

Localization tensor : Metal-Insulator Transition

Bohshiang Jong

Definitions

Insulator

Zero dc conductivity at 0k

Sustains bulk macroscopic polarization with or without electric field

Metal-insulator transition

Gap opens as a competition between the kinetic energy and potential energy

Types of Insulator

Single-particle

Band

electron interactions with the periodic potential of the ions

Peierls

electron interactions with static lattice deformation

Anderson

electrons interaction with impurities

Many-Particle

Mott

electron-electron interactions

Modern theory of insulating state

[Kohn]: Insulating state arises when the electron many-body wave function is localized

Wave function Ψ breaks into disconnected parts in the configuration space

$$\Psi(x_1, \dots, x_N) = \sum_{M=-\infty}^{\infty} \Psi_M(x_1, \dots, x_N)$$

The large supercell Ψ_M and $\Psi_{M'}$ have an exponentially small overlap for $M \neq M'$. **Kohn proved that dc conductivity vanishes.**

Localization tensor

Macroscopic polarization and localization are expectation value of many-body phase operator

$$Z_N^{(\alpha)} = \left\langle \Psi \left| e^{-i \frac{2\pi}{L_\alpha} \sum_{i=1}^n X_i} \right| \Psi \right\rangle$$

Phase defines the macroscopic polarization

$$\gamma_N^{(\alpha)} = \text{Im} \log z_N^{(\alpha)} \quad \text{where} \quad P_\alpha = \frac{eN}{L^3} \langle r_\alpha \rangle_c$$

Modulus defines the localization tensor $\langle r_\alpha \rangle_c = \frac{1}{N} \frac{L}{2\pi} \text{Im} \log z_N^{(\alpha)}$

$$\langle r_\alpha^2 \rangle_c = \frac{-1}{N |G_\alpha|^2} \ln |Z_N^{(\alpha)}|^2$$

R. Resta and S. Sorella, 1999 Phys. Rev. Lett. 82 370

I. Souza Wilkens T and R. Martin, 2000 Phys. Rev. B 62 1666

Localization tensor

- Localization tensor is related to the conductivity

$$\xi_i^2(N) = \frac{\hbar}{\pi q_e^2 n_0} \int_0^\infty \frac{d\omega}{\omega} \operatorname{Re} \sigma_{ii}(\omega)$$

