then expand,

$$\exp[\mathrm{i}\boldsymbol{k} \cdot \boldsymbol{\xi}'] = 1 + \mathrm{i}\boldsymbol{k} \cdot \boldsymbol{\xi}'$$

because ψ_0 is generally appreciable only in regions considerably smaller than the typical wave length in the packet. Then noting that $\psi_0(\boldsymbol{\xi}')$ and $\psi_n(\boldsymbol{\xi}, 0)$ are orthogonal, we obtain

$$\begin{aligned} V_{n0} &= \lambda \int \psi_0^*(\boldsymbol{\xi}') \sum_{\boldsymbol{k}} \mathrm{i}\boldsymbol{k} \cdot \boldsymbol{\xi}' \exp[\mathrm{i}\boldsymbol{k} \cdot \boldsymbol{\xi}_0]\, \psi_n(\boldsymbol{\xi}', 0)\, \mathrm{d}V' \\ &= \lambda \int \psi_0^*(\boldsymbol{\xi}')\, \boldsymbol{\xi}'\, \psi_n(\boldsymbol{\xi}', 0)\, \mathrm{d}V' \cdot \nabla K(\boldsymbol{\xi}_0) . \end{aligned} \tag{62}$$

This means that the matrix element depends on the mean field K through its gradient. The fact that it depends on the gradient has come about because we are working with a scalar theory. With a vector theory, e.g., the electromagnetic field, the matrix element is indeed proportional to the mean field itself. For our purposes, e.g., in discussing the interference pattern, this difference is not of fundamental significance because it means only that the probability of transition depends on the gradient of the mean field function (so that, for example, the maxima will occur at points of steepest gradient rather than at points of greatest intensity of the wave).

When the ground state is spherically symmetrical, the probability of transition, P, will also be spherically symmetrical so that

$$P \propto |\nabla K|^2 . \tag{63}$$

We thus complete the demonstration that, although the field is essentially continuous, the possibility

of transfer of energies is discrete and is quantized according to the usual rules. Evidently, similar results will be obtained for emission, excitation, etc. As we have already explained, the energy is swept up through the non-local and non-linear effects of the quantum potential. The fact that such a process is "quantized" is explained, in effect, by the existence of discrete stable limit cycles for the whole system of particles and fields. A transition takes place whenever the whole system passes a bifurcation point. And, as in our previous paper [1], such a transition is very rapid compared with the mean life time of the state.

It is interesting now to consider the process of inelastic scattering of quanta. Roughly speaking, this is accomplished by the absorption of the quantum which is swept up into the atom and the re-emission of the quantum slightly lower energy moving in a somewhat different direction. If the initial energy of the quantum is large compared with that absorbed by the atom, then a series of such inelastic scatterings will define something resembling a jagged track. This is as close as one can get to the track of the radiation field.

5.2. The absence of photon trajectories

Actually, it is not possible to make a causal interpretation of the field in which there would be a well-defined particle trajectory. This can be done only by systems whose basic structure is that of a particle, e.g., electrons, and which approach an exact particle behaviour in the classical limit (where the quantum potential can be neglected). On the other hand, the quantized field approaches a classical field in its classical limit where its quantum potential can be neglected and so there is no room to talk of it as a particle except in the crude sense used in our description of scattering processes.

From the mathematical point of view one can exclude the possibility of classical trajectories by noting that for relativistic boson fields, there is no conserved four-vector whose time component is positive-definite. Where there is such a four-vector, the time component represents the probability density for particles. And the space component represents the probability current. Because the density current is a four-vector it is covariant in the sense that if the probability is normalised in one Lorentz frame, then it will be normalised in every frame. Thus it is consistent to equate the time component of this four-vector to a probability density.

In the complex Klein–Gordon field there is indeed a conserved four-vector but this is not positive definite. This has therefore been interpreted as a mean charge current vector rather than as a probability for a particle. In the electromagnetic field as well as in other boson theories, there is an energy momentum tensor T^{00} which is conserved at least in the absence of interaction. Some authors [10, 11] have proposed that T^{00} can be taken as the probability density for a particle, while the velocity of the particle would be

$$V^i = T^{0i}/T^{00}.$$

This means that if the same definition of velocity is used in different frames it will lead, in general, to particle velocities that are not Lorentz transformations of each other and to normalisations of probabilities that are not the same in different frames.

