Non-Hamiltonian Quantum Mechanics
(Some simple ideas)

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Heisenberg's equation for interacting systems and its generalization

A simple Ising-like model

Does quantum mechanics make sense without a Hamiltonian?

Heisenberg's Equation(HE)

$$- i \hbar \, \partial/\partial t \, \boldsymbol{A}(t) = [\boldsymbol{H}, \boldsymbol{A}(t)]$$

Where ***H*** is the Hamiltonian and ***A***(t) is some observable

Non-singular in macroscopic limit and local – unlike the Schrödinger equation

Find examples of interacting ***H*** and ***A*** for which HE can be solved analytically

No known orthogonal polynomial systems related to interacting HE

Commutator makes HE complicated even if ***H*** is simple

Generalize Heisenberg's Equation

$$- i \hbar \, \partial/\partial t \, \boldsymbol{A}(t) = \mathscr{L} \boldsymbol{A}(t)$$

where $\mathscr{L}$ is a super-operator – linear operator on operators
$\mathscr{L}$ often called Quantum Liouvillian

Properties of $\mathscr{L}$:

Hermitian – unitary evolution of $\boldsymbol{A}(t)$ ($tr\{\boldsymbol{A}^\dagger \boldsymbol{A}\}$ invariant)

Acts like a time derivative $[\mathscr{L} \boldsymbol{A} \boldsymbol{B}] = [\mathscr{L} \boldsymbol{A}] \boldsymbol{B} + \boldsymbol{A} [\mathscr{L} \boldsymbol{B}]$

Conserved quantities are stationary $\mathscr{L} \boldsymbol{S} = 0$ where $\boldsymbol{S}$ is some conserved quantity like energy or angular momentum

Time-reversal symmetry: $(\mathscr{L} \boldsymbol{A})^\dagger = \mathscr{L} \boldsymbol{A}^\dagger$

A Quantum Ising Model

Simplification of spin-1/2 Heisenberg model (Excitons?)

Sites – on a line, lattice, or some other structure

Neighbors – pairs of interacting sites

Site Operators – identity $\boldsymbol{i}$ is no change, $\boldsymbol{a}$ is a change (flip spin)

Total Operator – linear combination of products $\boldsymbol{o}_1\boldsymbol{o}_2\ldots\boldsymbol{o}_N$