- We will actually use

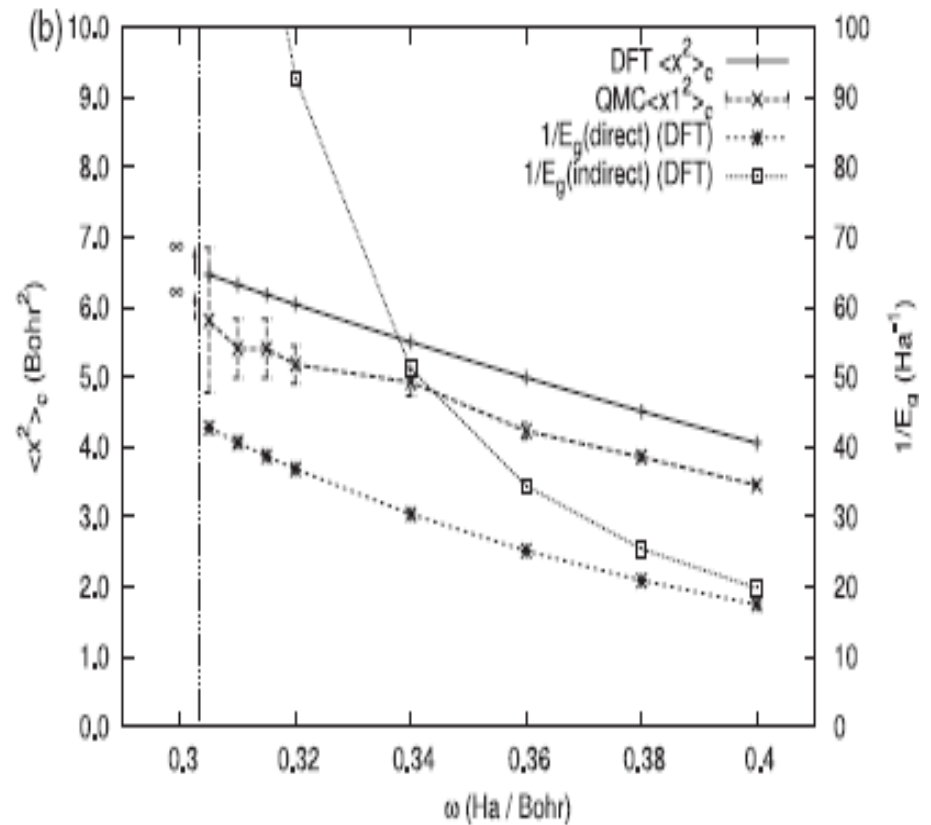
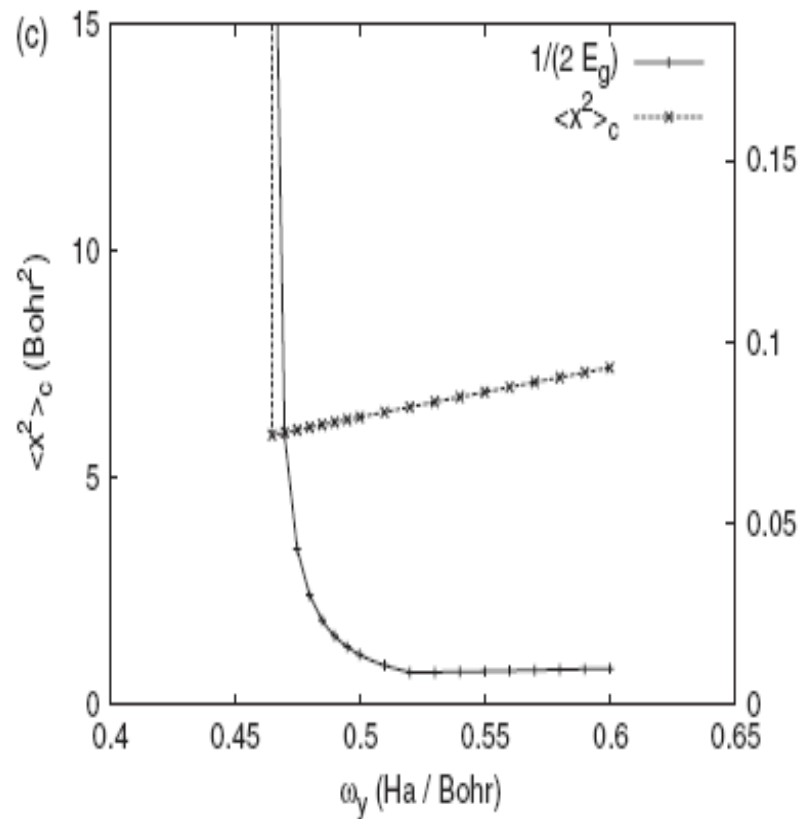
$$\langle r_\alpha r_\beta \rangle_c = \frac{L^2}{4\pi^2 N} \ln \frac{\langle \Psi_0(\mathbf{\kappa}_\alpha) | \Psi_0(\mathbf{\kappa}_\beta) \rangle}{\langle \Psi_0(\mathbf{\kappa}_\alpha) | \Psi_0(0) \rangle \langle \Psi_0(0) | \Psi_0(\mathbf{\kappa}_\beta) \rangle},$$

