TCM

BEC-BCS Crossover in Cold Atoms

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Outline

- Theory
  - *- Cold Atoms*
  - *- BEC-BCS Crossover*
  - *- Feshbach Resonance*
  - *- Universal number, ξ*
- Previous Work
- Our method
  - *- Modelling the interaction*
  - *- Pairing wavefunctions*
- (Results)
  - *- Total Energy*
  - *- Condensate fraction*
- Future work

Theory

Cold Atoms

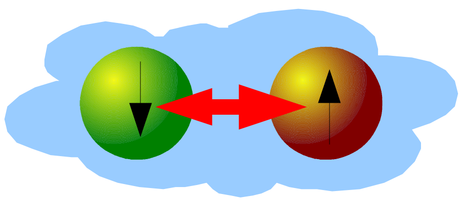
- Bose Gas
  - *- BEC (1995)*
  - *- Quantised Vortices*
  - *- Propagation of solitons*

- Fermi Gas
  - *- e.g.* ^{6}Li, ^{40}K, ^{2}H

- Vary the interaction strength between fermionic atoms...

BEC-BCS crossover

- Strong pairing :

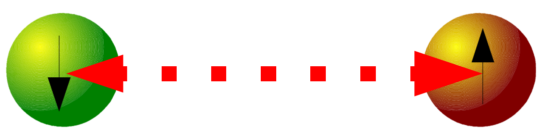


- Atoms form molecules of up and down spin

- These molecules are bosonic

- Bosonic molecules condense into BEC

- Weak pairing :



- Atoms interact over a long range

- BCS theory

- Interesting point at *unitarity* :

- Dilute : Interatomic potential range << Interparticle distance

- Strongly interacting : Scattering length >> Interparticle distance

- How would this occur?

Feshbach Resonance

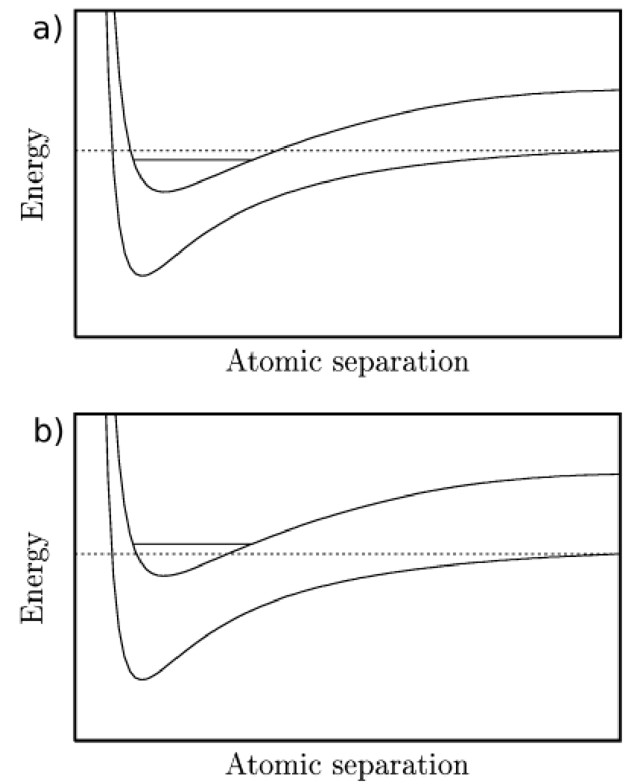
• 2 channels corresponding to different spin states

• *Open channel* (scattering process)

• *Closed channel* (bound state)