Let us, for example, consider the case of the electromagnetic field. If we begin with a plane wave moving at the speed of light, the energy density is $\frac{1}{2}(E^2 + H^2)$ and the Poynting vector is $\boldsymbol{E} \times \boldsymbol{H}$. $\boldsymbol{E}$ and $\boldsymbol{H}$ are perpendicular to each other and perpendicular to energy flux $\boldsymbol{E} \times \boldsymbol{H}$ with $|E| = |H|$. Since

$|\boldsymbol{E} \times \boldsymbol{H}| = \frac{1}{2}(E^2 + H^2)$ the energy flux would define a velocity $V_i = 1$ which corresponds to the speed of light. And since all these conditions are Lorentz invariant, it follows that such a field, when transformed into new coordinates with new values $\boldsymbol{E}'$, $\boldsymbol{H}'$, will correspond to a flux also at the speed of light. At least in this case it might seem at first sight that such a definition of trajectories could be made consistently. Nevertheless, as indicated above, the normalisation will not be invariant under a Lorentz transformation. Consider, for example, the component of the energy-momentum tensor $T^{00} = \frac{1}{2}(E^2 + H^2)$ which would have to correspond to the probability density of particles. Let us suppose the energy flux is in the z-direction and that we make a Lorentz transformation in the x-direction. In the new frame the corresponding component of the energy-momentum tensor is

$$T'^{00} = T^{00}/(1 - V^2)$$

where V is the boost velocity in the Lorentz transformation. Let us now normalize T'^{00} by computing the integral

$$N' = \int T'^{00}\, \mathrm{d}x'\, \mathrm{d}y'\, \mathrm{d}z' .$$

Using the Jacobian of the transformation, we obtain

$$\mathrm{d}x'\, \mathrm{d}y'\, \mathrm{d}z' = \sqrt{1 - V^2}\, \mathrm{d}x\, \mathrm{d}y\, \mathrm{d}z$$

$$N' = \int \frac{T^{00}}{\sqrt{1 - V^2}}\, \mathrm{d}x\, \mathrm{d}y\, \mathrm{d}z .$$

This evidently does not agree with the original normalisation

$$N = \int T^{00}\, \mathrm{d}x\, \mathrm{d}y\, \mathrm{d}z .$$

What this means is that if we interpret the flow lines in any one frame as particle trajectories, we will not get the right corresponding probabilities for particles in the other frame. (But if we had replaced T^{00} by the time component of a four-vector, V^0, we would have had $N = N'$, and the trajectory interpretation would have been consistent.)

If we go on to situations in which the field does not correspond to a plane wave, we will not even get the same trajectories in different frames. For example, consider an electrostatic field E in the x-direction. The Poynting vector is zero, and this would therefore correspond to particles at rest. Let us consider a Lorentz transformation in the x-direction. Under this transformation E remains in this direction and there is no field H. Therefore the Poynting vector is still zero in the new frame. This would mean that the particle would have to be at rest in both frames which is clearly impossible.

The reason for this result is, of course, that the energy momentum is a tensor and so cannot be regarded as describing a flux of particles.

6. Interference experiments

6.1. The treatment of interference

We are now ready to discuss how interference is to be treated in the causal interpretation of the quantum theory. We shall do this by assuming that quanta of the field are detected by atoms that absorb discrete units of energy in the way described in the previous section. We shall see that in an interference experiment with two slits, for example, the probability of absorption will be proportional to what would be thought of classically as the field intensity at the point where the atom is located. Of course, this probability will vary in just such a way as to explain the observed interference patterns.

We shall then go on to discuss the Pfleegor–Mandel experiment [12] in which two independent laser sources are excited in such a way that there is only one quantum of energy in the whole system. If the two lasers can interfere in a certain region of space, this gives rise to a paradox when we try to think of where the photon that is absorbed "comes from". In our treatment this situation presents no serious problem. The energy is distributed continuously between the sources in a statistically varying way, while in the process of absorption it is swept into the single atom by the action of the non-local, non-linear quantum potential.