in CASINO

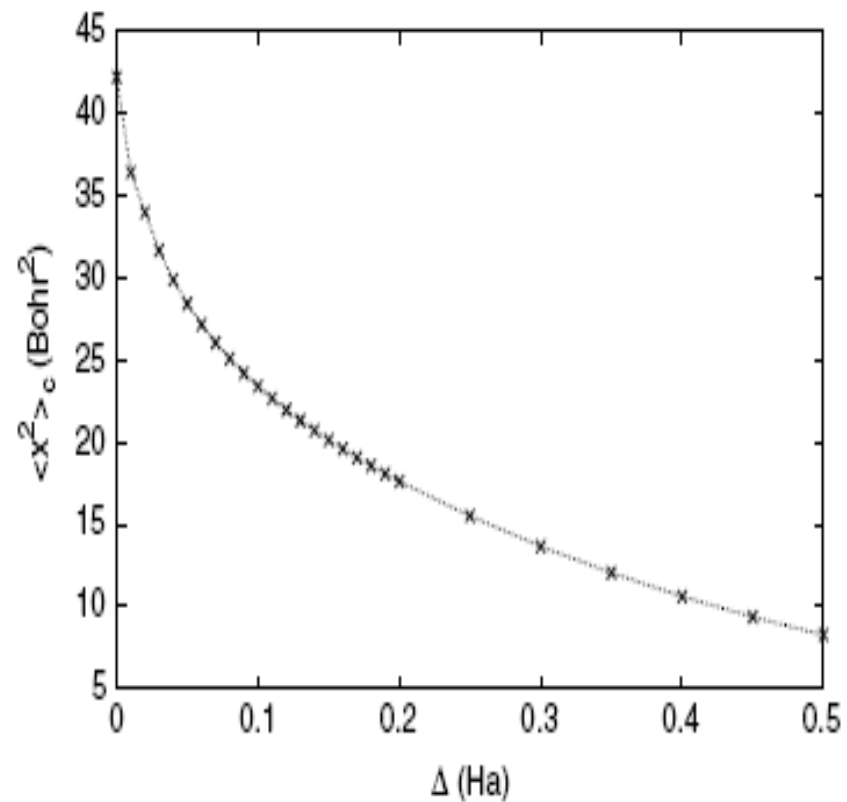
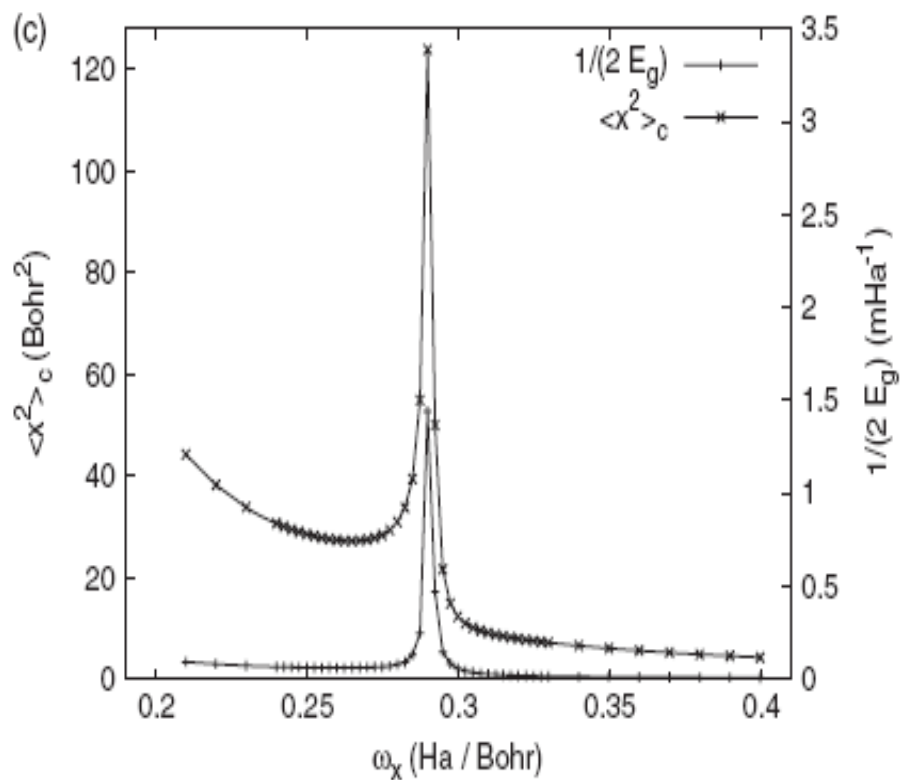
Numerical implementation

1. Sum over the electron positions
2. Add up the $e^{(i.k.x)}$ for each configurations then average over the weight to get Zn
3. Evaluate localization tensor and polarization tensor