• *Resonance* occurs when Open and Closed channel energies are close

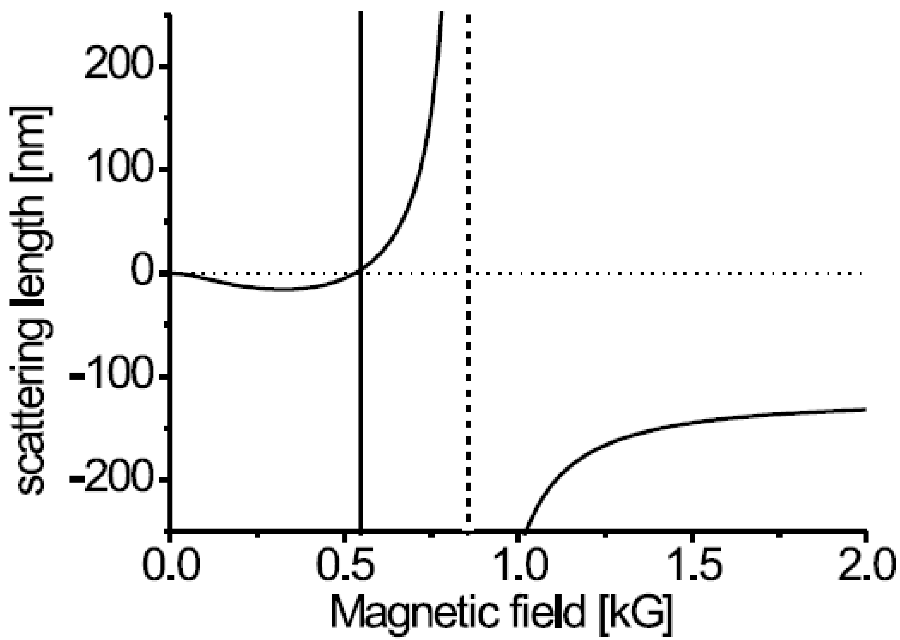
• Channel energies are *tuned* by a magnetic field



From Giorgini et al eprint cond-mat 0706.3360v1

Feshbach Resonance (2)

- The *s-wave scattering length*, *a*, diverges at resonance



Resonances in ^{6}Li from Bourdel et al PRL 93 050401

Universal Number, ξ

- At resonance the only relevant energy scale is that of a non-interacting gas

$$E_I = \xi E_{FG} = \xi \frac{3 k_F^2}{10 m}$$

- This value, ξ, is believed to be universal when $k_F R_0 << 1$ where R_0 is the effective range of interaction

- Throughout we measure the interaction strength in units of $1/k_F a$

Previous Work

- 2 previous studies using QMC,

  - *J. Carlson et al, PRL 91 050401: $\xi = 0.44(1)$*
  - *G.E. Astrakharchik et al, PRL 93 200404: $\xi = 0.42(1)$*

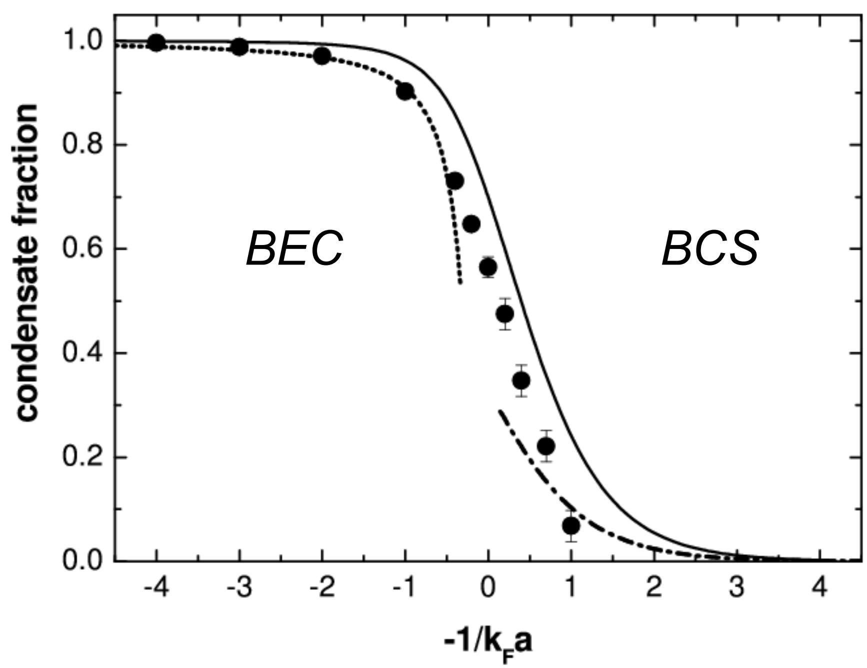
Symbols - QMC

(Dot - Bogoliubov

Dot-Dashed BCS theory

Black line SC-MF)

From Astrakharchik et al PRL 95 230405.