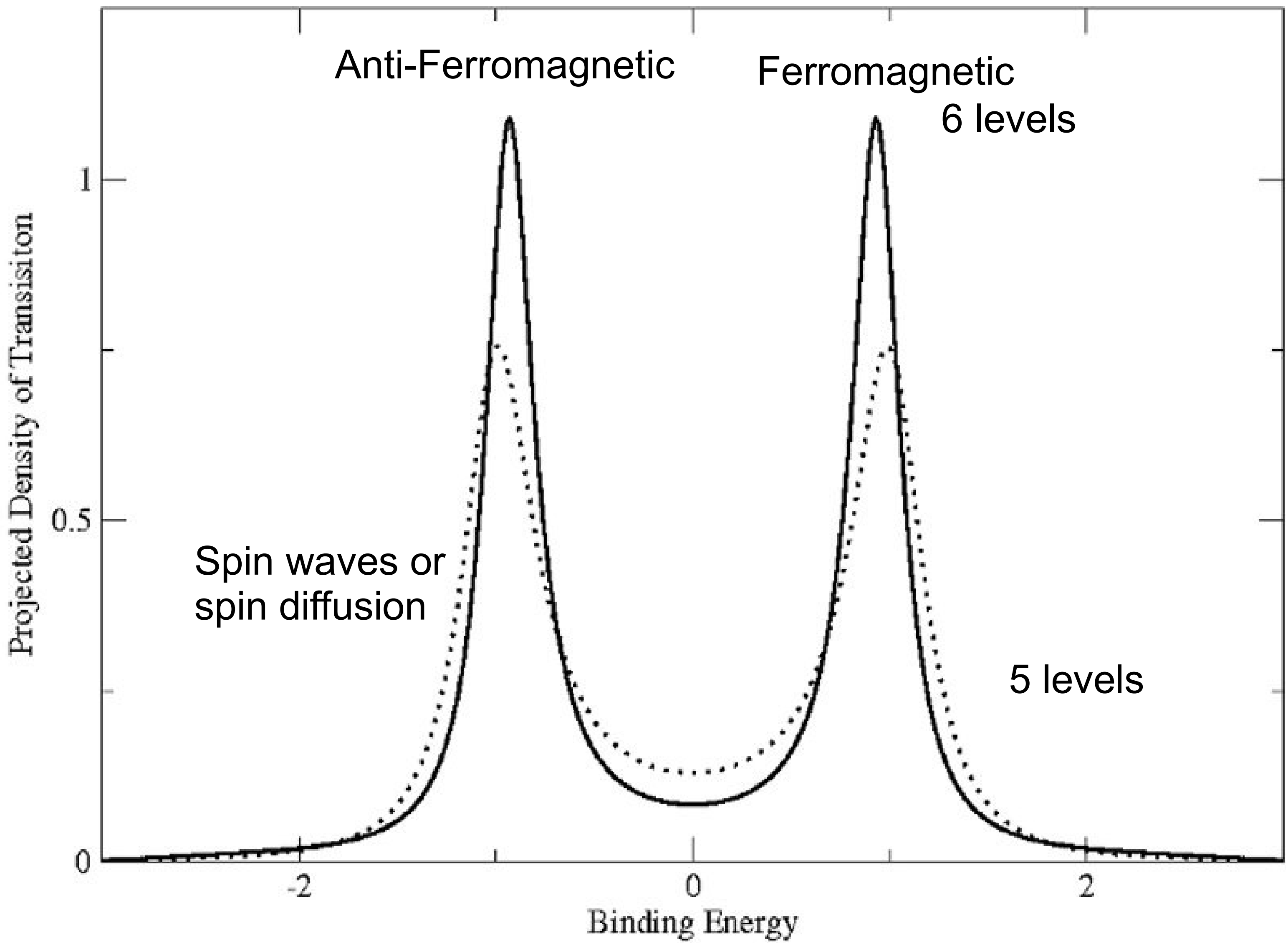
Operators of different sites commute

Rule – definition of $\mathscr{L} = \sum \mathscr{L}_{j,k}$ over pairs of neighbors j and k

$$\mathscr{L}_{j,k}\, \boldsymbol{o}_1\boldsymbol{o}_2\ldots\boldsymbol{o}_N = \boldsymbol{o}_1\boldsymbol{o}_2\ldots[\mathscr{L}_{j,k}\, \boldsymbol{o}_j\boldsymbol{o}_k]\ldots\boldsymbol{o}_N$$

$$\mathscr{L}_{j,k}\, \boldsymbol{i}_j\, \boldsymbol{i}_k = 0,\ \mathscr{L}_{j,k}\, \boldsymbol{i}_j\, \boldsymbol{a}_k = \mathscr{L}_{j,k}\, \boldsymbol{a}_j\, \boldsymbol{i}_k = \boldsymbol{a}_j\, \boldsymbol{a}_k,\ \mathscr{L}_{j,k}\, \boldsymbol{a}_j\, \boldsymbol{a}_k = \boldsymbol{i}_j\, \boldsymbol{a}_k + \boldsymbol{a}_j\, \boldsymbol{i}_k$$

Quantum Ising Model Binding Energy



Classical Systems without Hamiltonians

Ball rolling on a plane – equations of motion not determined by the energy

Non-holonomic constraints

Mike Godfrey has studied a quantum version of the ball rolling on a plane

Is there a Hamiltonian?

Eigenvalues of $\mathscr{L}$ are transition energies and eigen-operators are stationary transitions $\boldsymbol{A}(t) = \exp\{i\omega t\}\, \boldsymbol{A}(0)$

$\mathscr{L}$ is the commutator of a Hamiltonian if Conservative in the sense that there are loops of transitions for which the product around a loop is the identity

Quantum Ising model violates this

Why should physical systems be conservative in this sense?

Wouldn't it be better to have equations of motion for observables directly, rather than trying to define states?

Summary

Heisenberg's equation is good for interacting systems

Can generalize to equation of motion for observables

Quantum Ising model is a simple interacting model without a Hamiltonian

Perhaps some quantum systems don't have Hamiltonians