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A few-fermion BCS superconductor

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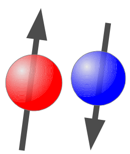
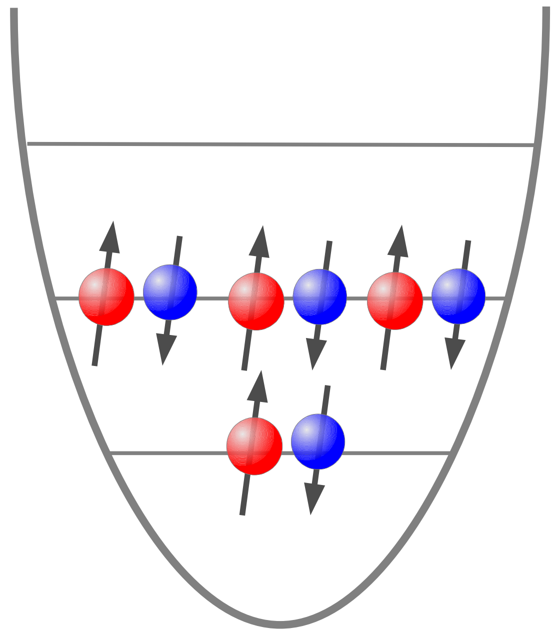
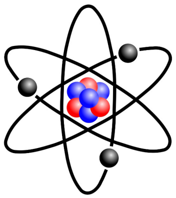
Theory of Condensed Matter Group, Department of Physics, Cambridge

Superconductivity and Scale in Quantum Systems

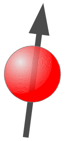


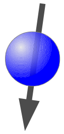


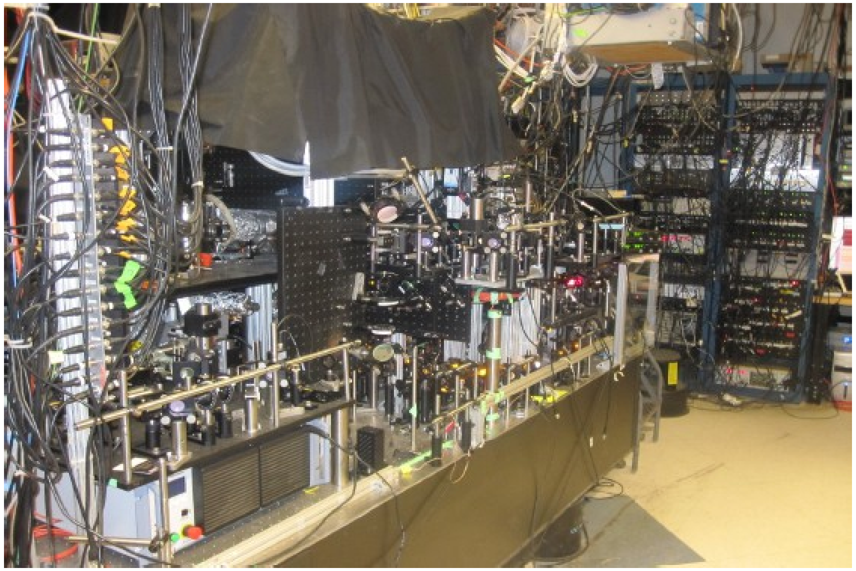
Experimental Realization of a Cold Atom Gas

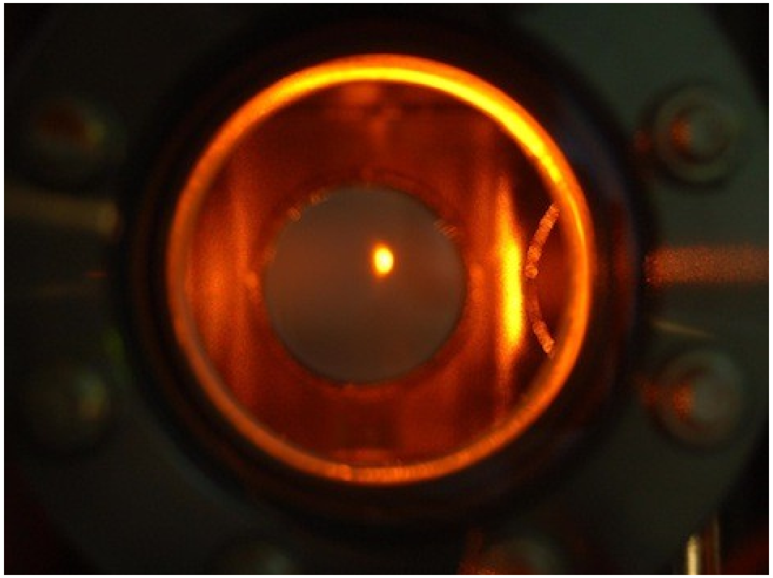


^{6}Li atom

$|F = 1/2, m_F = 1/2\rangle$ →  Up spin electron

$|F = 1/2, m_F = -1/2\rangle$ →  Down spin electron



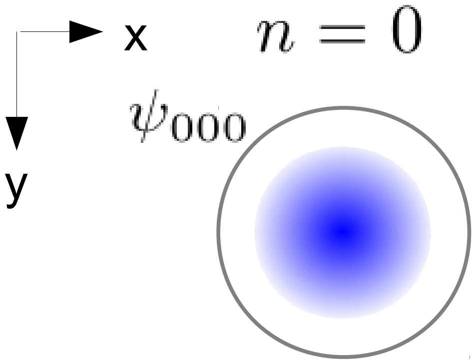


A Model of a Cold Atom Gas

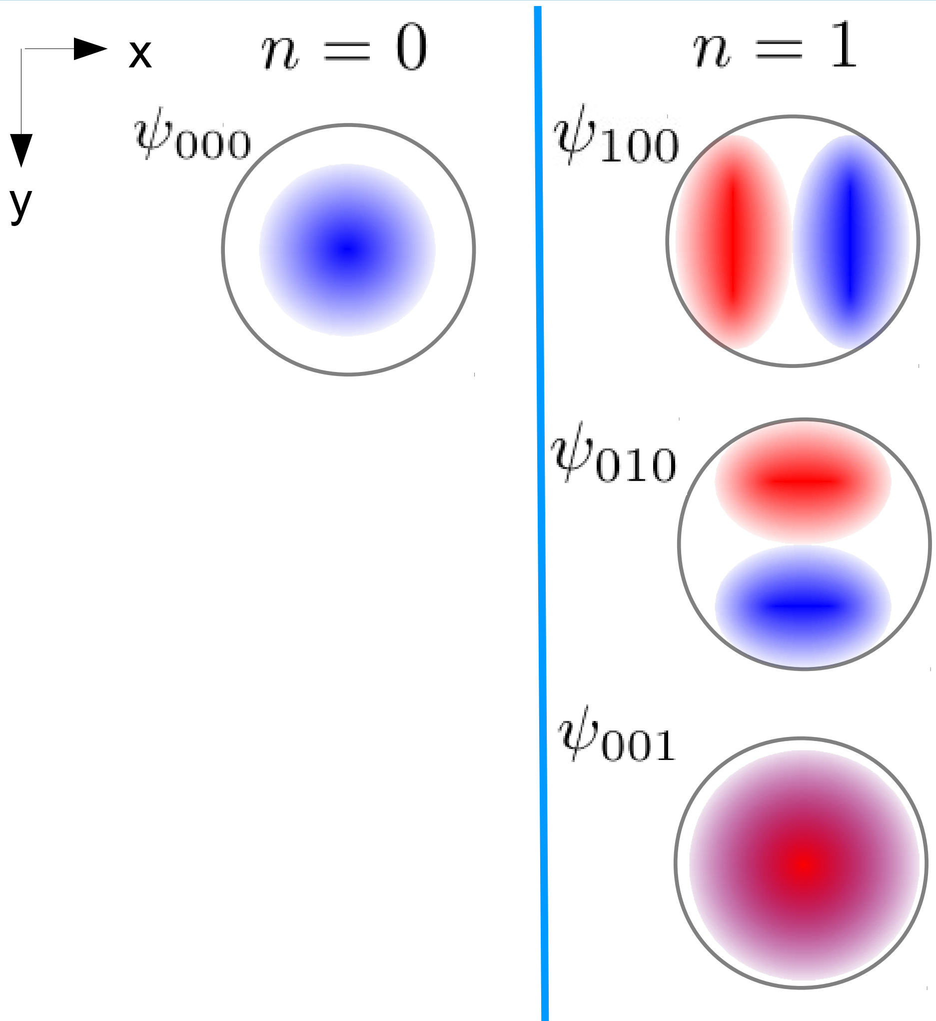
- Optical trap modeled by 3-D QHO

$$\hat{H}^{(0)} = \frac{-\hbar^2}{2m}\nabla^2 + \frac{1}{2}m(\omega_x^2 x^2 + \omega_y^2 y^2 + \omega_z^2 z^2)$$