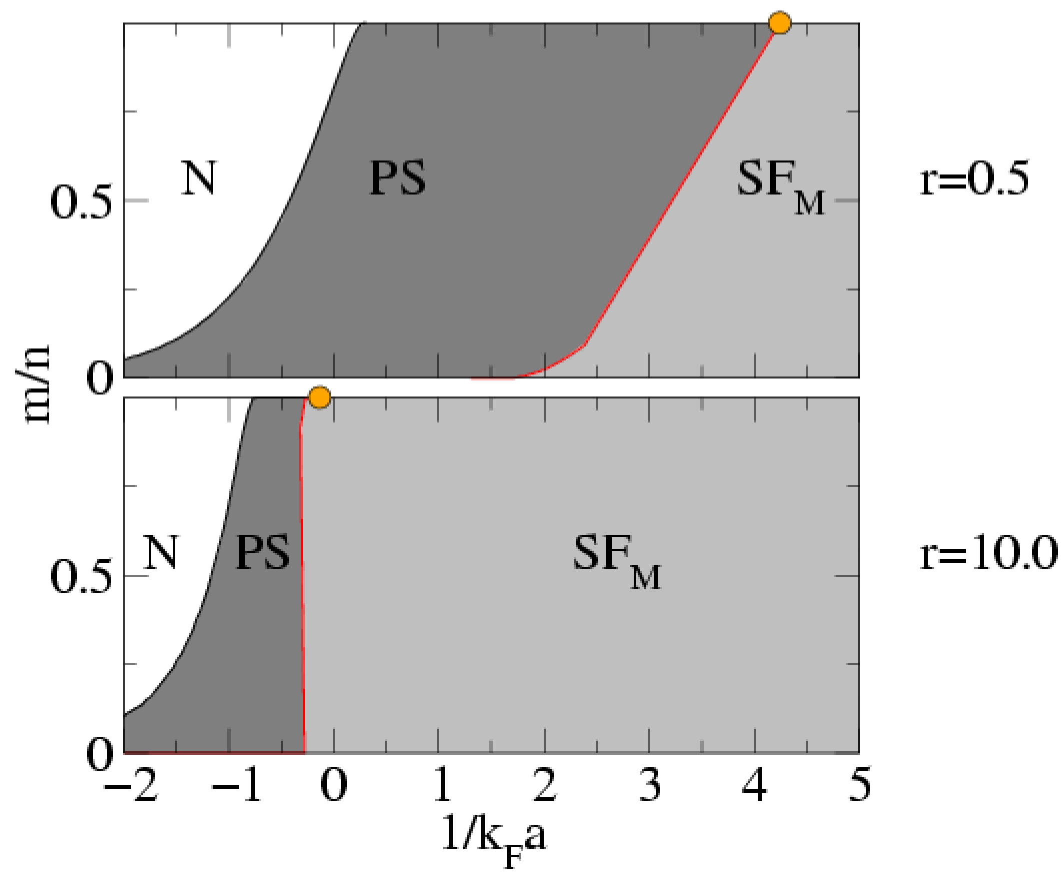
- Other methods,

  - *Nishida et al: eprint cond-mat/0607835 $\xi = 0.38(1)$*