6.2. The two-slit interference pattern

Let us now return to a consideration of the interference in the two-slit experiment. To simplify the discussion we shall assume a resonant cavity with one of its normal modes excited to its first quantum state. Let us suppose that two small holes are made in the cavity close to each other, as shown in fig. 1. We assume that this will make a negligible change to the normal modes inside the cavity. In the classical description, waves will radiate out from the holes 1 and 2, and will interfere in the region A. We will treat this by representing the field inside the cavity by Φ_1 and the field outside by ϕ_0. It will be convenient to treat ϕ_0 as the sum ϕ_1 and ϕ_2 of the contributions from slits 1 and 2 respectively. The effective normal mode is then $\phi = \Phi_1 + \phi_1 + \phi_2$. Of course, this is an approximation that will be valid if

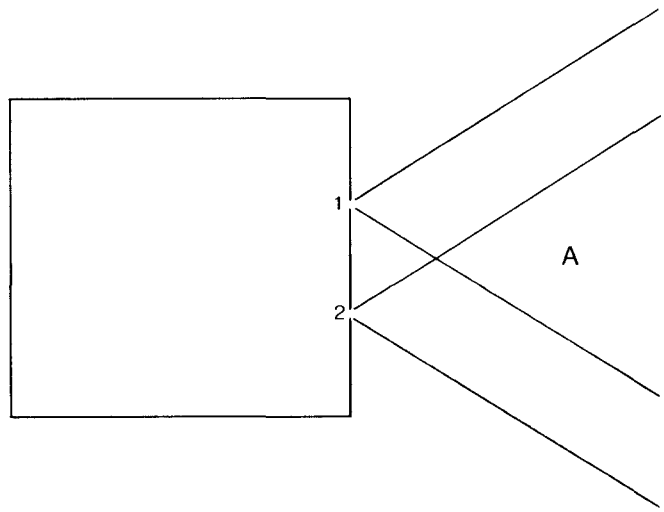

Fig. 1. Two-slit interference produced by resonant cavity.

the holes are small enough so the overall wave function will not change appreciably during the course of an experiment.

The probability of transition is not determined by ϕ alone but depends on K in the way given by (63). As we have said earlier, K is the spatial form of the mean wave packet as determined by the Fourier coefficients f_k,

$$K(\xi) = \sum_k f_k \exp[\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{\xi}] .$$

As with ϕ, K can be written as a sum with three terms

$$K = K_I + K_1 + K_2$$

where K_I represents the values of K inside the cavity and K_1 and K_2 represent the contributions from slits 1 and 2, as propagated from the cavity.

If we place an atom at point A where the contributions of the two slits overlap, the probability of transition will then be

$$P \propto |\nabla(K_1 + K_2)|^2 . \tag{64}$$

This shows the characteristic interference structure. Thus in a statistical ensemble of experiments done with the same super-quantum wave function, the usual fringes will be obtained. As we have indicated in the previous section, the fact that they depend on the gradient of the field is due to use of a scalar field (which would follow also in the usual interpretation). With a vector field it is readily shown that probability is indeed proportional to the field intensity [13].

6.3. The Pfleegor–Mandel experiment

Let us now go on to the Pfleegor–Mandel experiment [12]. We shall treat this in terms of two resonant cavities C_1 and C_2, each with a small hole in it. The beams coming out of the holes will cross at A as shown in fig. 2. Each cavity will have normal modes denoted respectively by the functions K_1 and K_2. As in the interference experiment, we divide each of these into a part that is inside and a part that is outside the cavity,

$$K_1 = K_{I_1} + K_{0_1} \qquad \text{and} \qquad K_2 = K_{I_2} + K_{0_2} .$$

Suppose that one of these lasers is excited to the first coherent state while the other is left in the ground state. According to (41) the overall wave function is then

$$\Psi_1 = \sum_k f_{1k} q_k \Psi_0$$

where Ψ_0 is the vacuum state and where f_{1k} is the Fourier coefficient of K_1. But if the second laser is excited then the overall wave function is