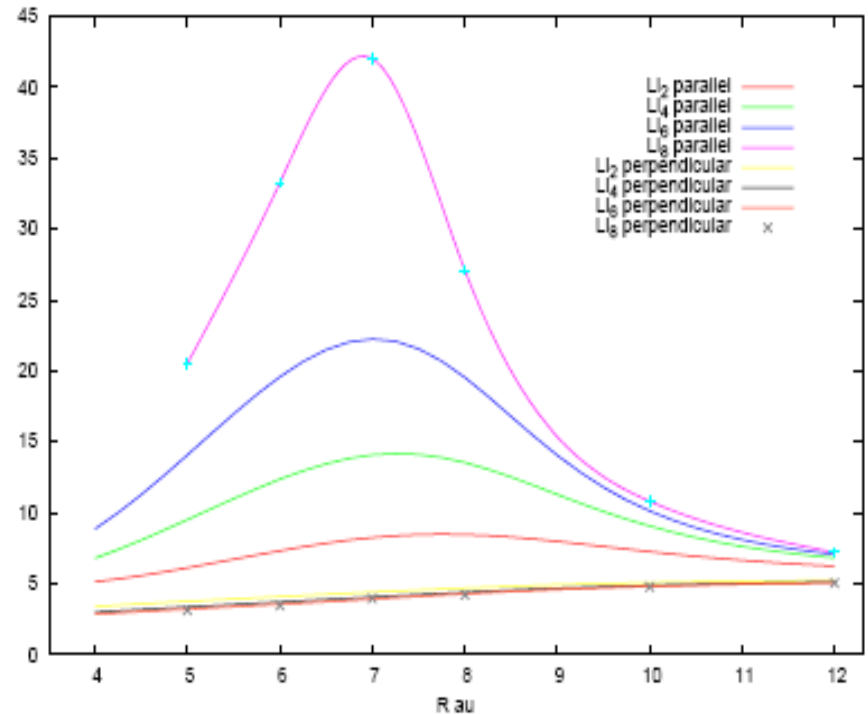
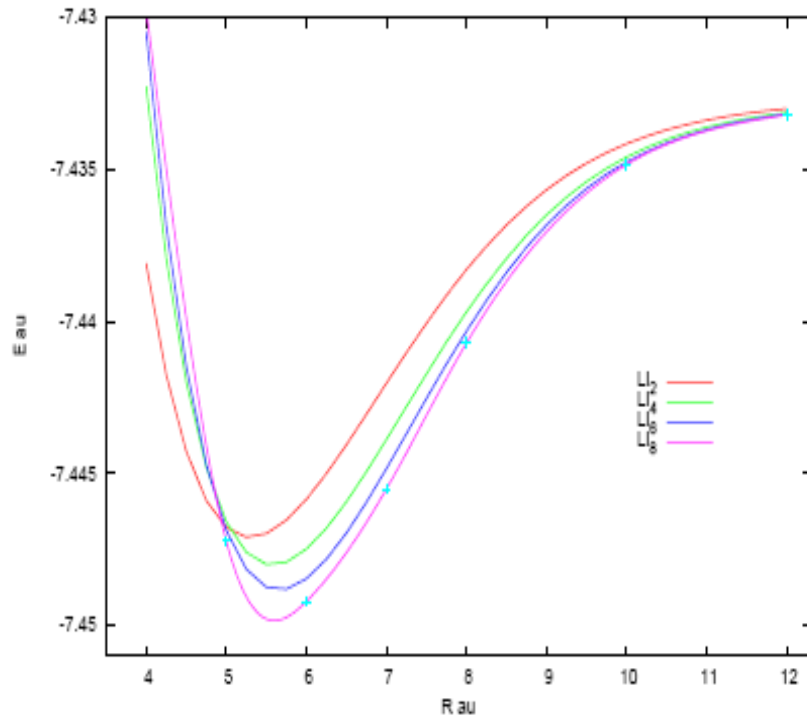
Localization tensor over metal to band insulator