This Work

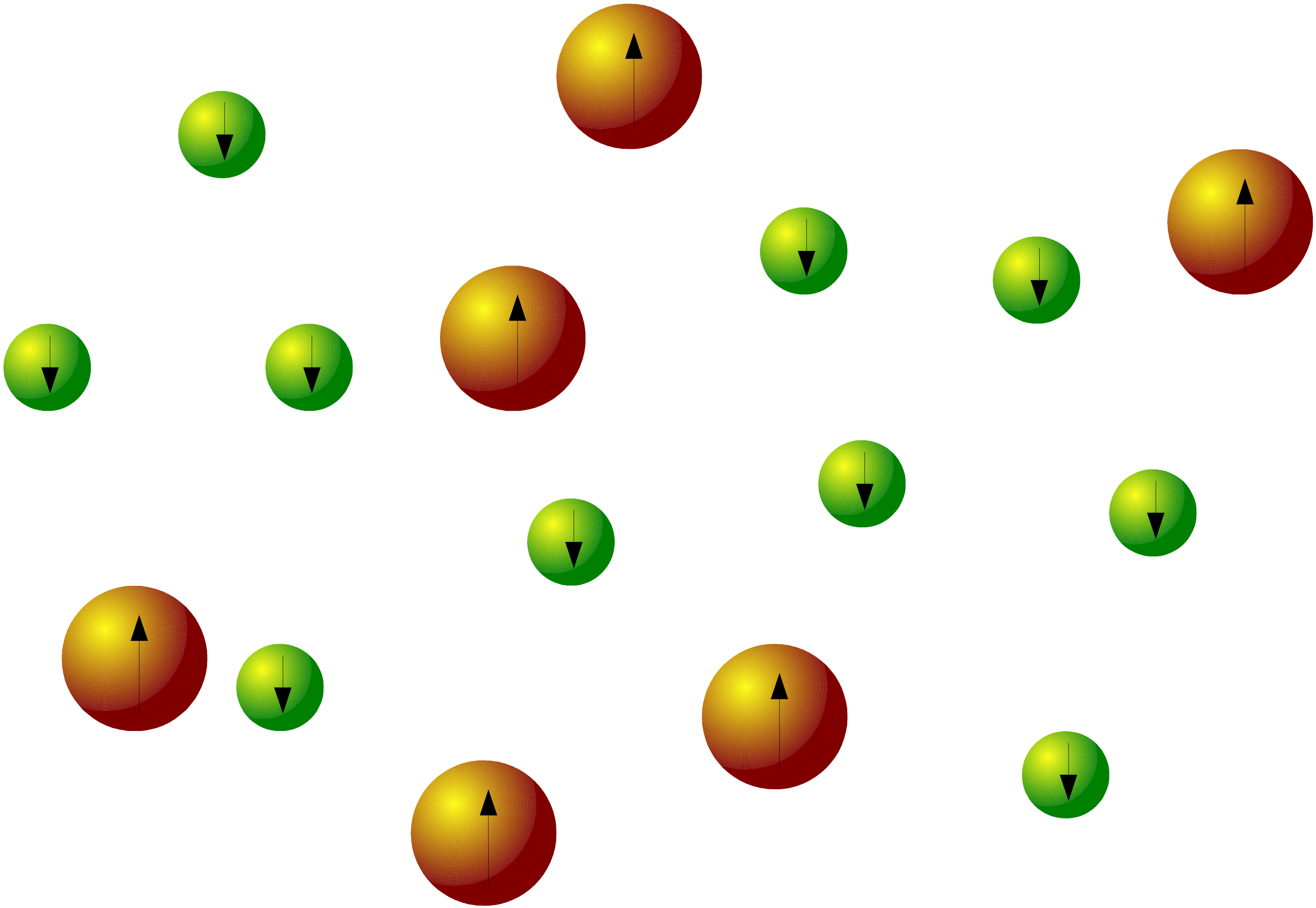
- Unequal particle numbers / unequal masses