$$\Psi_2 = \sum_k f_{2k} q_k \Psi_0 .$$

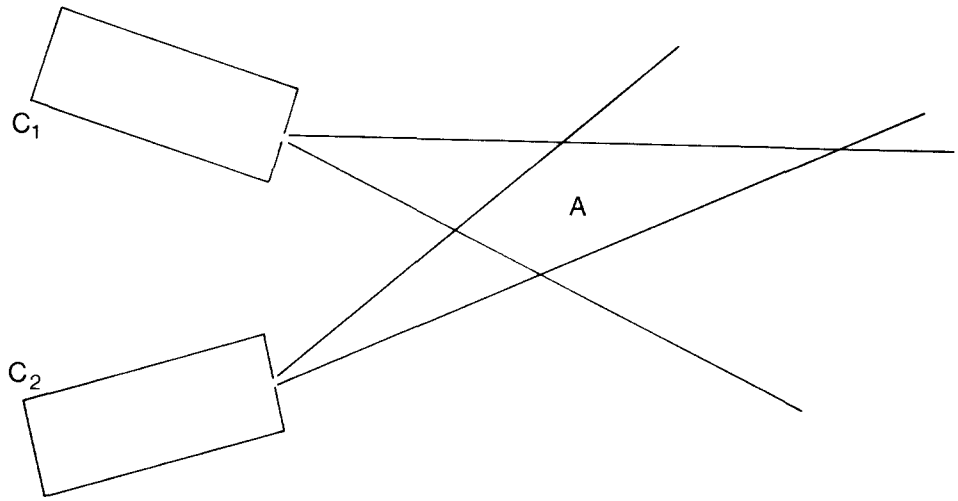

Fig. 2. Schematic diagram of interface of two independent lasers used in the Pfleegor–Mandel experiment.

However linear combinations of such states are possible corresponding to their being in phase or out of phase,

$$\Psi_{+} = \sum_{k} (f_{1k} + f_{2k}) q_k \Psi_0 \tag{65}$$

while the "out of phase" state is

$$\Psi_{-} = \sum_{k} (f_{1k} - f_{2k}) q_k \Psi_0 \,. \tag{66}$$

In these states it is not possible to attribute the quantum of energy to either laser.

To have two lasers with definite phase relations of this kind is, however, essentially the same as considering two wave packets with a definite phase relation as discussed in section 3. It follows from what we said there that the energy will be distributed between the two lasers in a way that varies statistically. Therefore the usual mode of talking about this, namely, that the entire quantum of energy is either in one laser or the other, with equal probability, is not valid.

If we now suppose that an atom is placed at A to absorb a quantum, the super-quantum potential would "sweep up" the energy from both lasers to constitute a full quantum. However, in a statistical ensemble of experiments of this kind one would obtain the usual interference pattern. In this way the causal interpretation provides a simple means of understanding what otherwise seems a very paradoxical situation in this experiment.

7. The EPR experiment

We now go on to apply the causal interpretation to the EPR situation. Because we are working in terms of a scalar field, we cannot discuss the usual example in terms of polarisation or spin. Rather, we shall return to something more like the original EPR experiment. This example was applied only to

particles. Here there was a wave function $\delta(\boldsymbol{x}_2 - \boldsymbol{x}_1 - \boldsymbol{a})$ representing a situation in which whenever particle 1 is found at $\boldsymbol{x}_1$, particle 2 will be found at $\boldsymbol{x}_2 = \boldsymbol{x}_1 + \boldsymbol{a}$. And, of course, whenever the momentum of particle 1 was measured, that of particle 2 would be the opposite. This meant that it was possible to measure both momentum and the position of 2 by making measurements on particle 1 only. EPR assumed there was no connection between the particles and therefore concluded that either the position or the momentum of particle 2 could be measured without it being disturbed.

But they further assumed that anything that could be measured with unlimited accuracy without being disturbed is an "element of reality". That is to say it exists objectively independently of our knowledge about it. This implied that both $\boldsymbol{x}_2$ and $\boldsymbol{p}_2$ are such elements of reality. But quantum mechanics has no way of defining two non-commuting observables simultaneously and therefore in its usual interpretation it cannot attribute independent reality simultaneously to both of them. From this EPR concluded that "quantum mechanics was incomplete" in the sense that it lacked the concepts needed to describe and understand the underlying reality. They did not wish to imply the quantum mechanics was empirically wrong. Rather they regarded the formalism as a sort of probability calculus which gave correct statistical predictions but which contained no adequate reflection of the underlying reality of the individual system.