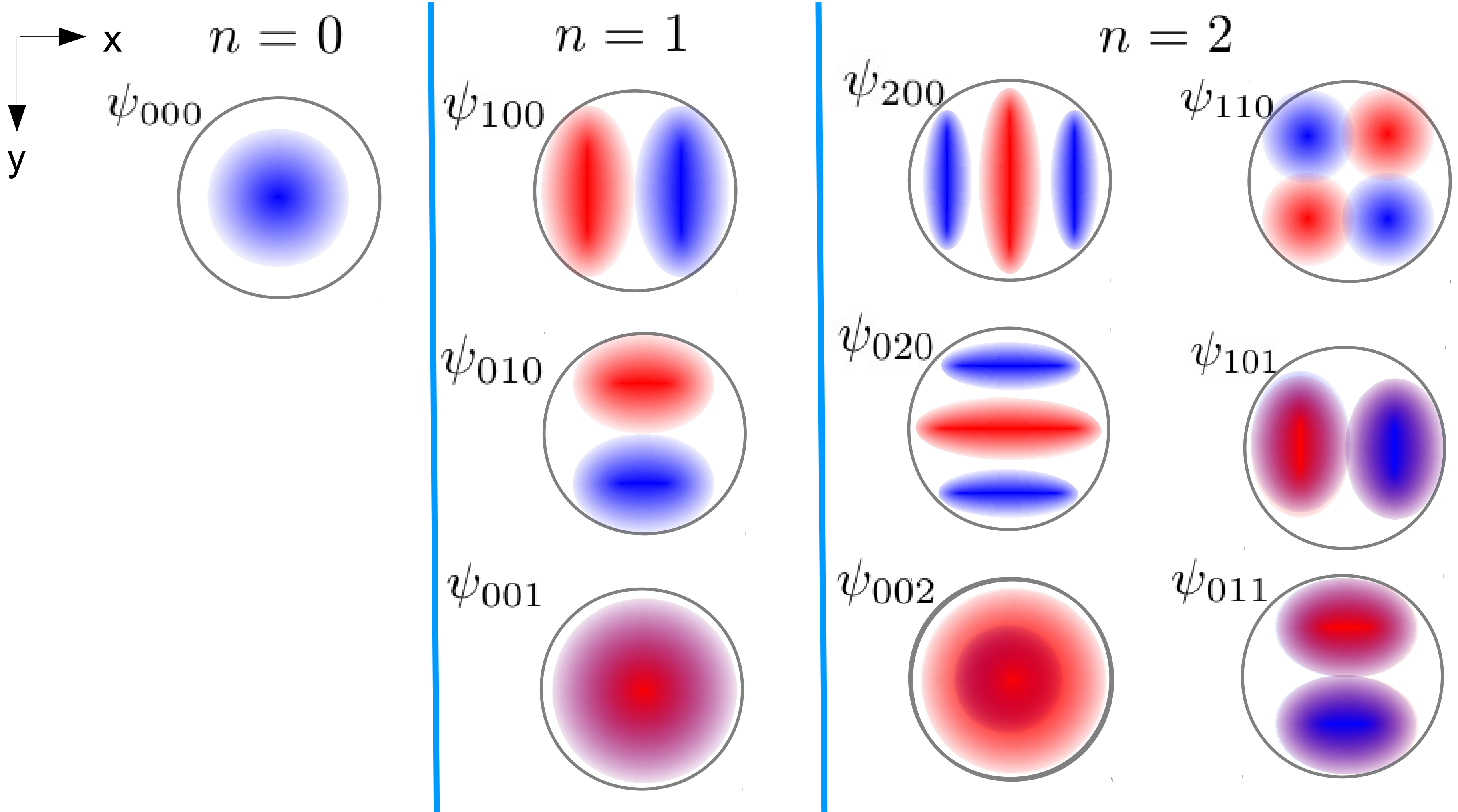
3-D QHO Orbitals



3-D QHO Orbitals



3-D QHO Orbitals

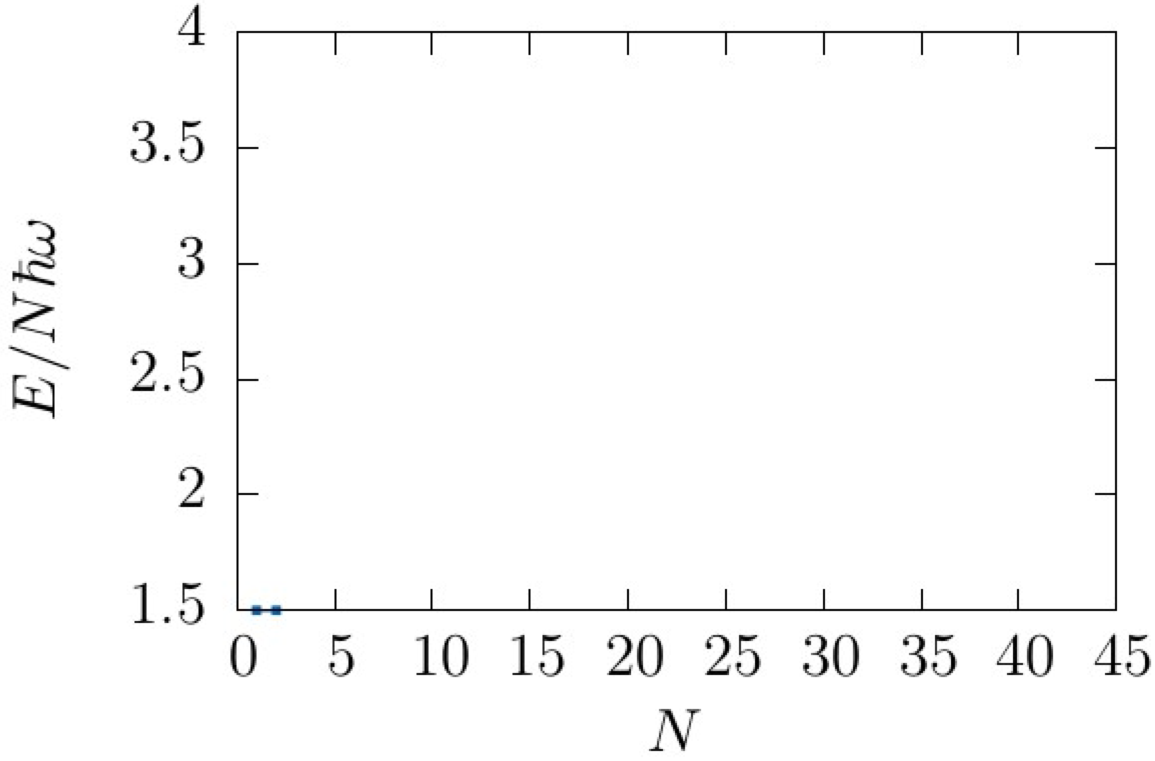


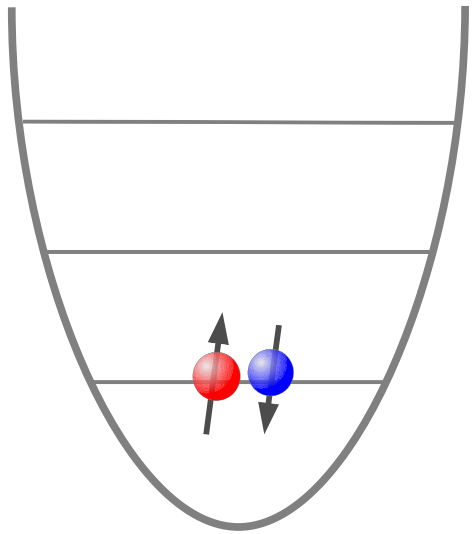
3-D QHO Orbitals

$$E = \hbar\omega\left(n + 3/2\right)$$

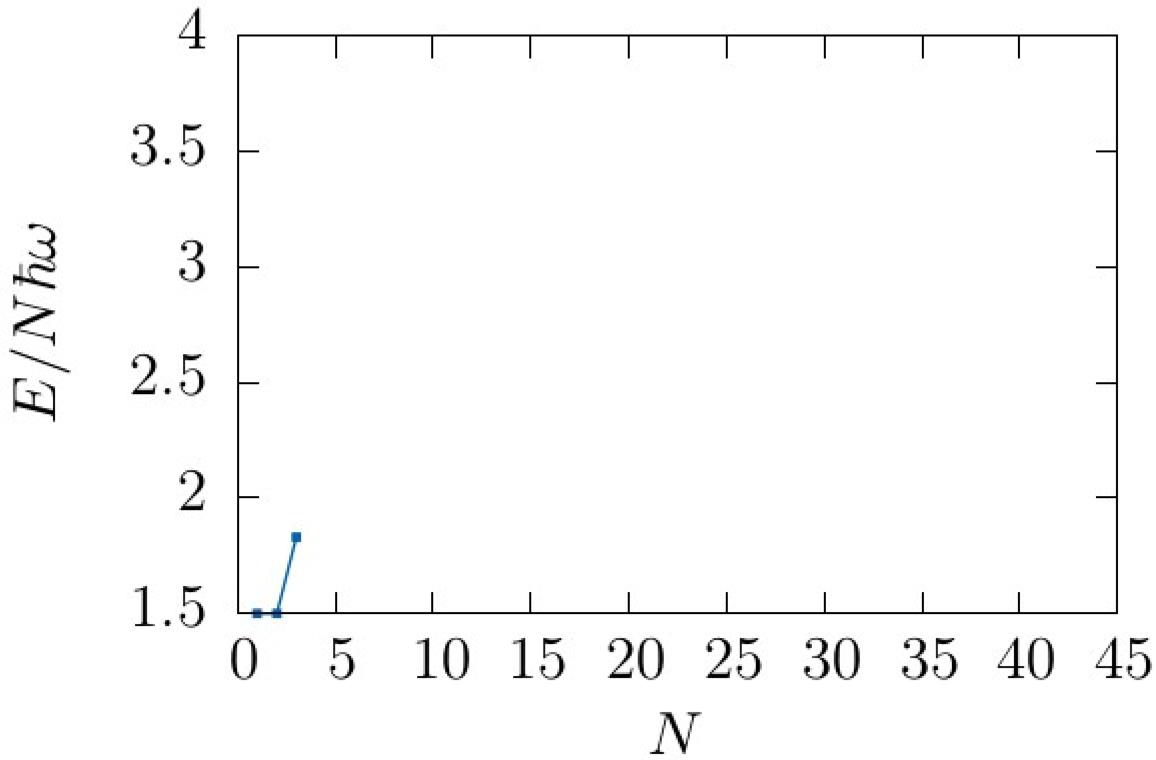
$$n = n_x + n_y + n_z$$

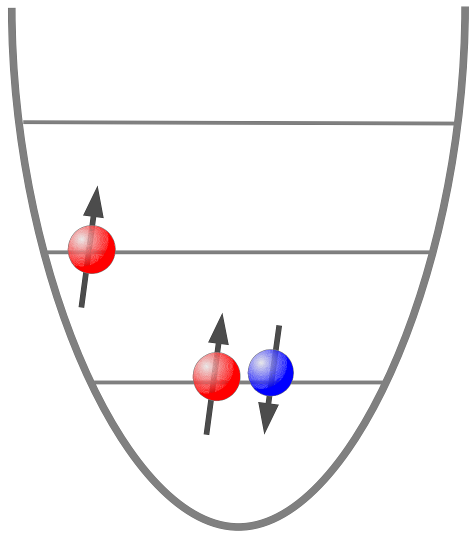
Non-Interacting System Ground State



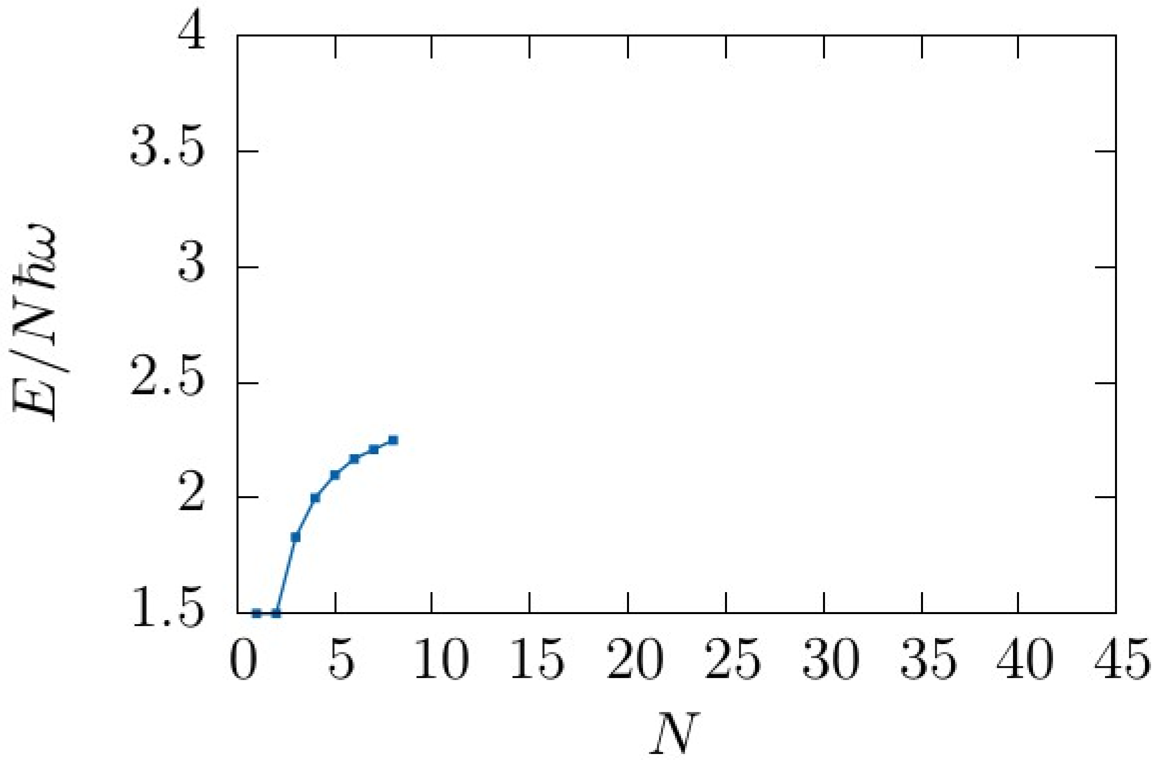


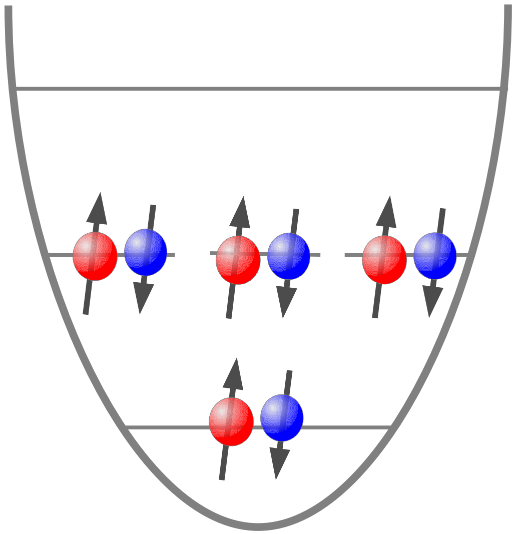
Non-Interacting System Ground State



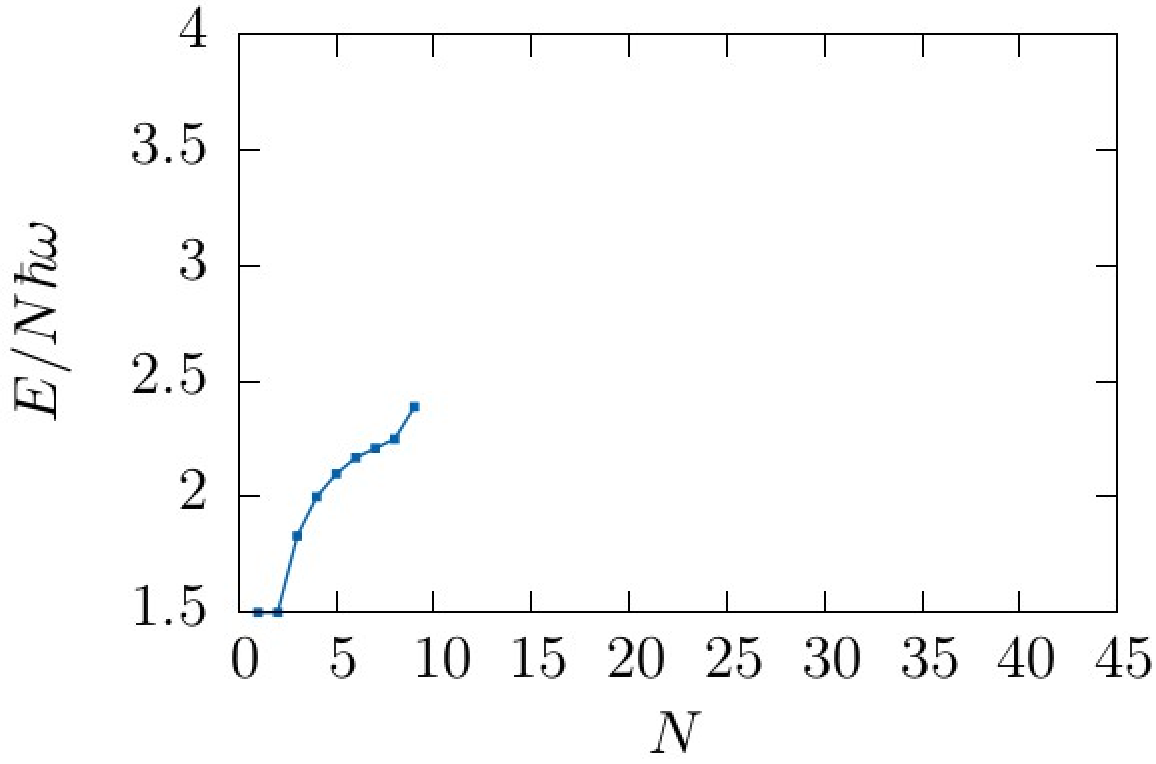


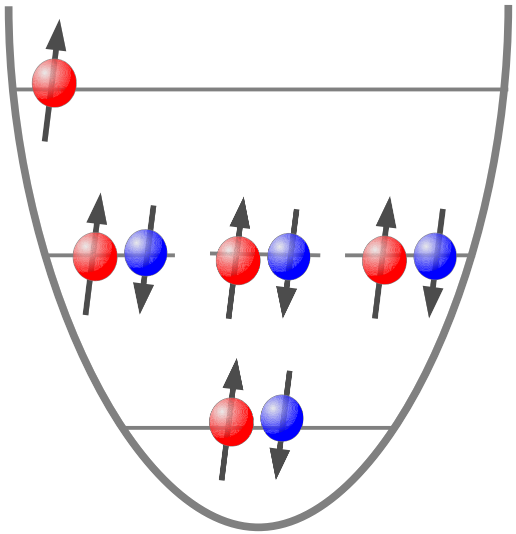
Non-Interacting System Ground State



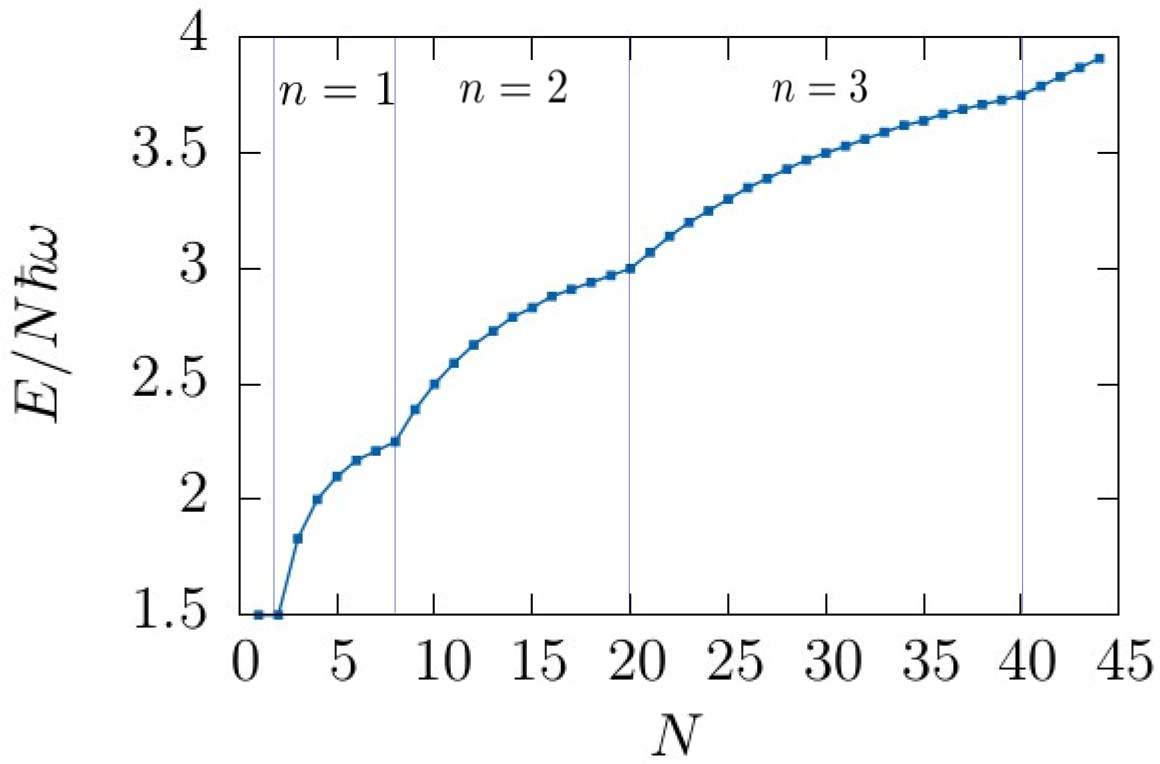


Non-Interacting System Ground State





Non-Interacting System Ground State



Quantum Monte-Carlo

- Non-interacting system orbitals suitable basis for weak interactions

$$\psi_{n_x n_y n_z}(\mathbf{r}) = H_{n_x}\left(\sqrt{\omega_x}x\right)e^{-\frac{1}{2}\left(\sqrt{\omega_x}x\right)^2} \times H_{n_y}\left(\sqrt{\omega_y}y\right)e^{-\frac{1}{2}\left(\sqrt{\omega_y}y\right)^2} \times H_{n_z}\left(\sqrt{\omega_z}z\right)e^{-\frac{1}{2}\left(\sqrt{\omega_z}z\right)^2}$$