Localization tensor over metal to band insulator

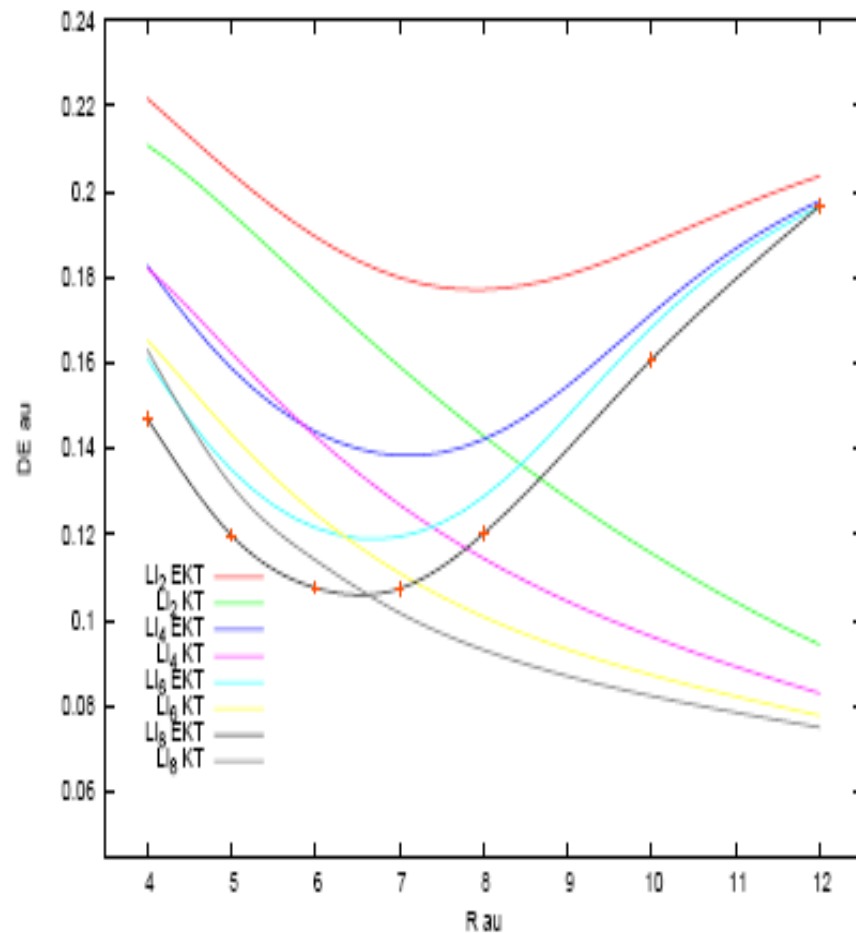
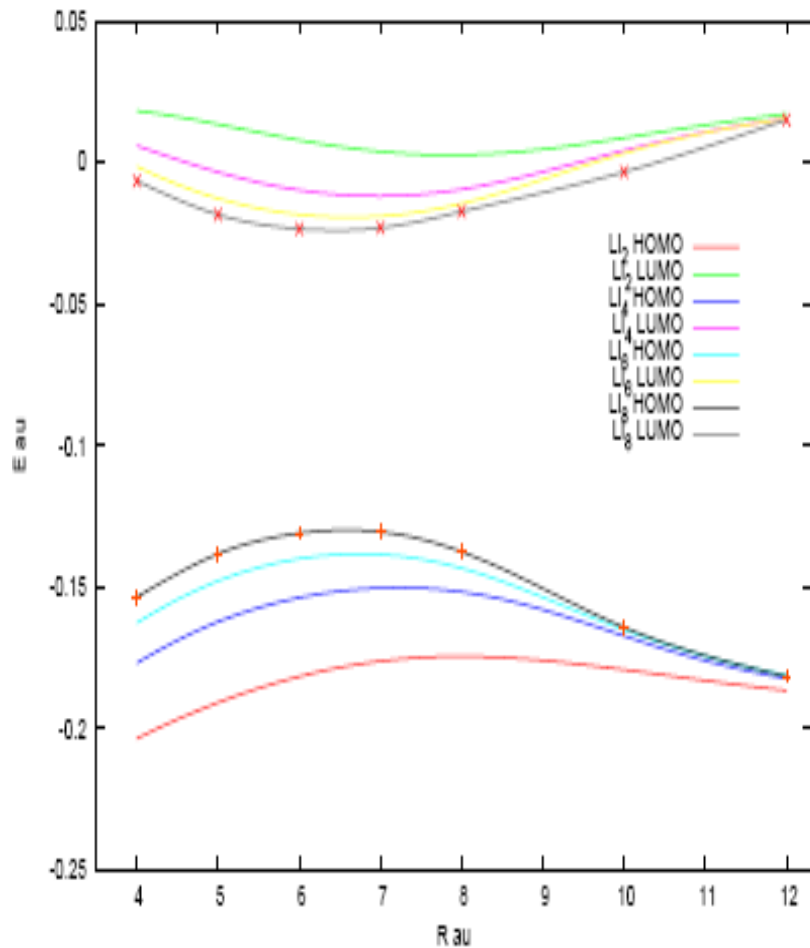