r = m↓/m↑
n = density of particles
m = magnetisation

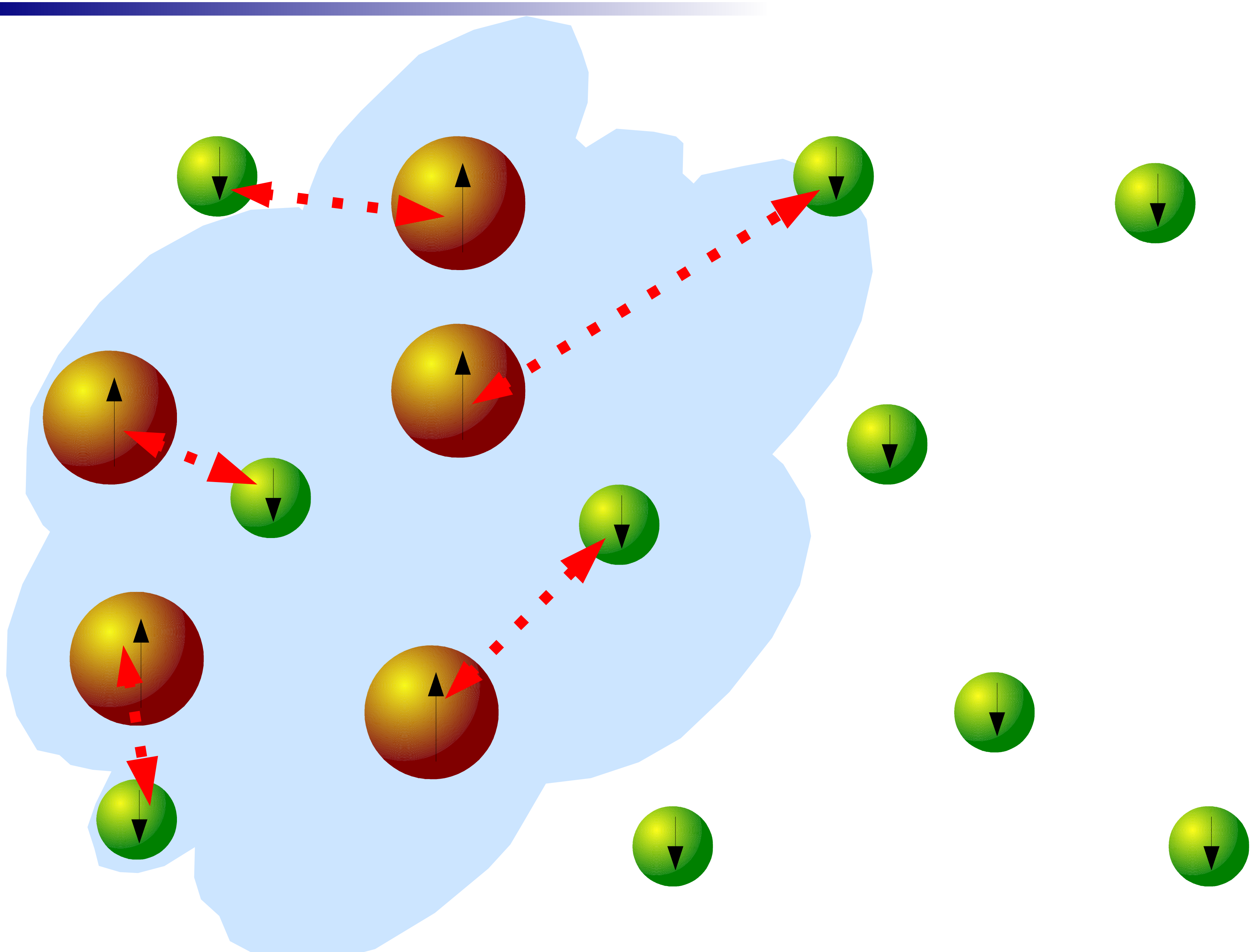


From Parish et al, PRL 98 160402 (2007)