- G.S. Energy Monte-Carlo integration

$$E^{\text{est}} = \frac{\int d^{3N}\mathbf{r}\Psi^*_{\text{trial}}(\mathbf{r_1},\ldots,\mathbf{r_N})\hat{H}\Psi_{\text{trial}}(\mathbf{r_1},\ldots,\mathbf{r_N})}{\int d^{3N}\mathbf{r}\Psi^*_{\text{trial}}(\mathbf{r_1},\ldots,\mathbf{r_N})\Psi_{\text{trial}}(\mathbf{r_1},\ldots,\mathbf{r_N})}$$

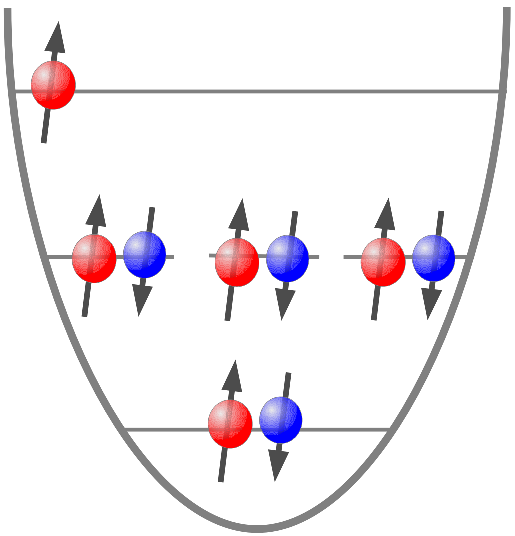
Superfluid Hamiltonian in External Trap

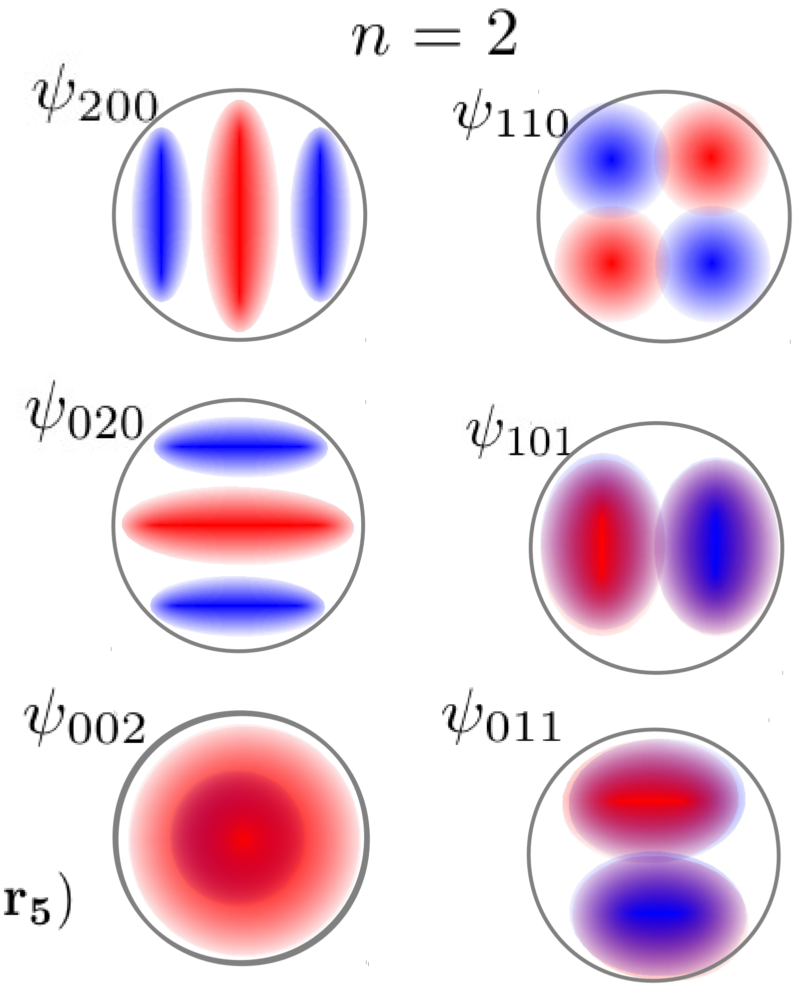
- Pairwise attractive interactions between up and down spins

$$\hat{H} = \hat{H}^{(0)} + \iint d^3\mathbf{r} d^3\mathbf{r}' c_\uparrow^\dagger(\mathbf{r}) c_\downarrow^\dagger(\mathbf{r}') V(\mathbf{r} - \mathbf{r}') c_\downarrow(\mathbf{r}') c_\uparrow(\mathbf{r})$$

$$V(\mathbf{r} - \mathbf{r}') = -V_0 \delta(\mathbf{r} - \mathbf{r}')$$

Effect of Interactions on Degeneracy

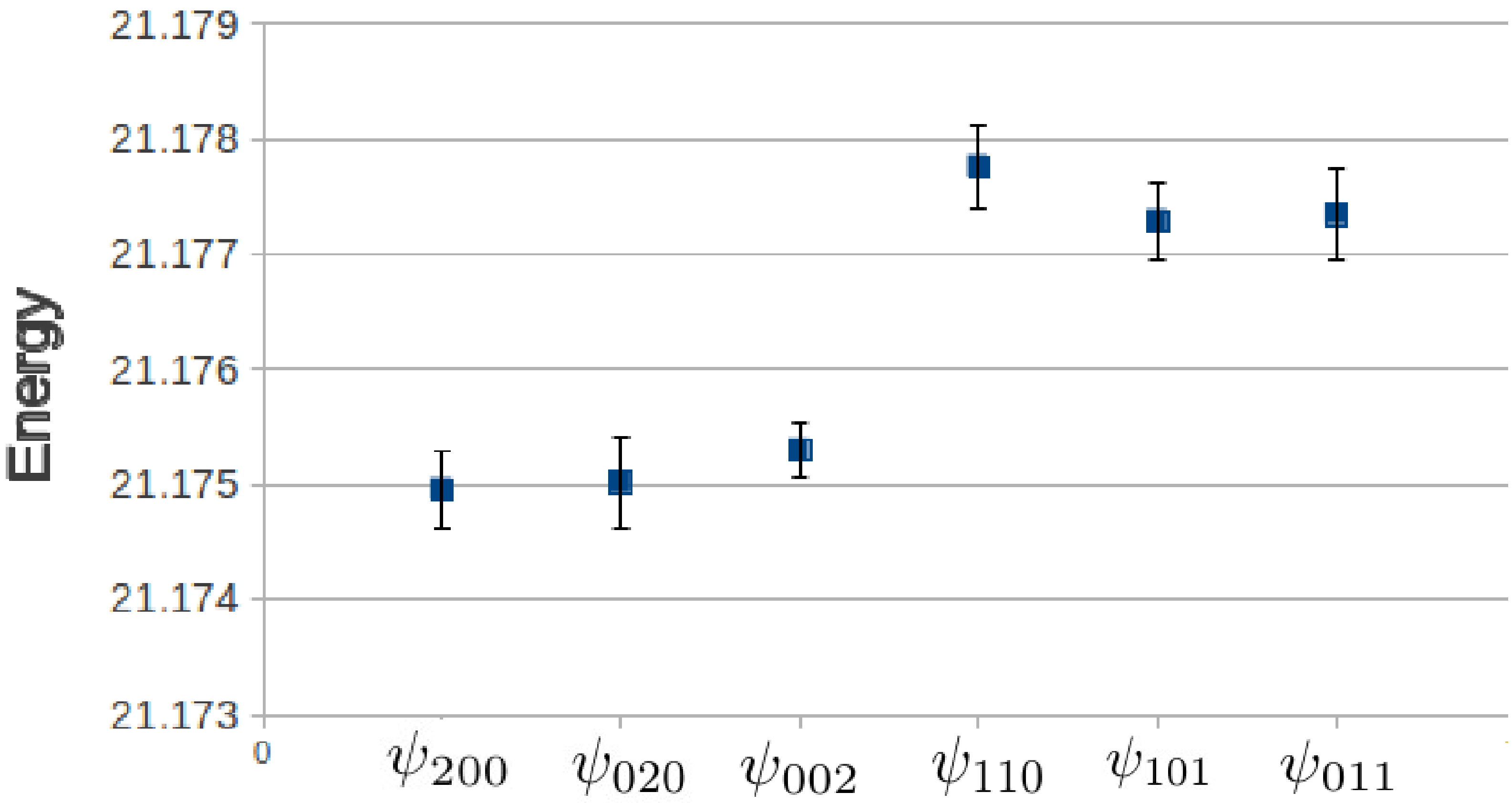




$$D_\uparrow(\mathbf{r_1}, \mathbf{r_2}, \mathbf{r_3}, \mathbf{r_4}, \mathbf{r_5}) =$$

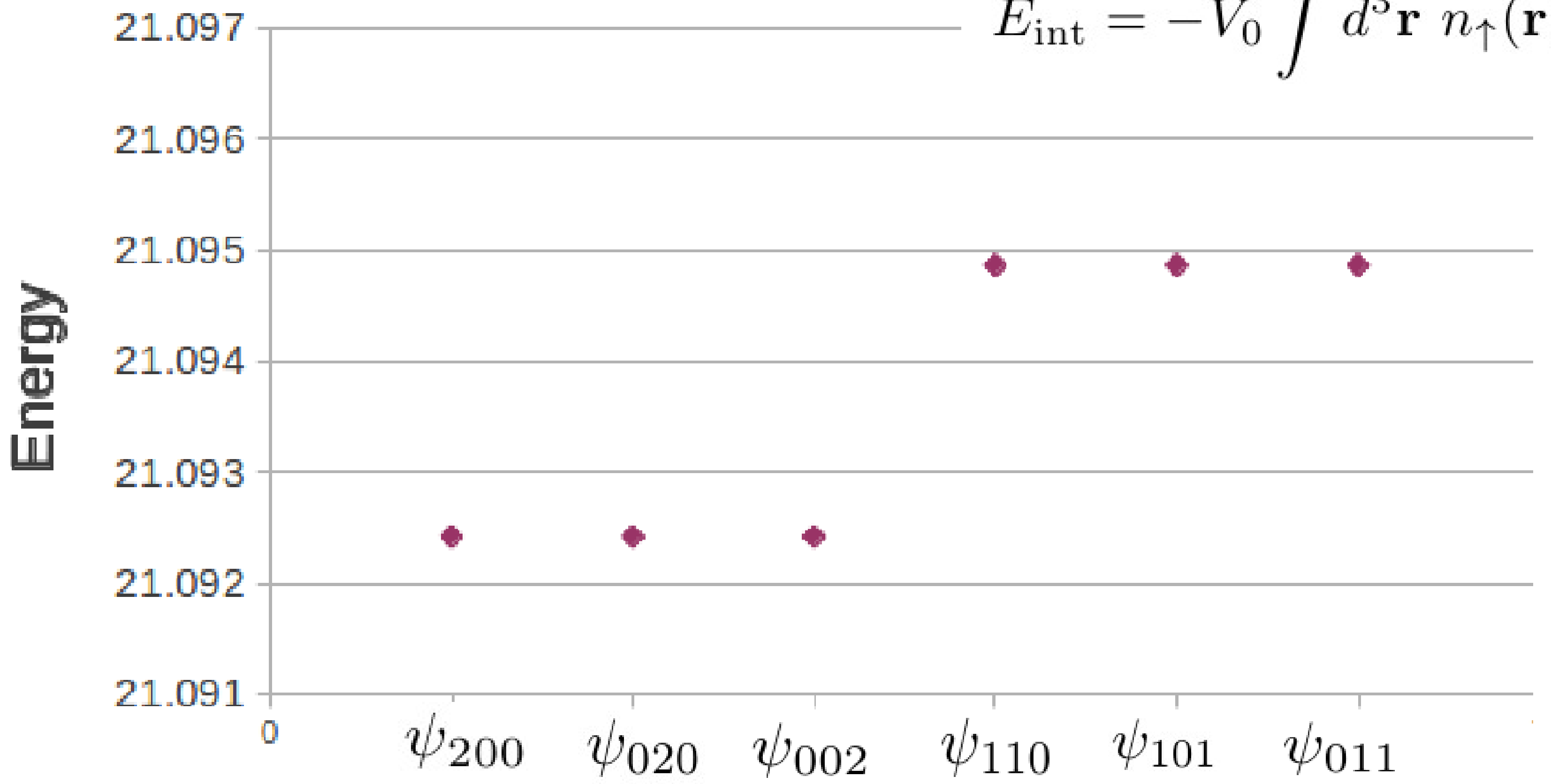
$$\mathcal{A} \prod \psi_{000}(\mathbf{r_1})\psi_{100}(\mathbf{r_2})\psi_{010}(\mathbf{r_3})\psi_{001}(\mathbf{r_4})\psi_j(\mathbf{r_5})$$

Effect of Interactions on Degeneracy



Effect of Interactions on Degeneracy

$$E_{\text{int}} = -V_0 \int d^3\mathbf{r}\ n_\uparrow(\mathbf{r}) n_\downarrow(\mathbf{r})$$