In order to transpose this experiment into a field-theoretical context, we first replace $\delta(\boldsymbol{x}_2 - \boldsymbol{x}_1 - \boldsymbol{a})$ by a function $S(\boldsymbol{x}_2 - \boldsymbol{x}_1 - \boldsymbol{a})$ which is sharply peaked but regular and can be Fourier analysed as

$$S = \sum_{\boldsymbol{k}} g_{\boldsymbol{k}} \exp[\mathrm{i}\boldsymbol{k} \cdot (\boldsymbol{x}_2 - \boldsymbol{x}_1 - \boldsymbol{a})] .$$

If the width of the function S is considerably less than $\boldsymbol{a}$, then clearly the conditions of the experiment are not significantly altered but we can avoid the mathematical difficulties that arise from the δ-function.

The EPR experiment will now correspond to exciting two normal modes of opposite momentum in a correlated way. The wave function of the combined system of field and particle will then be,

$$\psi = \sum_{k} g_k \exp[-\mathrm{i}\boldsymbol{k} \cdot \boldsymbol{a}]\, q_k q_{-k}\, \Psi_0\, \psi_0(\boldsymbol{\xi} - \boldsymbol{\xi}_0) \tag{67a}$$

$$= \int\int S(\boldsymbol{x}_2 - \boldsymbol{x}_1 - \boldsymbol{a})\, \phi(\boldsymbol{x}_2)\, \phi(\boldsymbol{x}_1)\, \Psi_0\, \psi_0(\boldsymbol{\xi} - \boldsymbol{\xi}_0)\, \mathrm{d}V_1\, \mathrm{d}V_2 \tag{67b}$$

where $\psi_0(\boldsymbol{\xi} - \boldsymbol{\xi}_0)$ is the initial wave function of the electron that absorbs (and therefore detects) one of the quanta near the position ξ_0. This clearly describes a situation in which there is a statistical tendency that whenever ϕ is excited at $\boldsymbol{x}_1$, a corresponding excitation of ϕ will take place at $\boldsymbol{x}_2 = \boldsymbol{x}_1 + \boldsymbol{a}$. On the other hand, whenever there is an excitation $\boldsymbol{k}$ there will be a corresponding excitation $-\boldsymbol{k}$. This is essentially what corresponds in field theory, to the EPR situation with regard to particles.

Let us now suppose we place an atom at $\boldsymbol{\xi}$, which can absorb a quantum of energy and liberate an electron in a photoelectric effect. To treat this process we introduce the interaction Hamiltonian

$$H_1 = \lambda \sum_{\boldsymbol{k}} q_{\boldsymbol{k}} \exp[\mathrm{i}\boldsymbol{k} \cdot \boldsymbol{\xi}] .$$

This Hamiltonian will absorb a quantum of energy, say $q_{\boldsymbol{k}}$, leaving the field with only a single quantum, $q_{-\boldsymbol{k}} = q_{\boldsymbol{k}}^*$, and it will introduce a factor $\exp(-\mathrm{i}\boldsymbol{k} \cdot \boldsymbol{\xi})$.

However, as shown by eq. (59) this term will also bring about a transition of the particle wave function from $\psi_0(\boldsymbol{\xi} - \boldsymbol{\xi}_0)$ to a final wave function which includes terms such as $\psi_n(\boldsymbol{\xi}, t)$. Each ψ_n determines a separate channel, and eventually the electron enters one of them ψ_{nf} after which the other channel may be dropped from consideration. The final wave function of field plus particle will then be

$$\Psi_f = \sum_k g_k \exp[-i\boldsymbol{k}\cdot(\boldsymbol{\xi} + \boldsymbol{a})]\, q_{-k}\, \Psi_0\, \psi_{nf}(\boldsymbol{\xi}, t)\,. \tag{68}$$