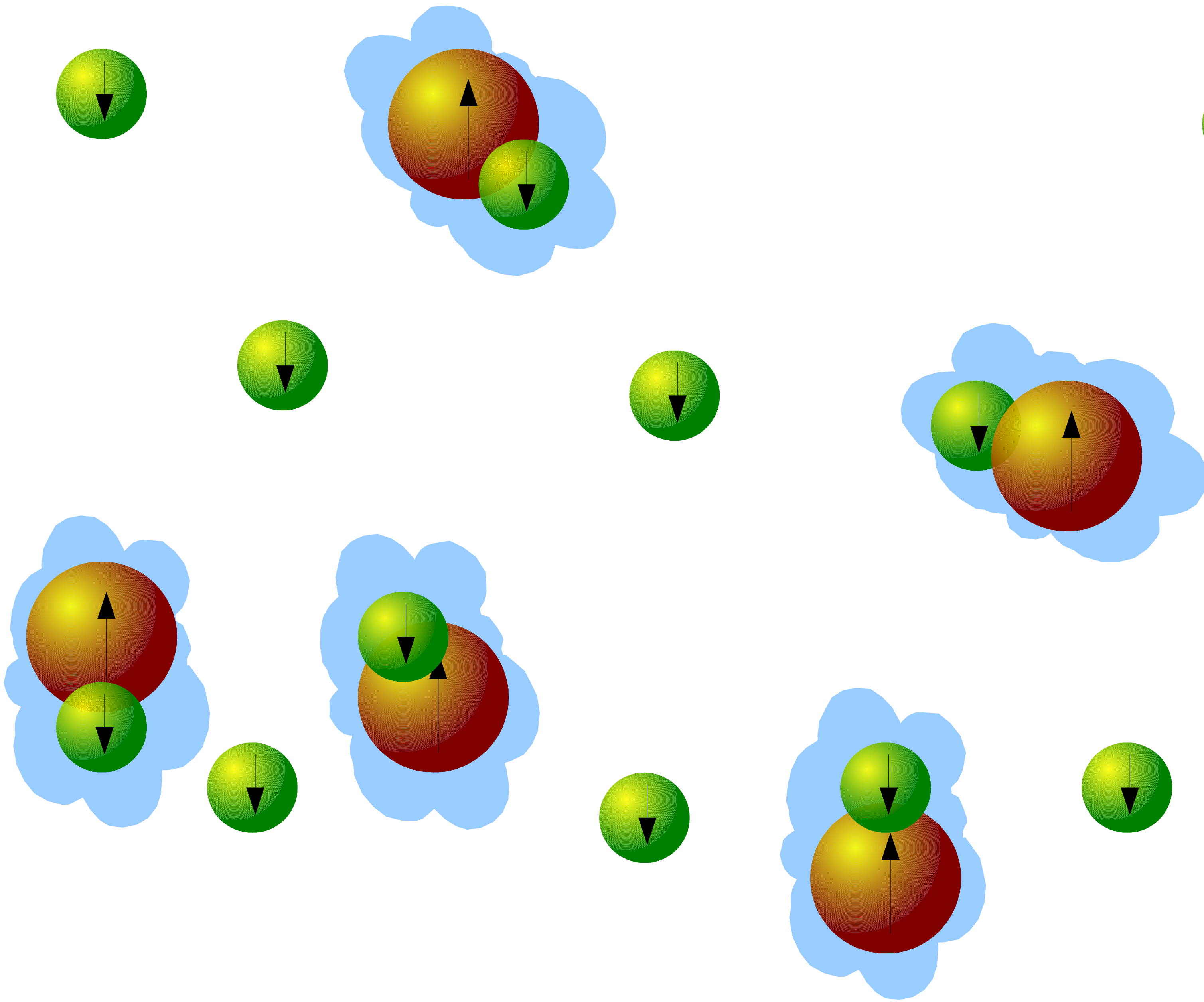
Normal Phase



Phase Separated



Magnetised Superfluid



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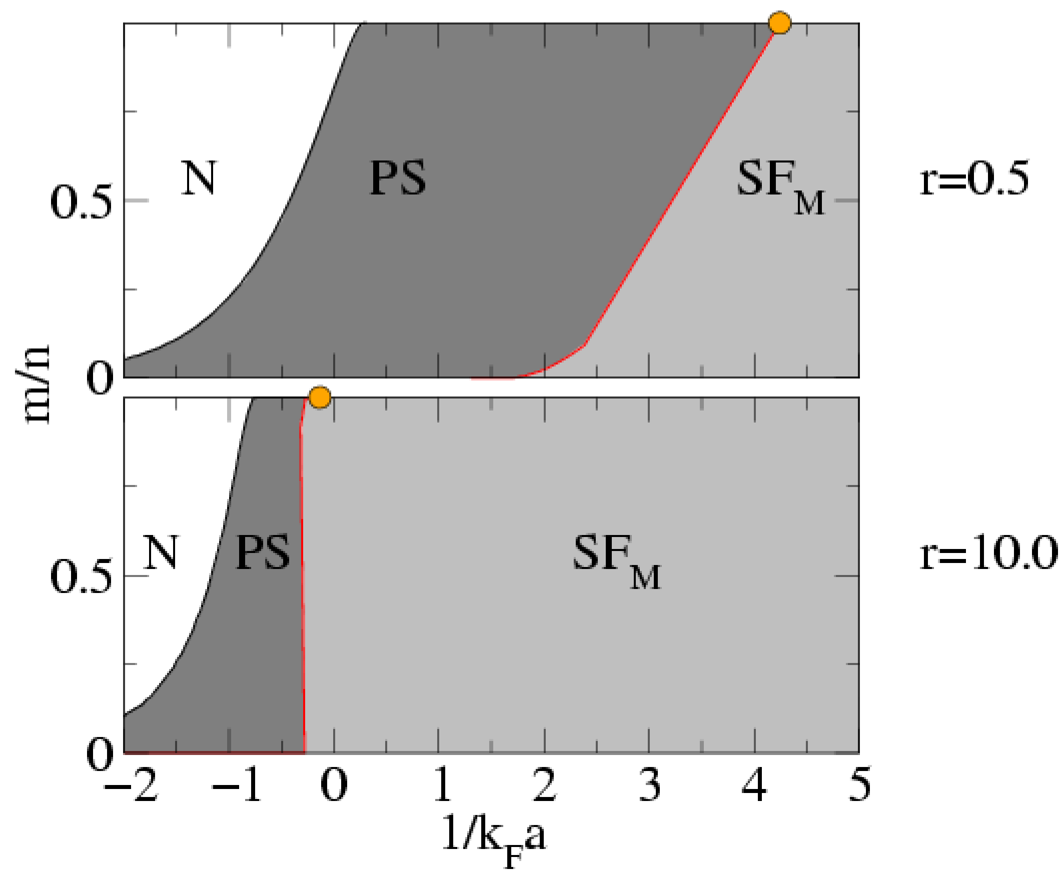