Interaction Energy

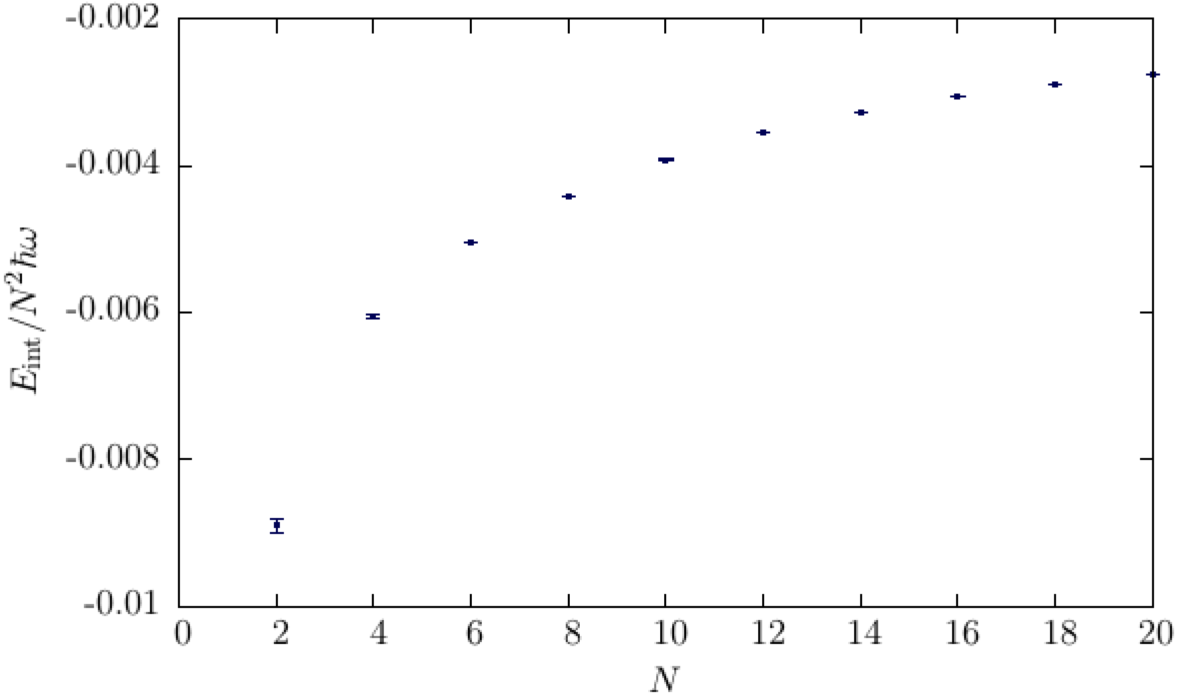
$$E_{\text{int}} = -V_0 \left\langle \int d^3\mathbf{r} \; c_\uparrow^\dagger c_\uparrow c_\downarrow^\dagger c_\downarrow \right\rangle$$

$$\rightarrow -V_0 \int d^3\mathbf{r} \;\; n_\uparrow n_\downarrow$$

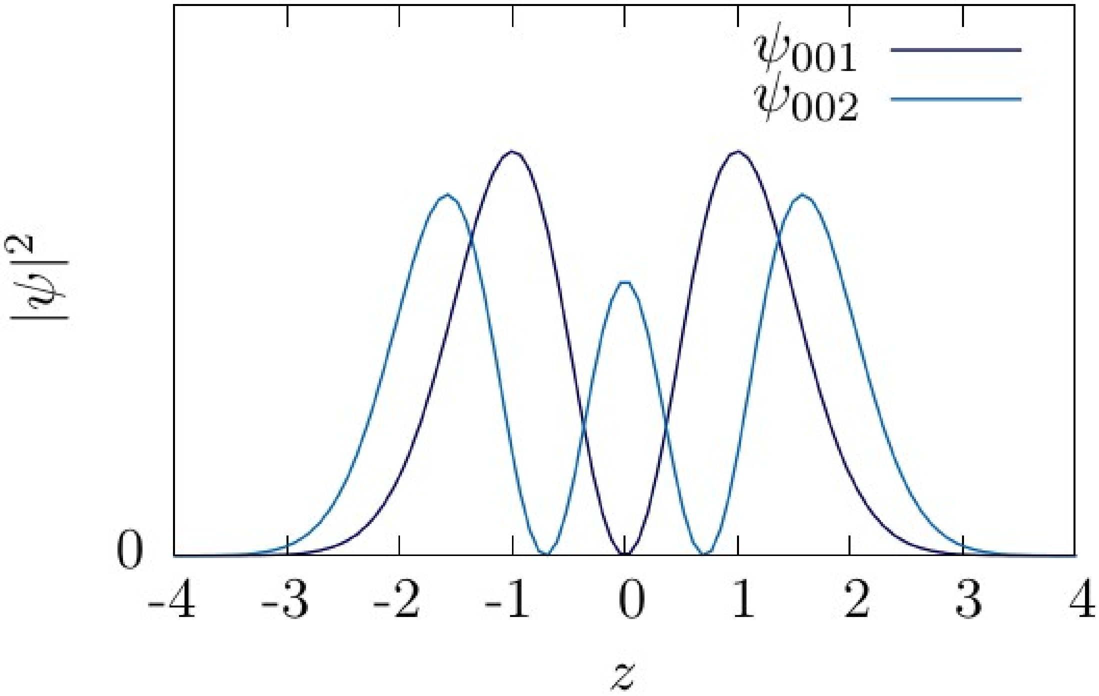
Interaction Energy

$$
\begin{aligned}
E_{\text{int}} &= -V_0 \left\langle \int d^3\mathbf{r}\; c_\uparrow^\dagger c_\uparrow c_\downarrow^\dagger c_\downarrow \right\rangle \\
&\to -V_0 \int d^3\mathbf{r} \;\; n_\uparrow n_\downarrow \\
&\backsimeq -V_0 \frac{N_\uparrow N_\downarrow}{L^3} \\
&= -V_0 \frac{N^2}{4L^3}
\end{aligned}
$$

Interaction Energy

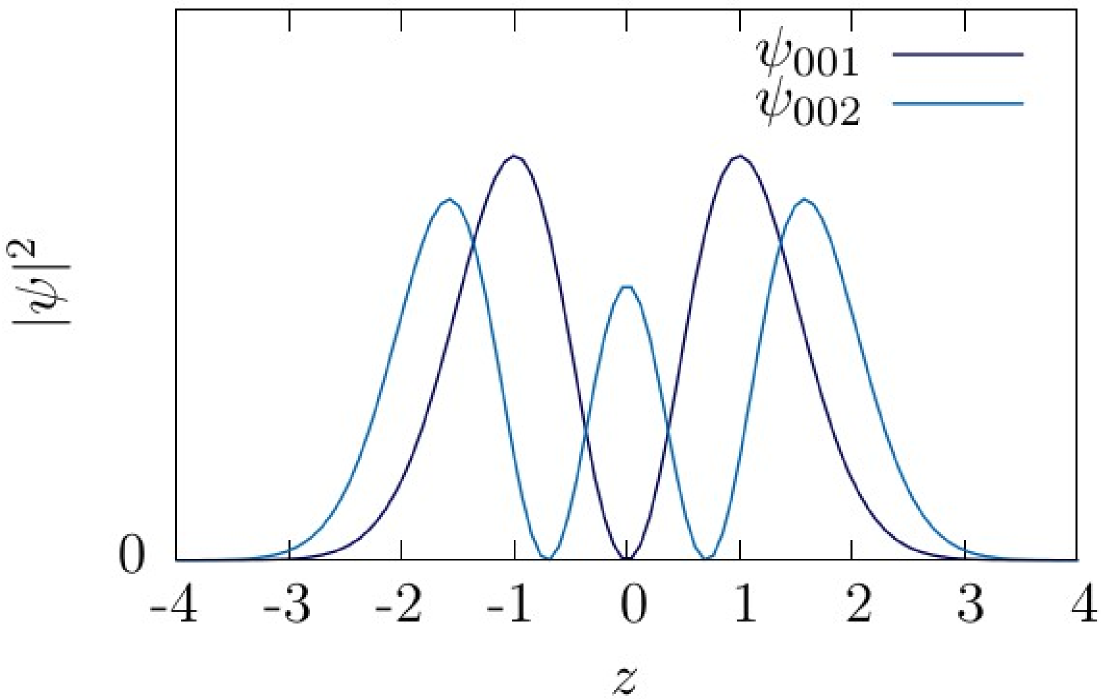


System Geometry



$$\frac{E_{\text{int}}}{N^2} = -\frac{V_0}{4L^3}$$

System Geometry

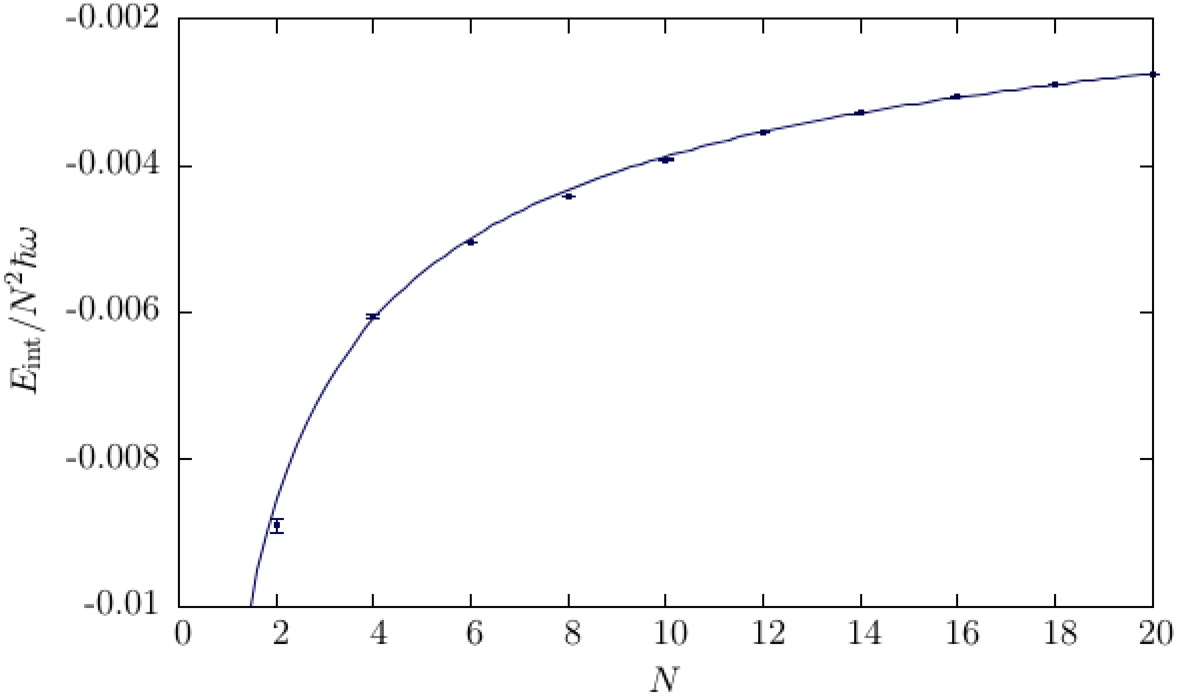


$$\frac{E_{\text{int}}}{N^2} = -\frac{V_0}{4L^3}$$

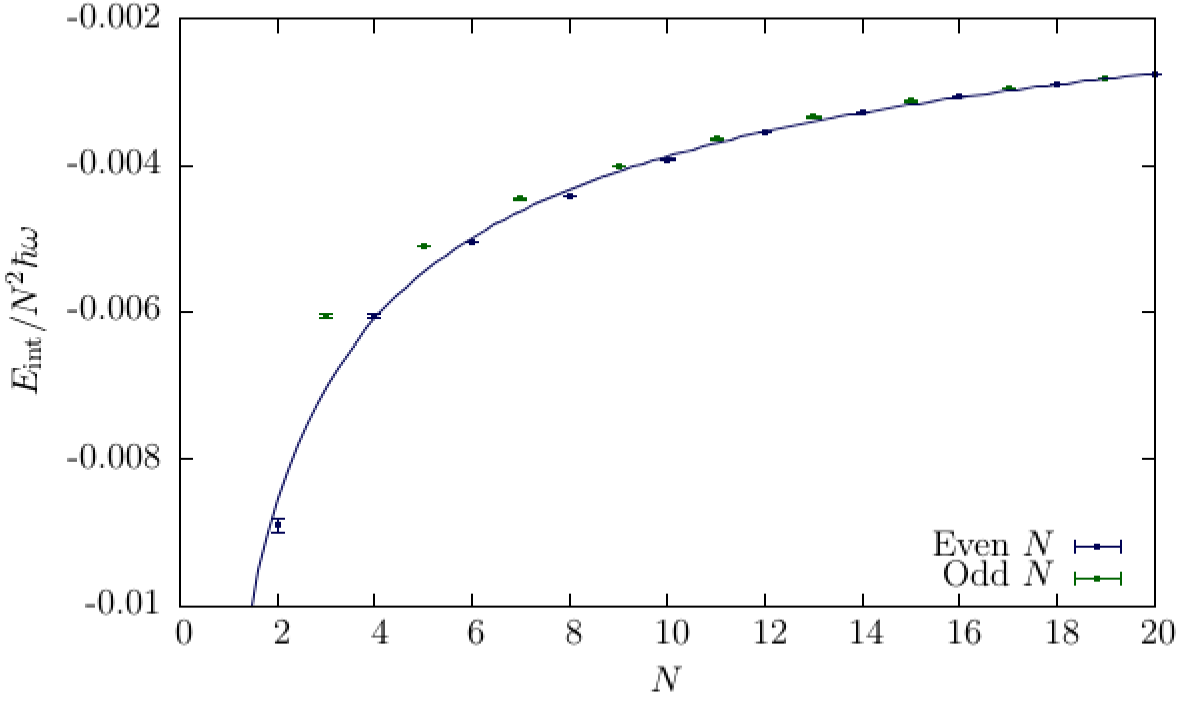
$$L^3 \sim N^{1/2}$$

$$\frac{E_{\text{int}}}{N^2} \sim -V_0 N^{-1/2}$$

System Geometry

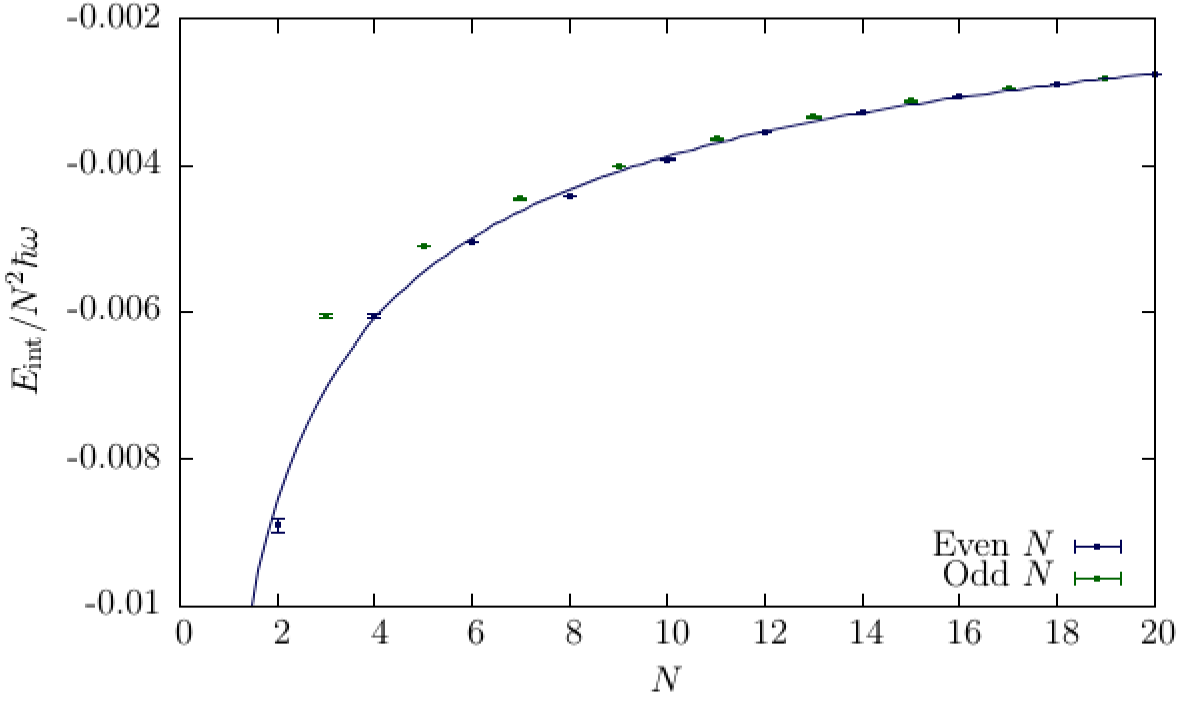


Pairing Effect



Pairing Effect

$$\frac{E_{\text{int}}}{N^2 \hbar\omega} \sim -\frac{V_0}{N^{\frac{1}{2}}}\left(1 - 4\frac{M^2}{N^2}\right)$$



Asymmetric Trap

$$\hat{H}^{(0)} = \frac{-\hbar^2}{2m}\nabla^2 + \frac{1}{2}m(\omega_\perp^2 x^2 + \omega_\perp^2 y^2 + \omega_{||}^2 z^2)$$