Recall $q_{-k} = q_k^*$ when ϕ is real. We are left with

$$\Psi_f = \int S(\boldsymbol{x} - \boldsymbol{\xi} - \boldsymbol{a})\, \phi(\boldsymbol{x})\, \Psi_0\, \psi_{nf}(\boldsymbol{\xi}, t)\, dV\,. \tag{69}$$

This result is the same as the one we would get using the usual quantum theory, which effectively contains the assumption of the collapse of the wave function to the final state. However, in the causal interpretation there has been no collapse, but an equivalent result has been brought about by the action of the super-quantum potential. The field now fluctuates in a region near $\boldsymbol{x} = \boldsymbol{\xi} + \boldsymbol{a}$. The absorption of the quantum of energy by the atom at $\boldsymbol{\xi}$ has therefore given shape and form to the remaining field at $\boldsymbol{x} = \boldsymbol{\xi} + \boldsymbol{a}$. This is a kind of non-linear and non-local effect of the super-quantum potential.

Let us now go on to discuss an experiment in which momentum would be determined rather than position. To bring this about we could consider the Compton effect. The quantum of frequency ν is to be scattered from a very weakly bound electron. This quantum is absorbed and a new quantum emitted with frequency ν' at an angle θ relative to that of the original wave vector. The electron absorbs the energy difference $\nu - \nu'$ and comes off at an angle ϕ with momentum p.

The relevant formulae are obtained from the detailed conservation of energy and momentum for each individual scattering process. The conservation of energy gives

$$\nu - \nu' = \sqrt{m^2 + p^2} - m\,. \tag{70}$$

It will be adequate for our purposes to assume the electron comes off in the non-relativistic range so that

$$\nu - \nu' = p^2/2m \qquad (\text{with } p/m \ll 1)\,. \tag{71}$$

Taking into account the conservation of momentum in the original and perpendicular directions, we obtain

$$\nu = \frac{p(1 - p^2/4m^2)}{2\cos\phi - p/m} \tag{72}$$

and

$$\cot\theta = \frac{\nu - p\cos\phi}{p\sin\phi}\,. \tag{73}$$

With the aid of equations (71), (72) and (73) we can calculate both the initial and final momentum of the field quantum from a knowledge of the momentum vector of the recoil electron. Therefore to each

value of this recoil momentum, $\boldsymbol{p}$, there is a corresponding quantum of energy ν and a momentum vector $\boldsymbol{k}'$ of the scattered quantum.

Let us represent the initial state of the electron by $\psi_0(\boldsymbol{\xi} - \boldsymbol{\xi}_0)$. This will be very poorly localised so that the range of momenta will be small. The limiting accuracy of the measurement will be determined by this range of momenta. To treat this problem in detail would involve time-dependent perturbation theory for the scattering process which includes the absorption of the original quantum and emission of the new quantum in another direction. However, for our purposes here we do not have to go into all the details. It is sufficient to take into account the fact that the initial wave function of the electron $\psi_0(\boldsymbol{\xi} - \boldsymbol{\xi}_0)$ goes into a linear combination of packet functions $\psi_n(\boldsymbol{\xi}, t)$. Each of these packets will constitute a channel and the electron will enter one of them so that thereafter the other channels can be neglected. We know from the more detailed mathematical treatment that these packets are very nearly plane waves with a range of wave numbers $\boldsymbol{k}$ corresponding roughly to the range in $\psi_0(\boldsymbol{\xi} - \boldsymbol{\xi}_0)$.

There will also be a matrix element which has to be worked out from second-order perturbation theory. Once again, the details are unimportant for us here. The only point that is of interest for this problem is that the second-order matrix element for the whole system implies the absorption of the original quantum q_k and its replacement by a narrow packet in momentum space. In this discussion we will ignore the width of the packet and say that the scattered quantum is represented by a definite state of momentum of $\boldsymbol{k}'$.