From Parish et al, PRL 98 160402 (2007)

The Model

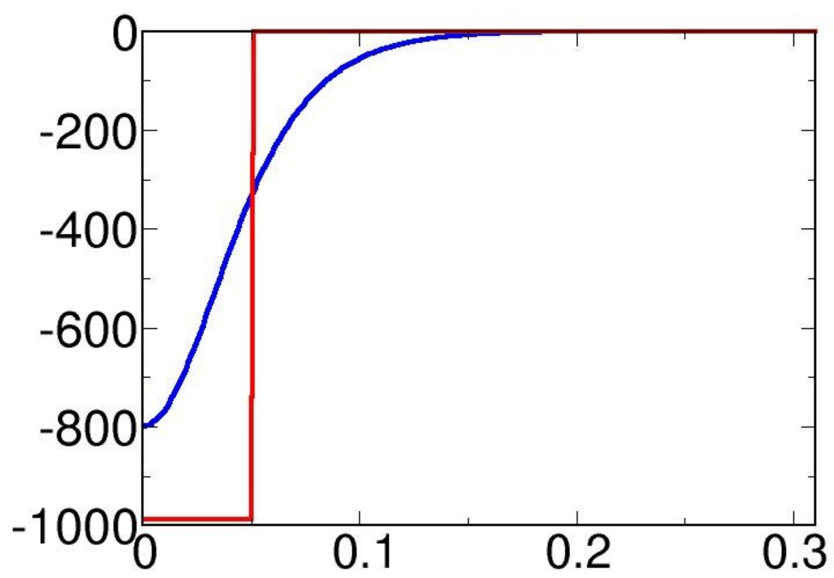
Modelling the Feshbach Resonance

- Pauli-Exclusion Principle for parallel spin

- As interatomic potential << atom spacing, the exact form of the interaction is unimportant

- 2 types of interaction normally used...

Square Well



Pöschl-Teller

- We use the Pöschl-Teller

Pairing Wavefunctions

- In QMC for a spin-independent operator we normally use a product of *Slater determinants,* one containing *n* up-spin and one *m,* down-spin one-particle orbitals, ϕ.

$$\Psi = e^{J} \begin{vmatrix} \phi_1(r_{1\uparrow}) & \cdots & \phi_n(r_{1\uparrow}) \\ \vdots & \ddots & \vdots \\ \phi_1(r_{n\uparrow}) & \cdots & \phi_n(r_{n\uparrow}) \end{vmatrix} \begin{vmatrix} \phi_1(r_{1\downarrow}) & \cdots & \phi_m(r_{1\downarrow}) \\ \vdots & \ddots & \vdots \\ \phi_1(r_{m\downarrow}) & \cdots & \phi_m(r_{m\downarrow}) \end{vmatrix}$$

- However, we want a wave function that explicitly describes pairing

Pairing Wavefunctions (2)

- We now use only one *Slater determinant*. It contains only one type of orbital, ϕ, which is a function of the distance between up and down particles

$$\Psi = \mathrm{e}^{J} \begin{vmatrix} \phi(r_{1\uparrow} - r_{1\downarrow}) & \cdots & \phi(r_{1\uparrow} - r_{n\downarrow}) \\ \vdots & \ddots & \vdots \\ \phi(r_{n\uparrow} - r_{1\downarrow}) & \cdots & \phi(r_{n\uparrow} - r_{n\downarrow}) \end{vmatrix}$$

- 3-types of ϕ have been tried

$$\phi = \sum_{i=1} C_i \exp\left(i\left(r\uparrow - r\downarrow\right)\right)$$ *- Pairing Plane-waves*

$$\phi = \sum_{i=1} g_i \exp\left(\beta_i \left(r\uparrow - r\downarrow\right)^2\right)$$ *- Pairing Gaussians*

$$\phi = \sum_{i=0} \alpha_i \left(r\uparrow - r\downarrow\right)^i$$ *- Pairing Polynomials*

- And combinations of the above

Jastrow factor + Backflow

- Jastrow factor of *Drummond et al PRB 70 235119 (2004)* *(CASINO users, that's a Jastrow U + P)*

$$J=\sum_{l=1}^{L}\alpha_l r_{ij}^{l}+\sum_{A} a_A \sum_{G_A}\cos\left(G_A\cdot r_{ij}\right)$$