Non-interacting energy of *N* = 8 state

$$E = 6\hbar(2\omega_\perp + \omega_{||})$$

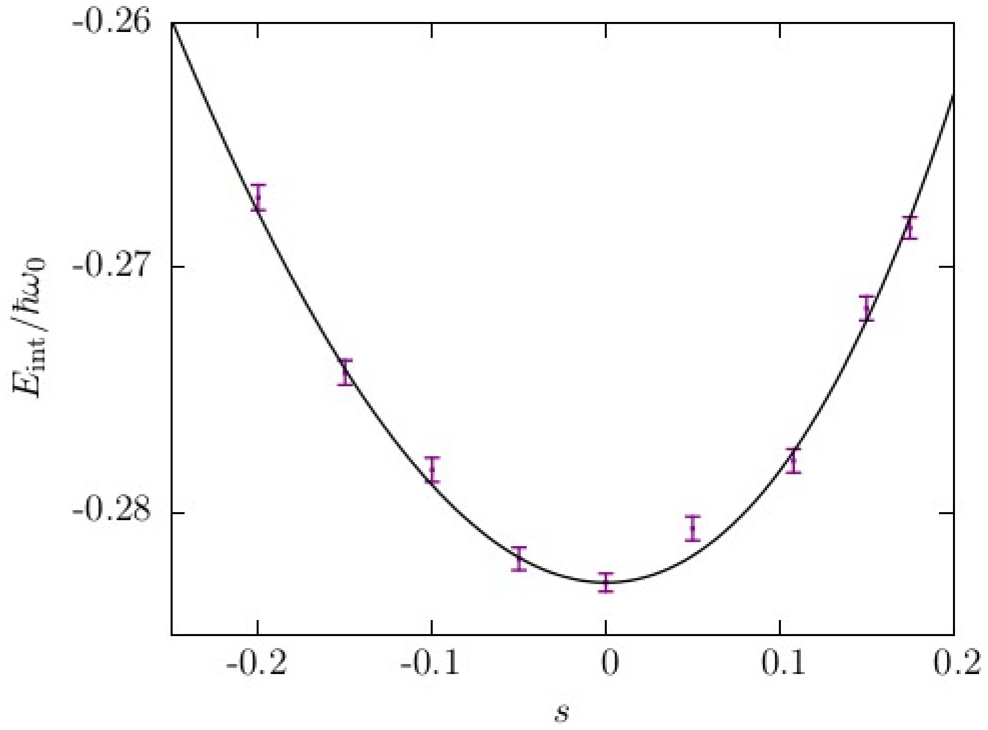
Kept constant for

$$\omega_{||}(s) = \omega_0(1 - 2s)$$

$$\omega_\perp(s) = \omega_0(1 + s)$$

Asymmetric Trap

$$\frac{E_{\mathrm{int}}}{\hbar\omega_0} \simeq -V_0\omega_0^{\frac{1}{2}}\frac{N^2}{\hbar^{\frac{3}{2}}}(1+s)\sqrt{1-2s}$$

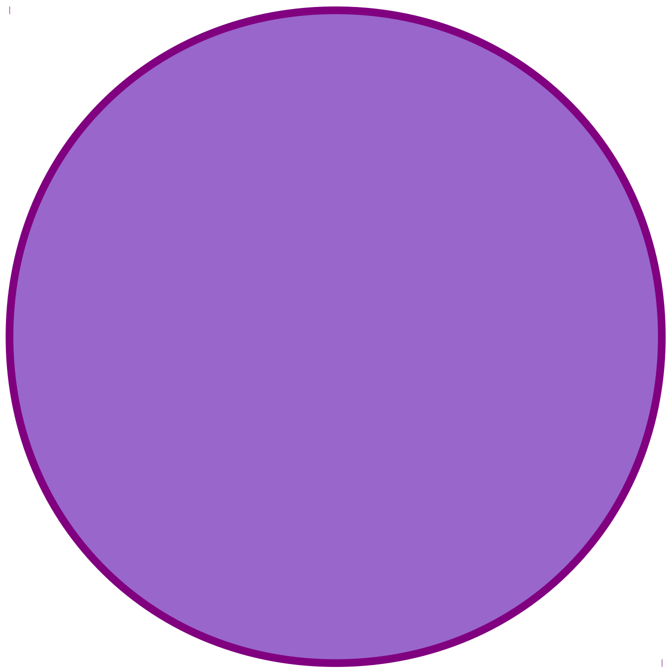


$$L^3 \simeq \sqrt{\langle\hat{x}^2\rangle\langle\hat{y}^2\rangle\langle\hat{z}^2\rangle} \sim \frac{1}{\omega_\perp\sqrt{\omega_{||}}}$$

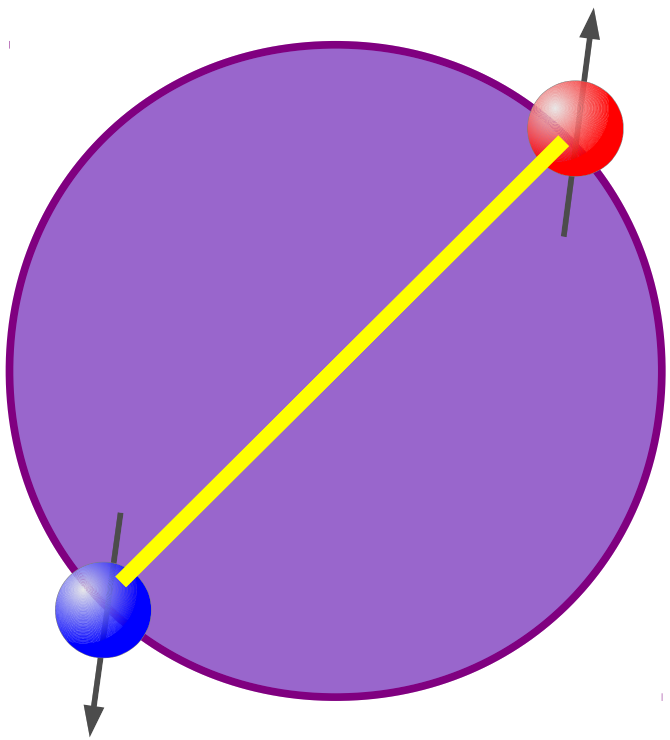
Summary

- Unique opportunity to realize a few body interacting system
- Link between microscopic physics and macroscopic phenomenology
- DMC simulations allowed us to probe attractive interactions