The original wave function (67a) will effectively reduce to

$$\Psi_f \propto g'_k \exp[-i\boldsymbol{k}' \cdot \boldsymbol{a}]\, q_{k'} q_{-k'}\, \Psi_0\, \psi_n(\boldsymbol{\xi}, t_0)\,. \tag{74}$$

The above result shows that after the Compton effect has taken place, not only does the scattered quantum go into a definite state q'_k but also the additional quantum goes into the corresponding state $q_{-k'}$. In this case the quantum potential arising during the Compton scattering interaction has produced a wave-like form of the distant quantum which corresponds to that produced in the directly scattered quantum. This shows how we explain the well-known result of the usual interpretation that any measurement profoundly affects the form of that which is measured. However, it should be emphasised here that the fact that the Compton effect is being used specifically to make a measurement is a side issue. The same thing would happen in any Compton scattering occurring of its own accord. In other words, what we are discussing here is, as in the previous paper, an ontology of the quantum process which does not require any further epistemological basis.

Finally, it is clear that the ontology proposed here differs significantly from that assumed by Einstein. For the latter does not admit non-local interactions and this is why Einstein found it natural to assume that the two particles were not connected in any way at all. However, in the causal interpretation they are connected by the quantum potential; and therefore they are not separate "elements of reality" in the sense defined by Einstein. Rather, as we have brought out in our previous paper, all these are participating in an undivided whole.

8. Generalization of the interpretation and extension to fermions

Thus far this paper has been discussing the causal interpretation in terms of the scalar field. But as we have already indicated earlier, this can be easily generalized to the vector field and this has indeed been done [3, 13]. Bosons in general thus seem to present no serious difficulty.

However, an extension of this interpretation to fermions cannot be carried out along the lines given in this paper. For if we regard the fermions as fields, they obey anticommutation relations which have no classical limit and which do not permit a picture of continuous field variables that we have used for bosonic systems.

We propose instead to treat fermions as particles which, in a many-body system, will be restricted to antisymmetric wave functions. Certainly in the non-relativistic limit this interpretation will work out in a straightforward way. To go to a relativistic theory we need an interpretation of the Dirac equation. Such an interpretation has in fact been proposed [14]. Basically, the idea is to assume the probability density for a single particle is given by $\psi^{+}\psi$ and the current density is

$$V_i = \psi^{+}\alpha_i\psi/\psi^{+}\psi .$$

Since the Dirac equation is covariant, this procedure will have the same meaning in every frame so that the interpretation is also covariant. In the above reference, it is shown that this will work consistently.

In the many-body system, the wave function will have a set of spinor indices for each particle. The probability density for the whole set of particles in their configuration space is obtained by summing the spinor indices over all the particles. A similar approach could be used to obtain the probability current density of the nth particle by taking the average of velocity operator $\boldsymbol{\alpha}_n$ summed over the spinor indices of all the particles. The velocity of the nth particle will then be the ratio of this current-density to the probability density.

In this approach, it is necessary to use Dirac's original suggestion that the negative energy states in the vacuum are filled. Pair creation is treated as a transition of a negative energy particle into a positive energy state. Of course, in principle, every problem will involve all the particles in the vacuum, though in most cases the wave function will factorise, so that it will be sufficient to consider a limited number of particles and ignore the rest. This is similar, in many ways, to what happens with the boson fields for which likewise the whole universe must be considered in principle, while in practice a limited number of Fock states may be adequate.

Although this theory provides a consistent interpretation, it is somewhat ad hoc and consequently is not likely to provide a great deal of further insight. However, it is our view that, at this stage, it is premature to put too much emphasis on the interpretation of relativistic quantum mechanics. This is because we feel that the theory in its current stages of development is probably not consistent enough to be given an overall coherent interpretation. First of all, there are the infinities which make it difficult even to see what the theory means. For example, the dressed particles are said to be in a different Hilbert space from that with which the theory starts. In general relativity these difficulties are even greater. Thus the quantization of the gravitation field implies that space and time would lose their customary meaning, not only at the Planck length of 10^{-33} cm, but also in black holes and near the "big bang" with which the universe is said to have originated. Moreover, near singularities of this type, there could be no atoms or other particles or any other structure of the kinds that we know. Therefore, there could be no measuring instruments and this means that the usual interpretation will have nothing to talk about in these domains. Even the causal interpretation could find no foothold if there is no clear significance for space-time. For these reasons we feel that a serious interpretation of the relativistic domain would best be deferred until we obtain a more consistent theory. For the present, we should regard the relativistic quantum theory mainly as a set of algorithms from which we can derive a large number of useful and interesting results.

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