- Backflow corrections of *López Ríos et al PRE 74 066701 (2006)*

$$\Psi^{BF}\left(\boldsymbol{R}\right)=\mathrm{e}^{J\left(\boldsymbol{R}\right)}\Psi_s\left(\boldsymbol{X}\right)$$

$$\boldsymbol{x_i}=\boldsymbol{r_i}+\xi_i\left(\boldsymbol{R}\right)$$

- We optimise the Jastrow, Backflow and orbital parameters using VMC and *energy minimisation* (Umrigar *et al PRL 98 110201 (2007))*

- *Conclude using DMC*

Pairing Wavefunctions (3)

- Polynomial of order 20 + Jastrow U (Polynomial) $E_{VMC} = 0.650(4)$
- Gaussian $E_{VMC} = 0.664(2)$
- Gaussian + Jastrow U $E_{VMC} = 0.5195(6)$
- Gaussian + Jastrow U + Backflow (Eta) $E_{VMC} = 0.4745(1)$
- Gaussian + Jastrow U + Backflow (Eta) + Jastrow P $E_{VMC} = 0.4605(2)$

E_{DMC} ~ 4% lower

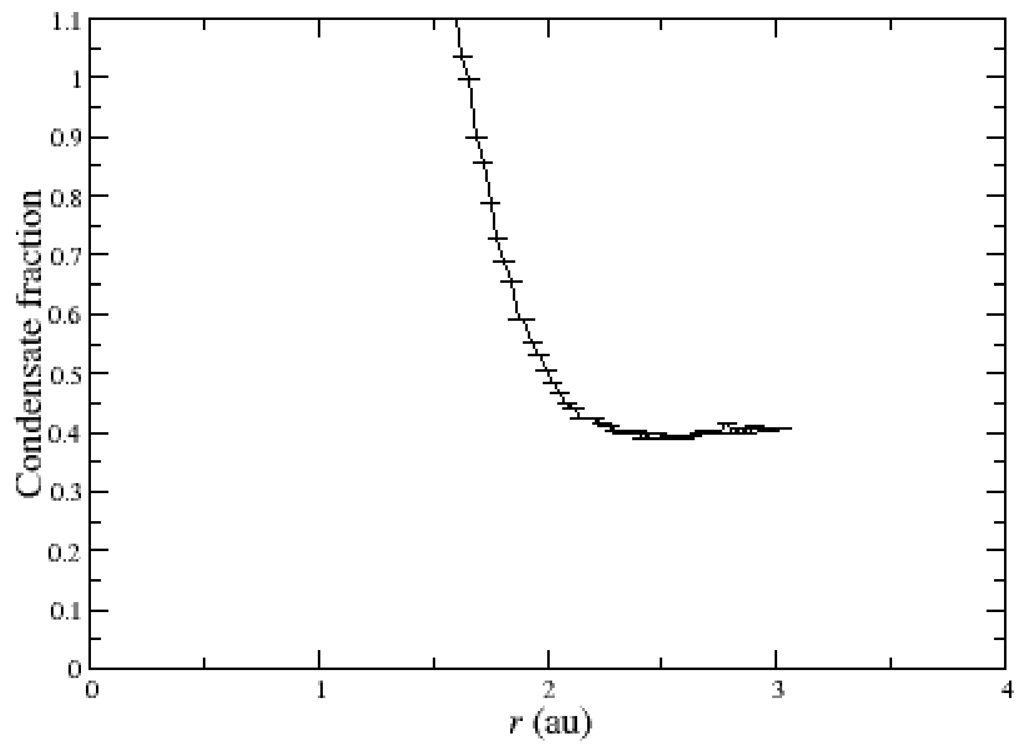
Results

Present state

- Testing our ideas and CASINO with reproducing the value of ξ
  - *J. Carlson et al PRL 91 050401:* $\xi = 0.44(1)$
  - *G.E. Astrakharchik et al PRL 93 200404:* $\xi = 0.42(1)$
  - *This work (to date)* *VMC* $\xi \sim 0.45$
    DMC $\xi \sim 0.44$ *(and still* ⇓ *)*

- *Condensate fraction:*

  ~ 0.4 – not right yet!



Where next?

- Verify / improve on ξ
- Calculate 1- and 2-body density matrices
- Calculate condensate fraction
- Move on to varied mass system

Acknowledgements

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