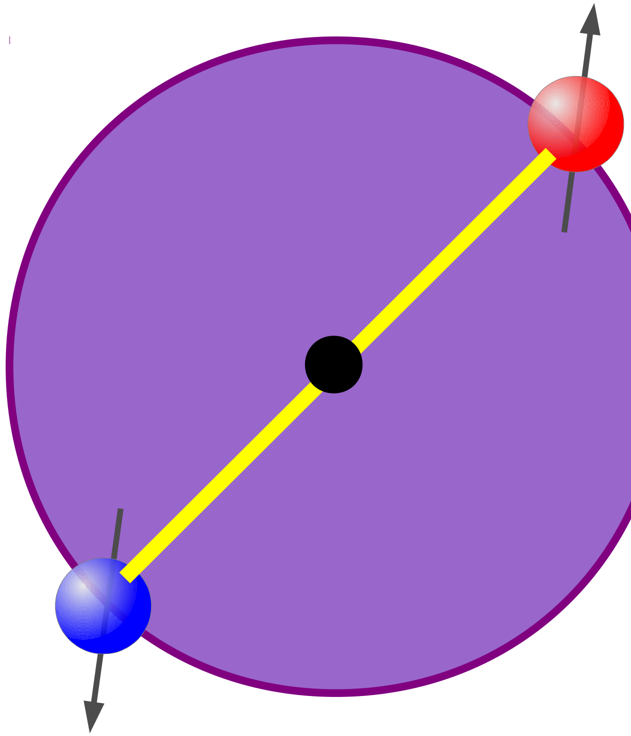
BCS state



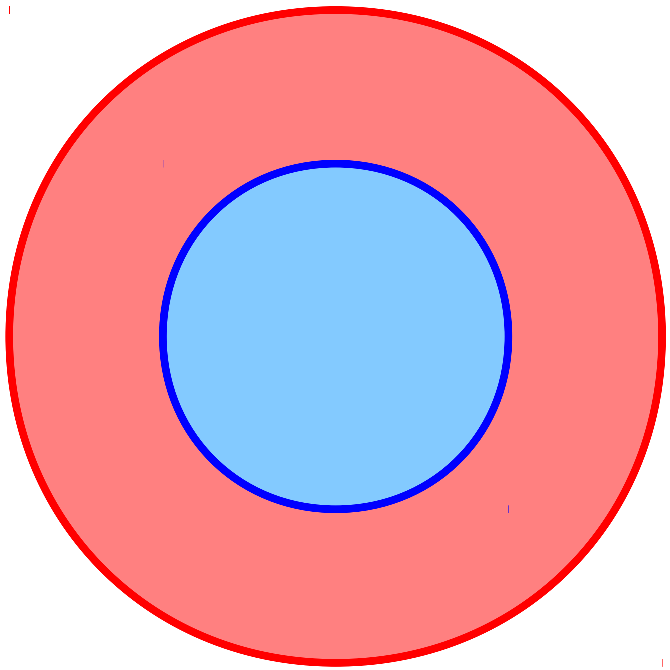
BCS state



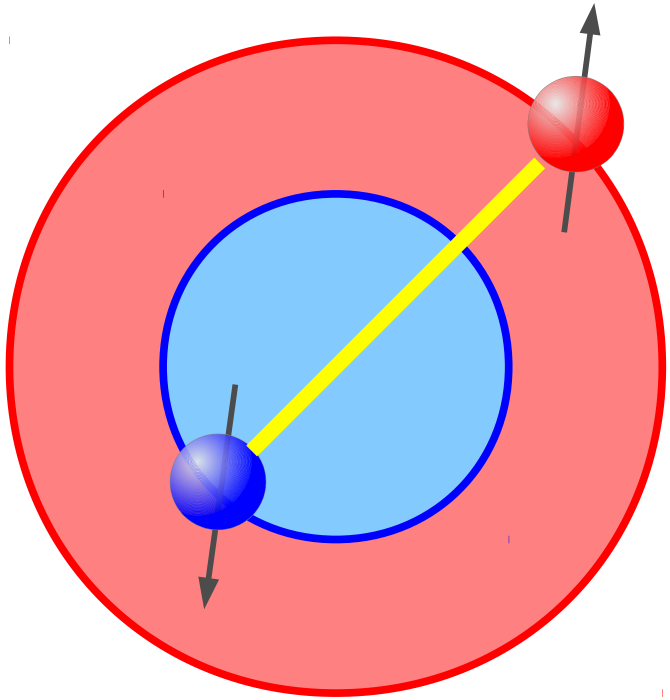
BCS state



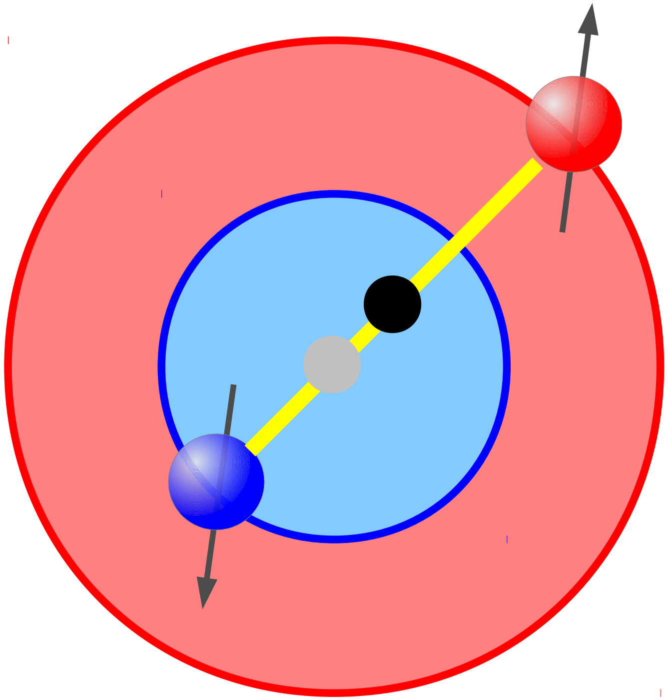
BCS FFLO state



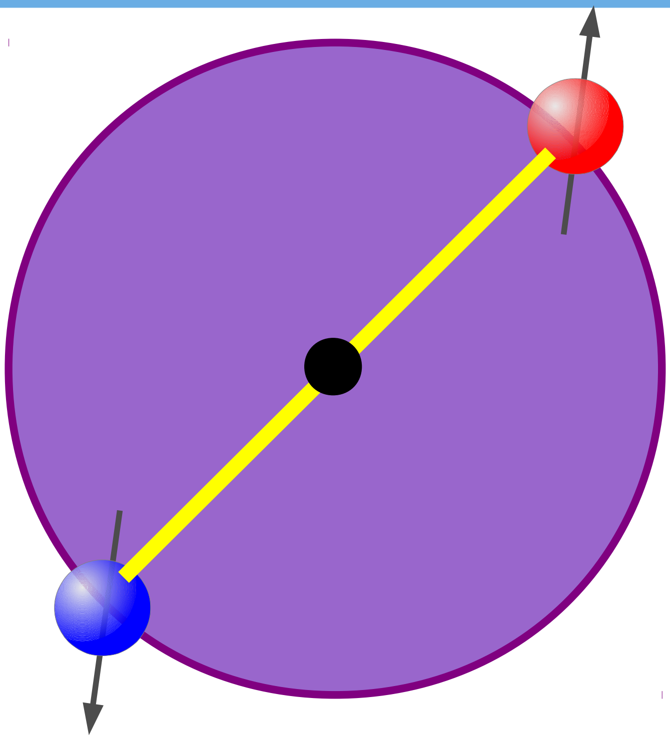
BCS FFLO state



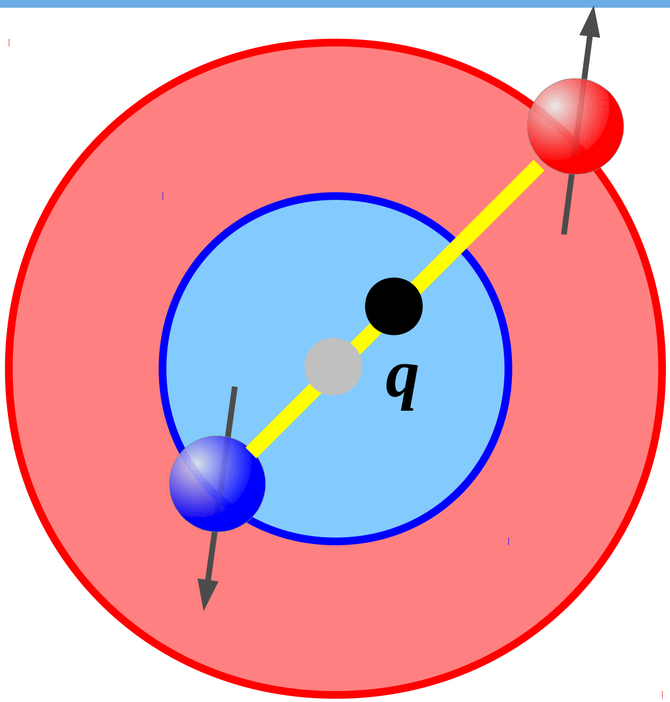
BCS FFLO state



BCS FFLO state

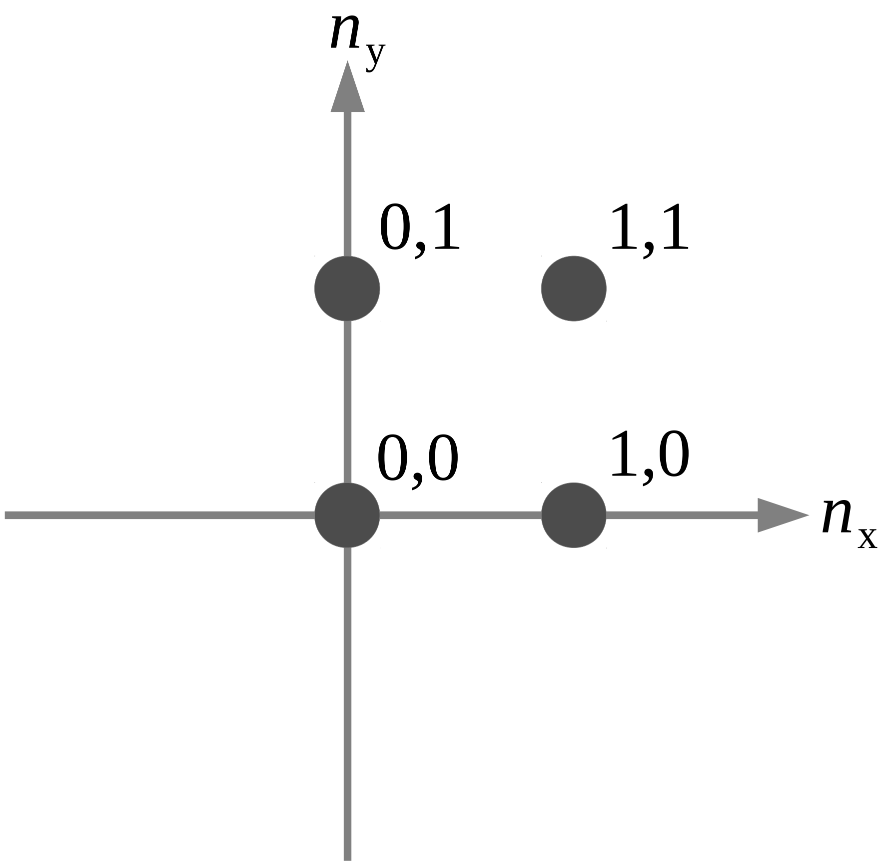


$$\Delta(\boldsymbol{r}) = \Delta_0$$

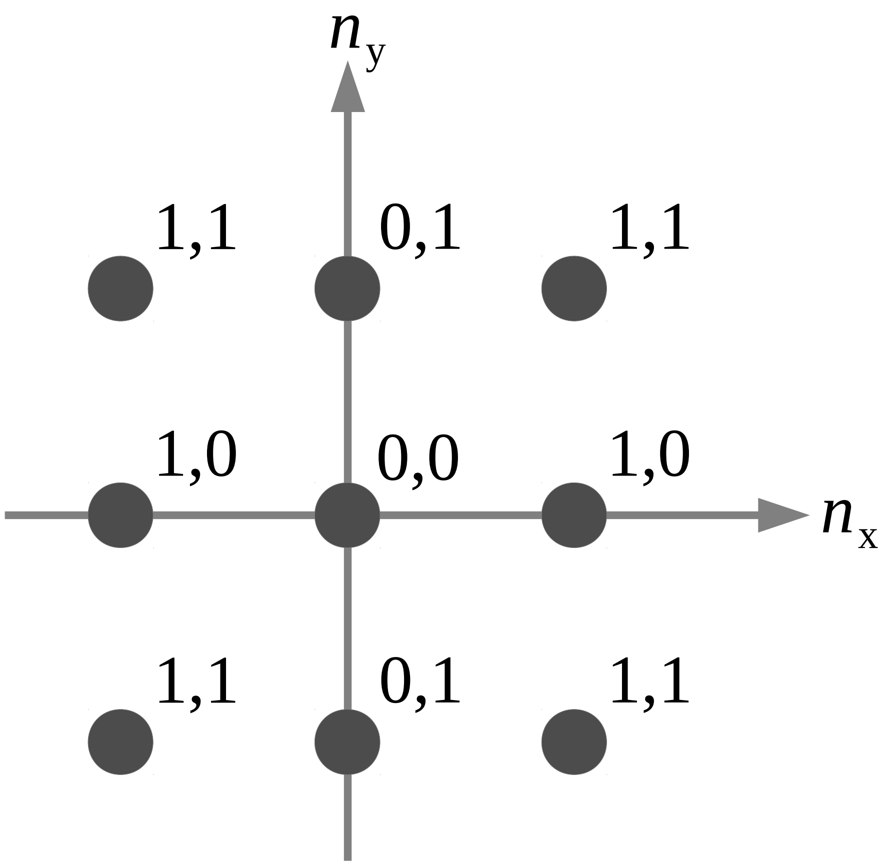


$$\Delta(\boldsymbol{r}) = \Delta_0 e^{\mathrm{i}\boldsymbol{q}\cdot\boldsymbol{r}}$$

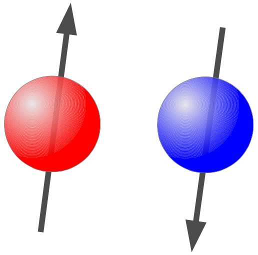
Available states

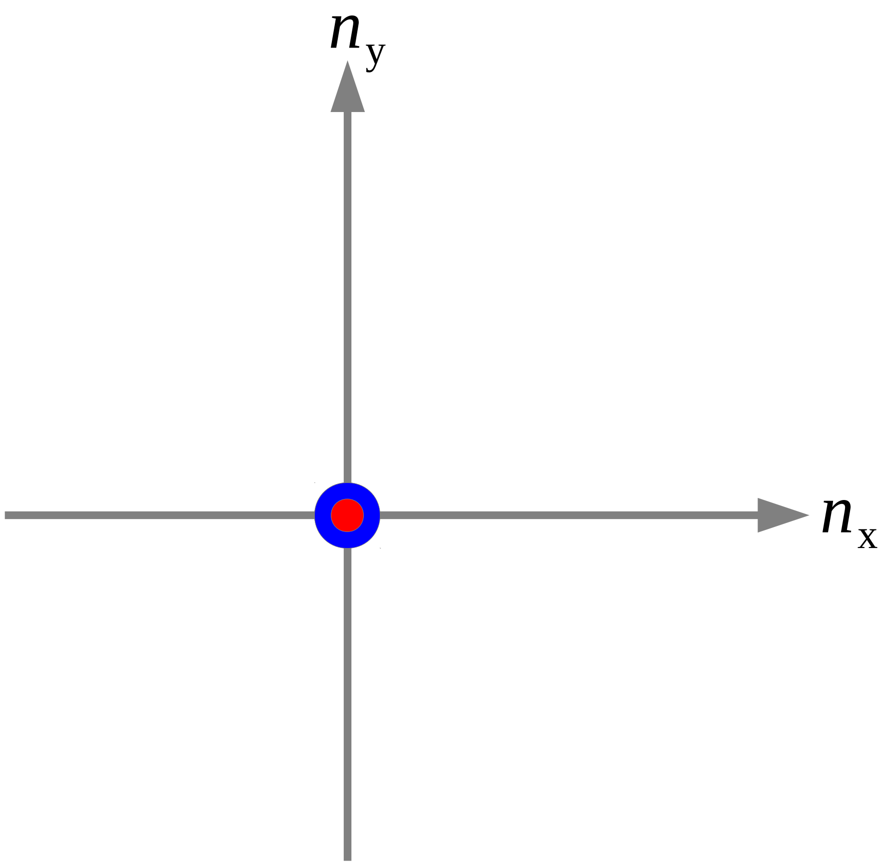


Available states



2 atoms in a spherical trap





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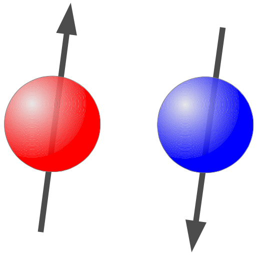
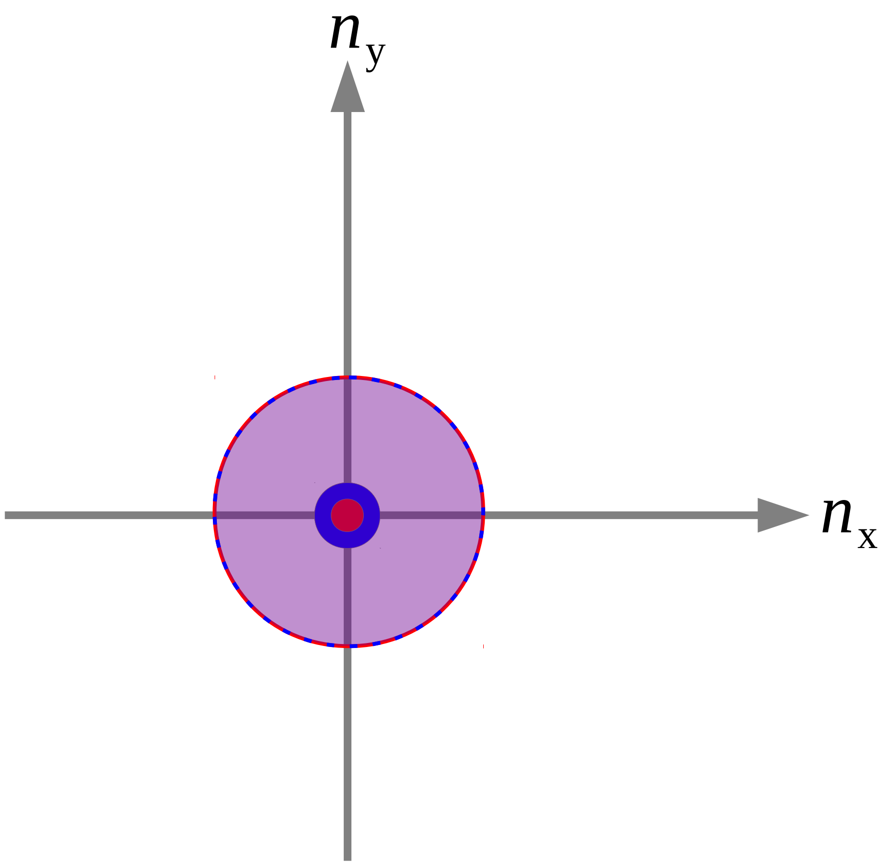
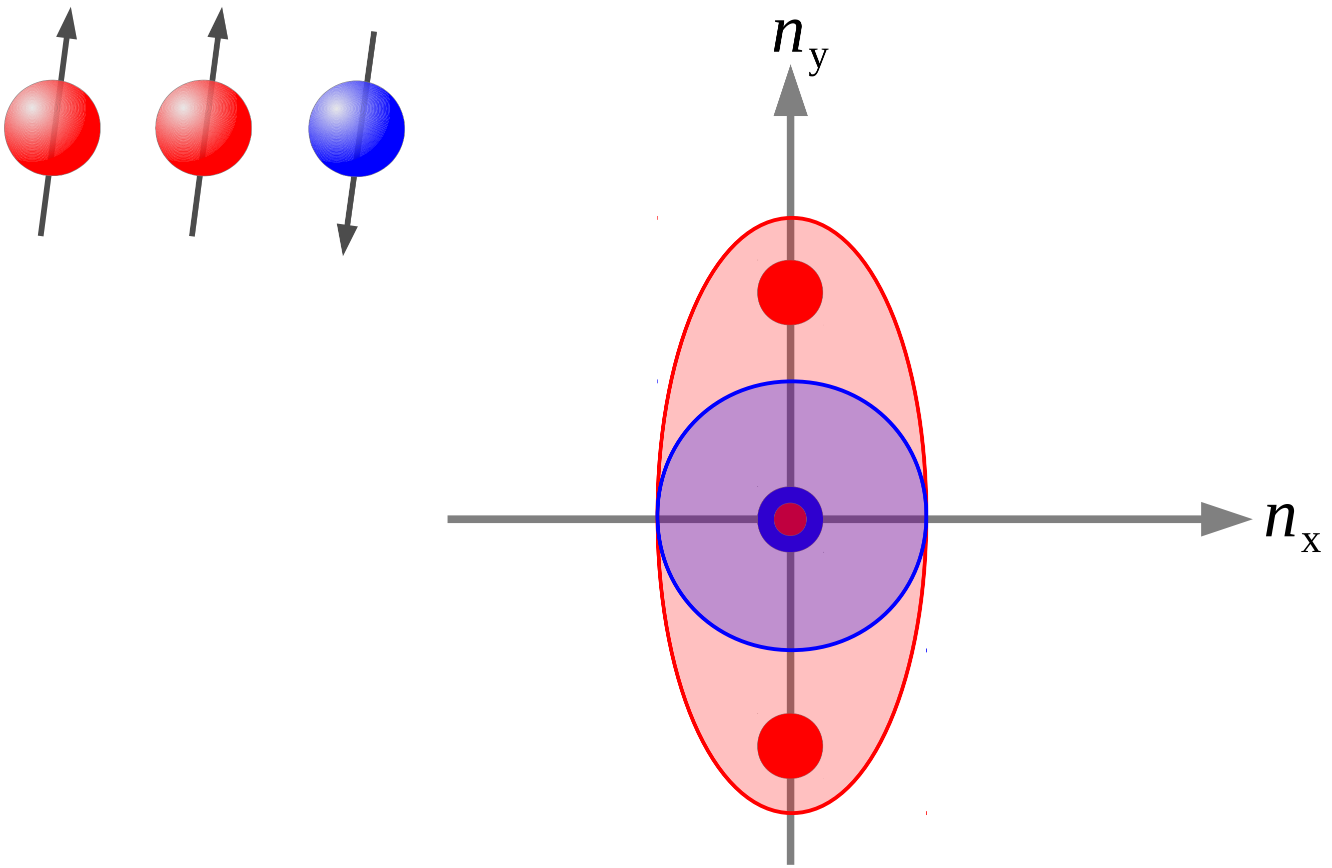