Metal-insulator transition of Lithium linear chain ($n=2,4,6,8$)

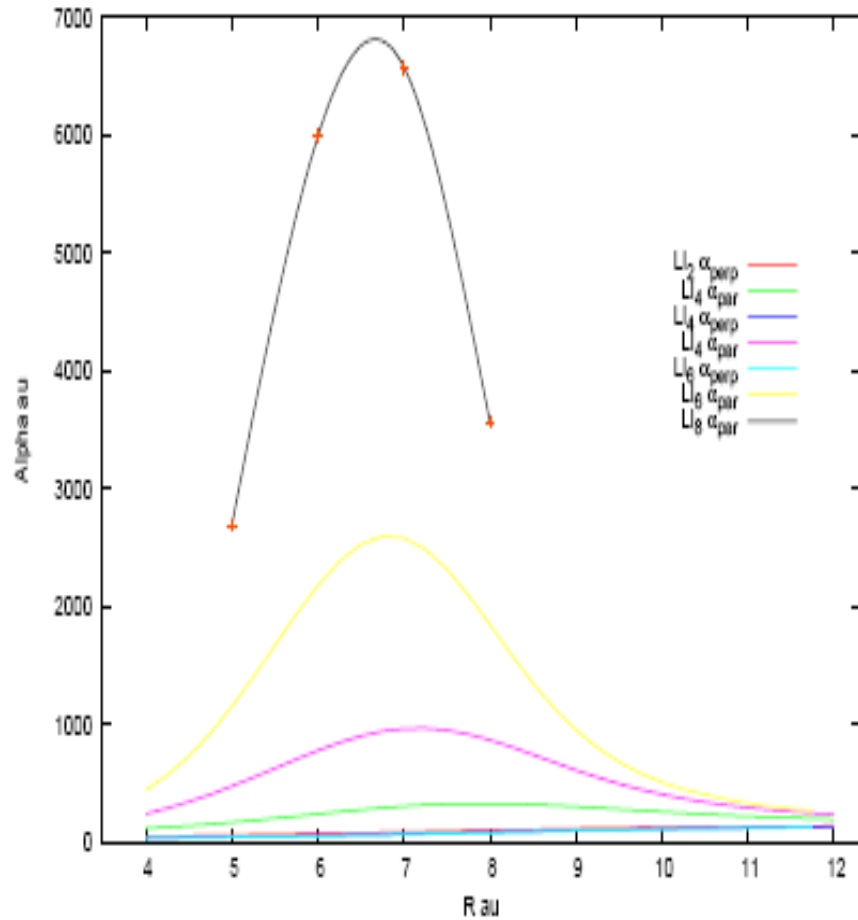


Full configuration-interaction study of the metal-insulator transition in model systems, G L Bendazzoli, S Evangelisti, A Monari, B Paulus, V Vetere, IOP Publishing, 17 (2008) 012005

Metal-Insulator transition of Lithium Linear Chain



Metal-Insulator transition of lithium linear chain



Plan

- Use Gaussian basis set with Jastrow factor
- K-point sampling of the expectation value
- Calculate linear hydrogen chain of atoms with larger system size ($n > 8$)
- See if we can locate a maximum

Acknowledgements

- Nick Hine
- Pablo Lopez
- Mike Towler
- Everyone else in the QMC group