3 atoms in a spherical trap



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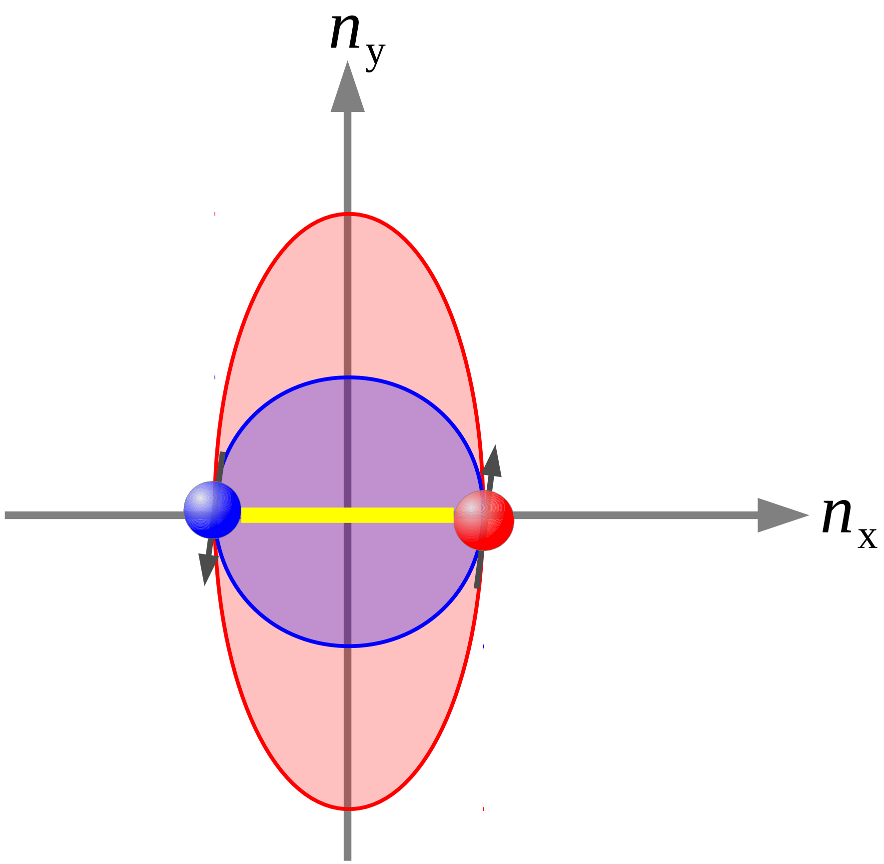
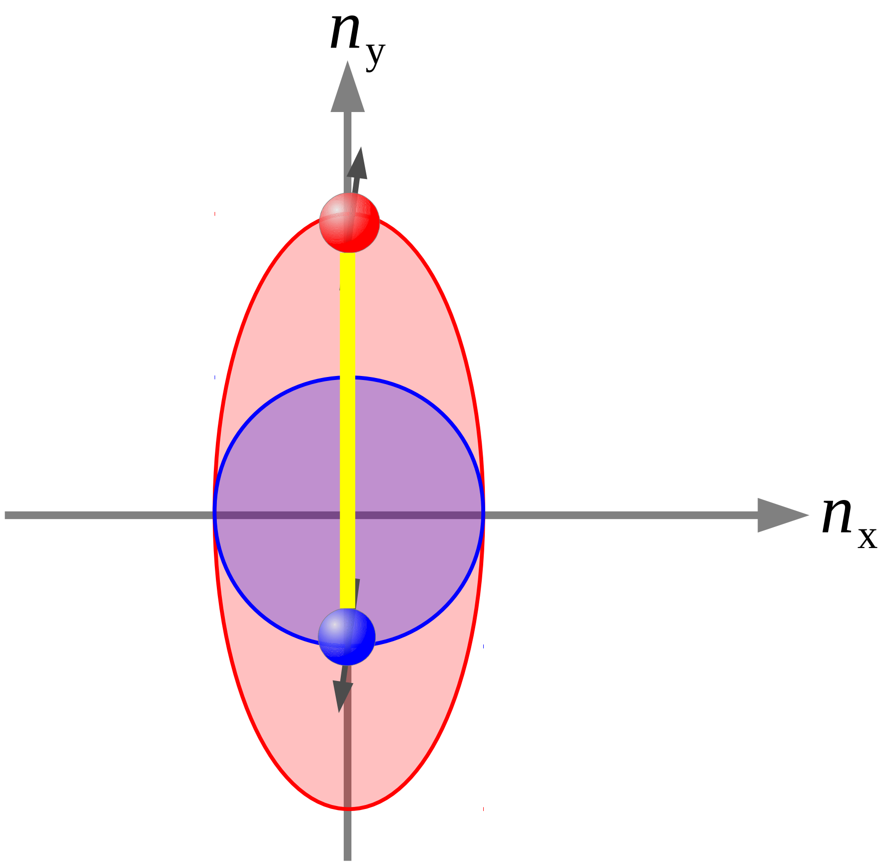
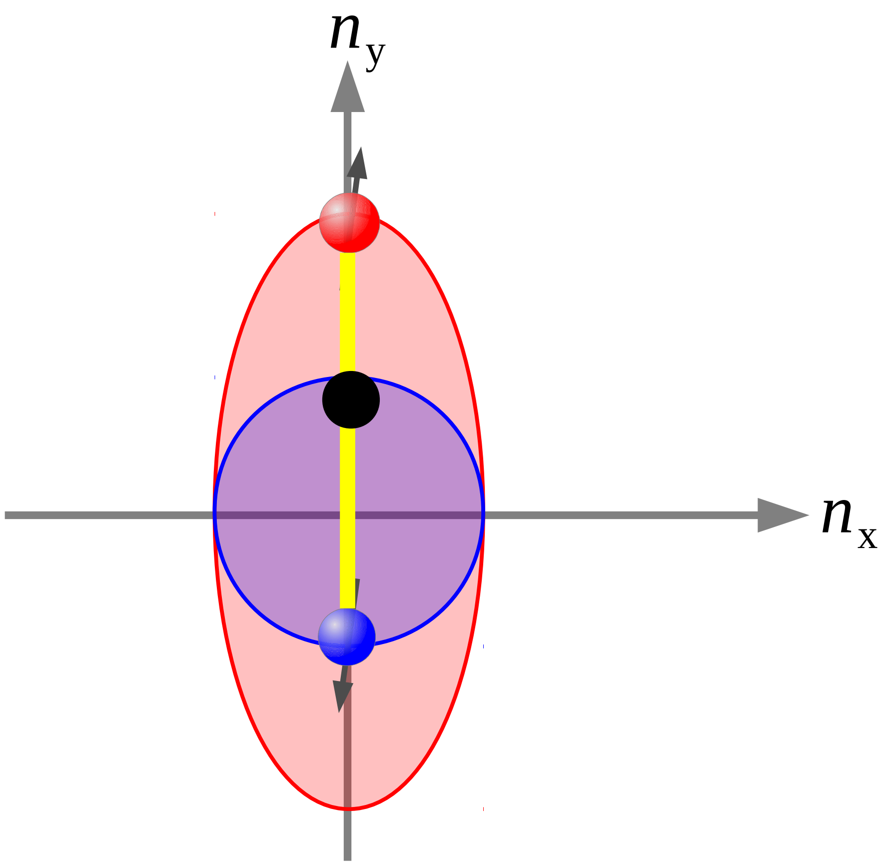
3 atoms in a spherical trap



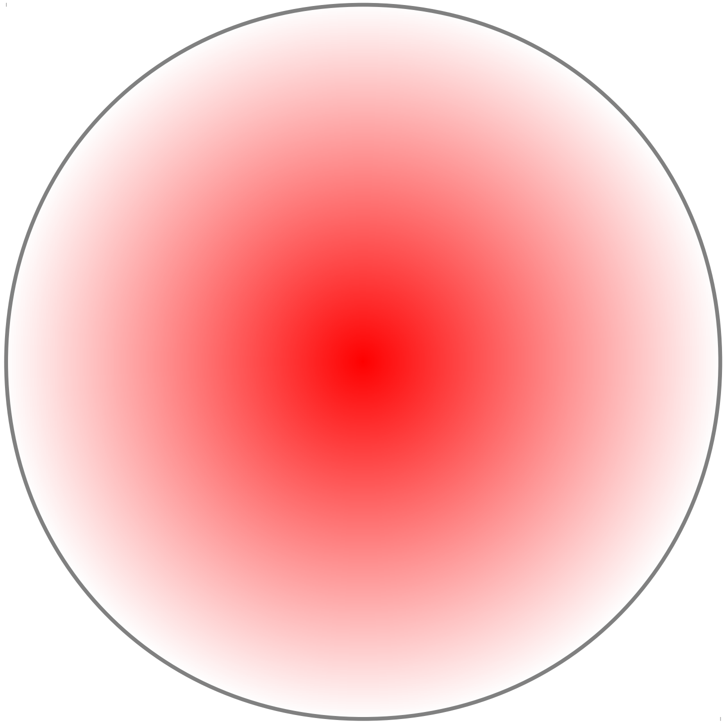
3 atoms in a spherical trap



3 atoms in a spherical trap

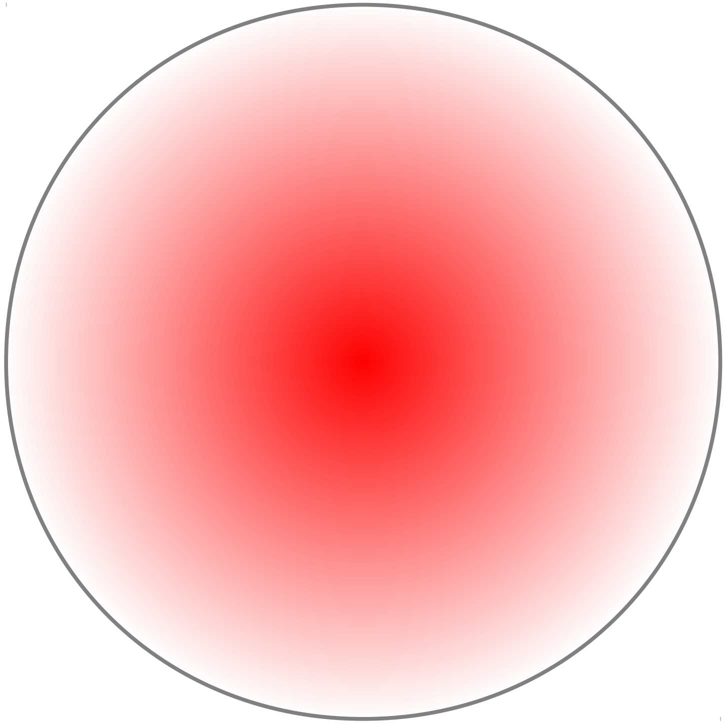


2 atoms in a spherical trap



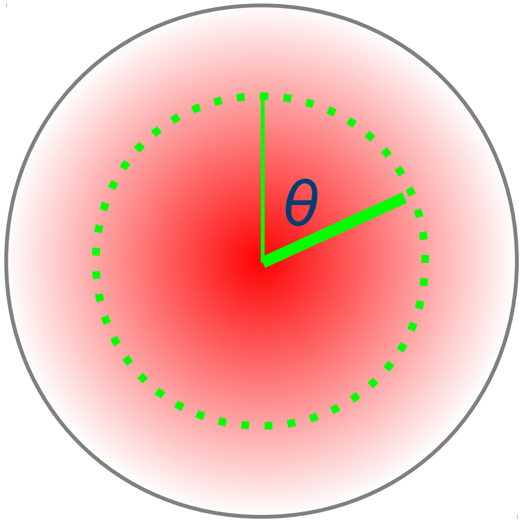
$$\Delta(\boldsymbol{r}) = \langle \psi | c_{\uparrow}(\boldsymbol{r}) c_{\downarrow}(\boldsymbol{r}) | \psi \rangle$$

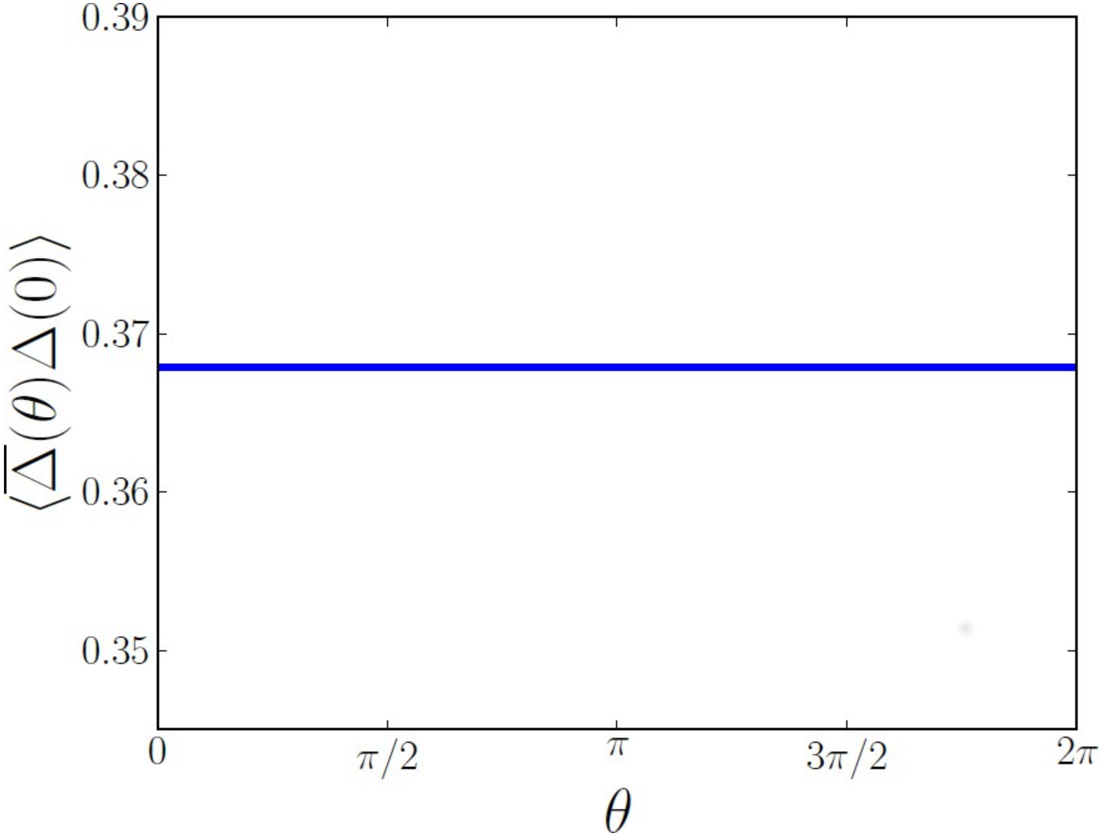
2 atoms in a spherical trap



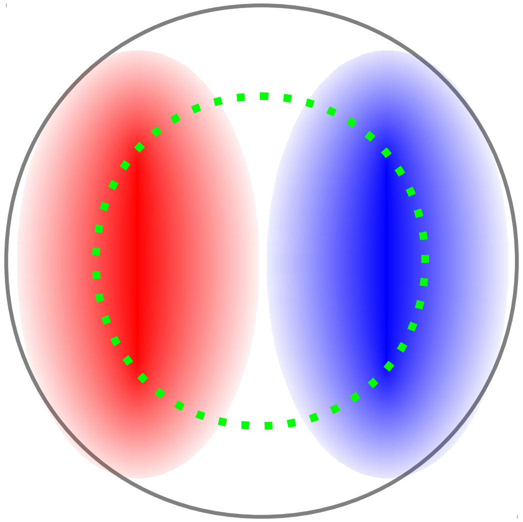
$$\bar{\Delta}(\boldsymbol{r})\Delta(\mathbf{0}) = \langle\psi|c_{\downarrow}^{\dagger}(\boldsymbol{r})c_{\uparrow}^{\dagger}(\boldsymbol{r})c_{\uparrow}(\mathbf{0})c_{\downarrow}(\mathbf{0})|\psi\rangle$$

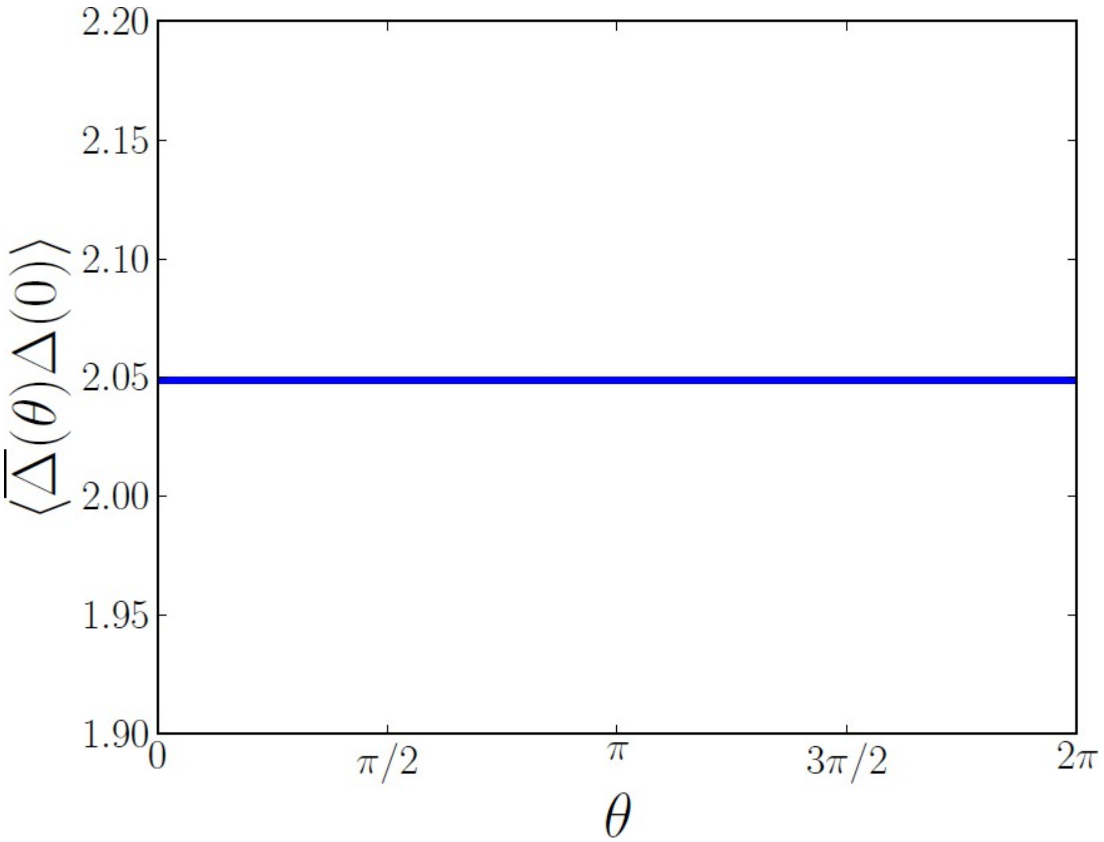
2 atoms in a spherical trap



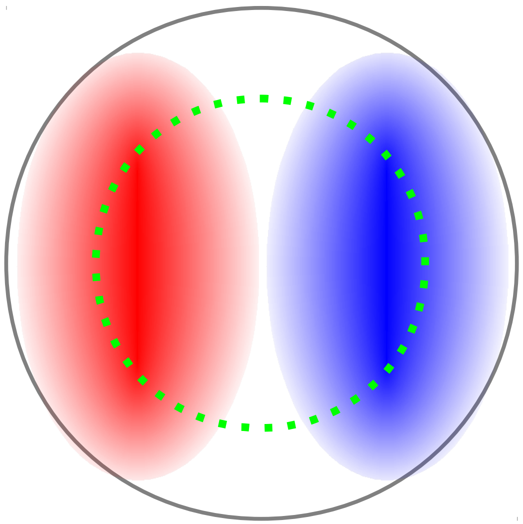


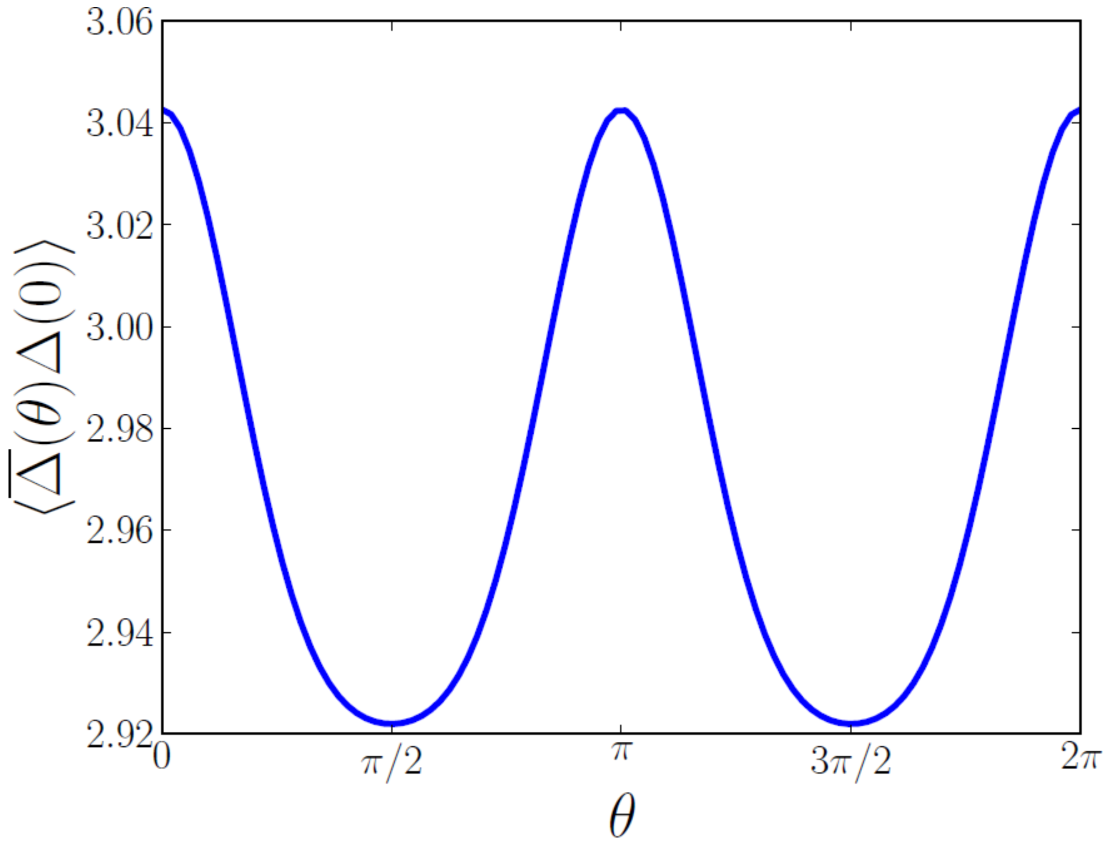
3 atoms in a spherical trap



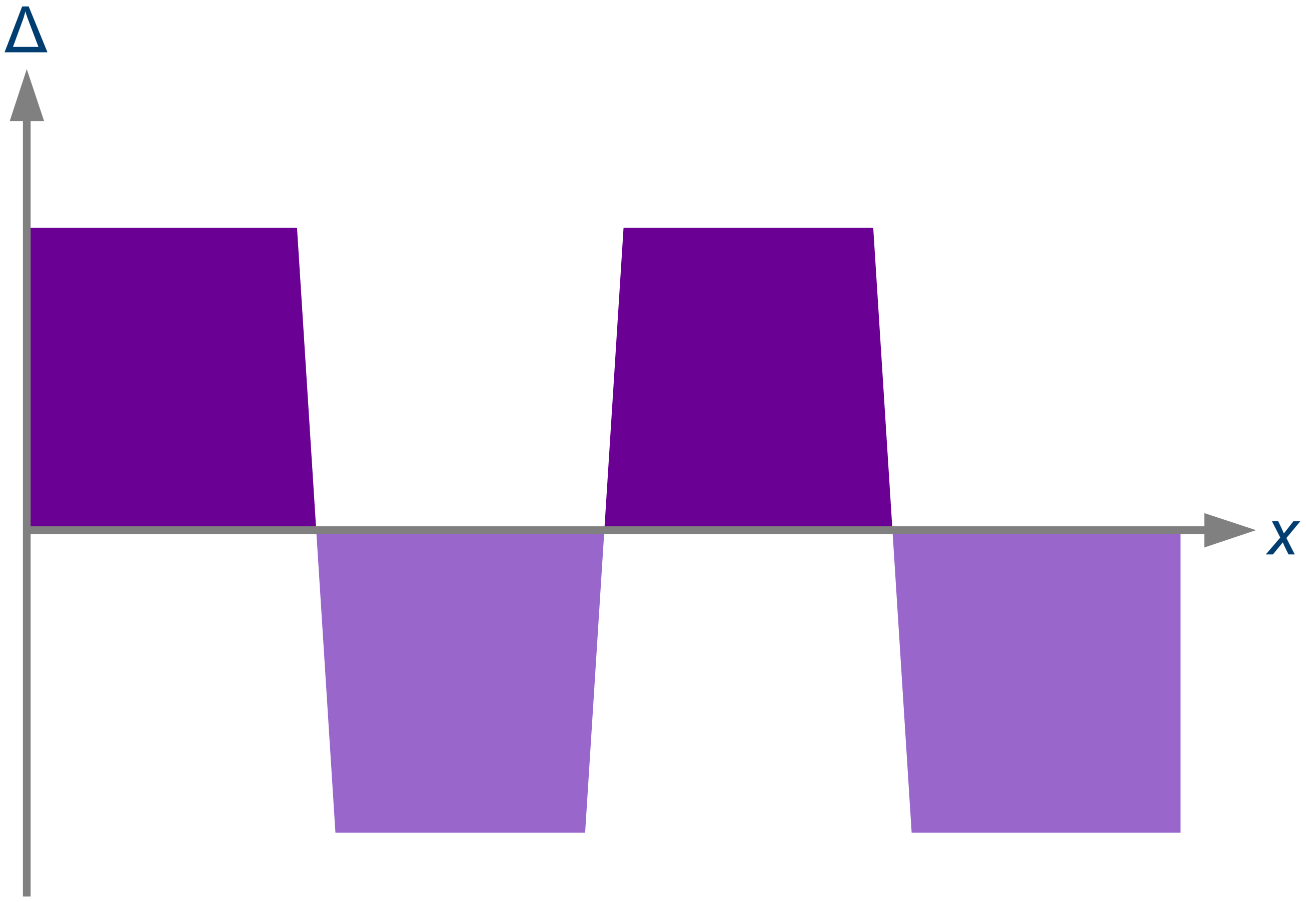


3 atoms in a spherical trap





Where does the extra majority spin reside?



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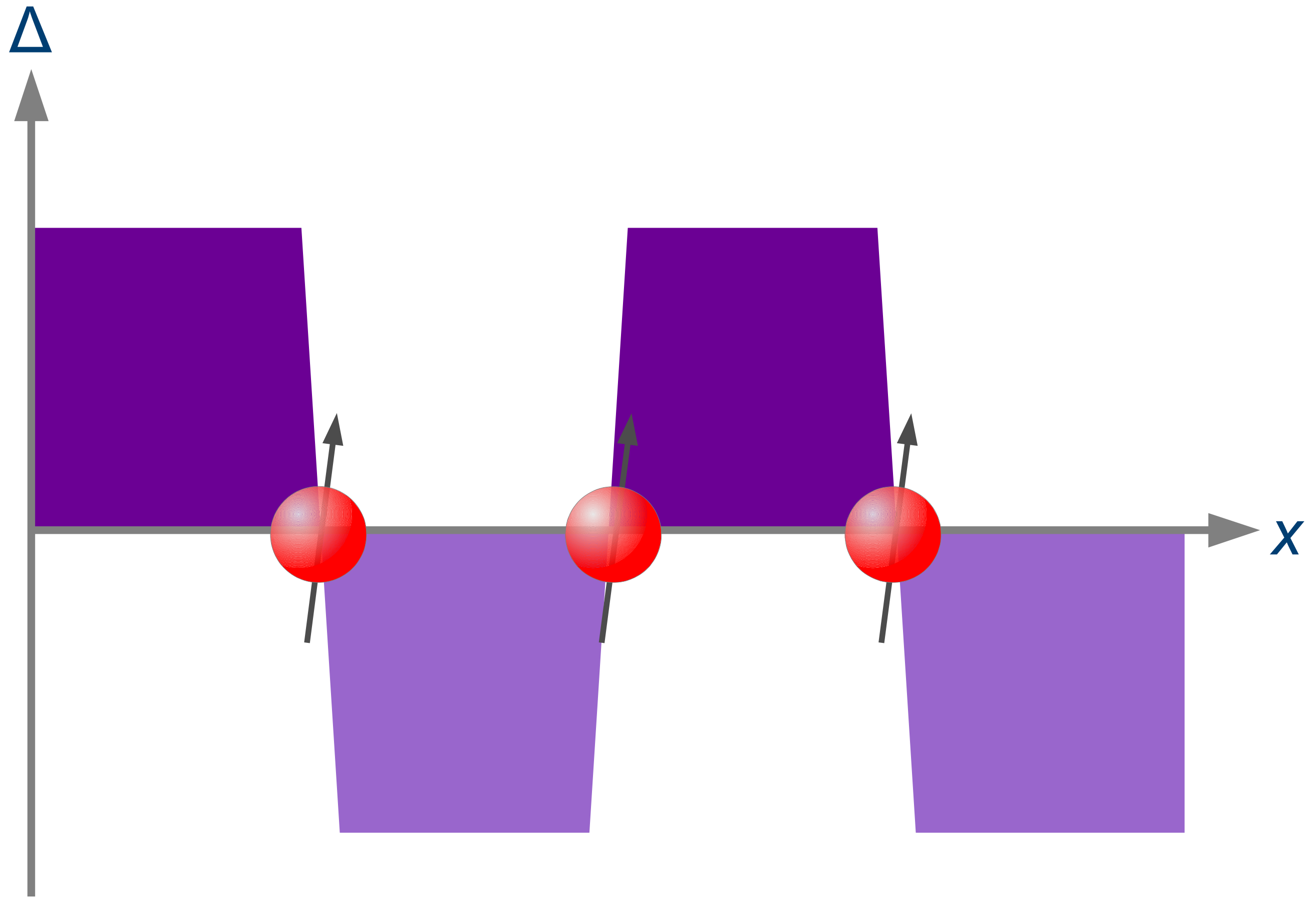
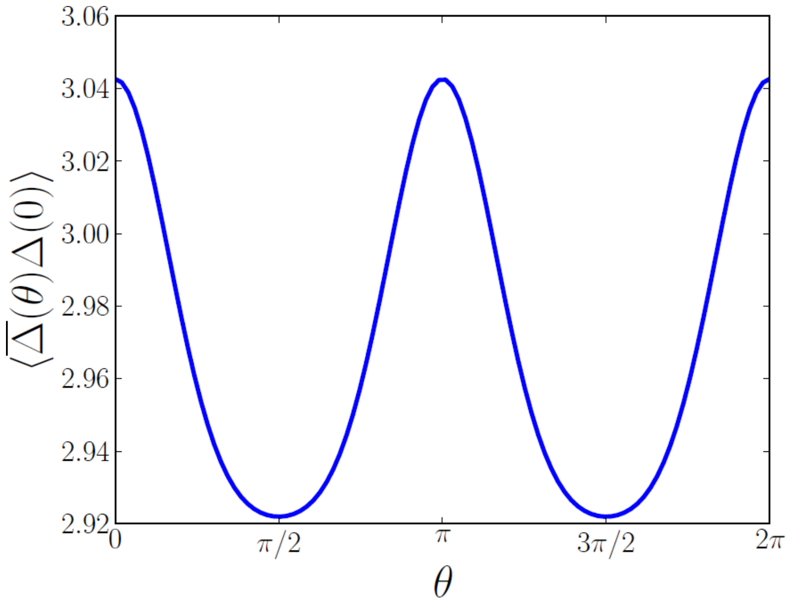
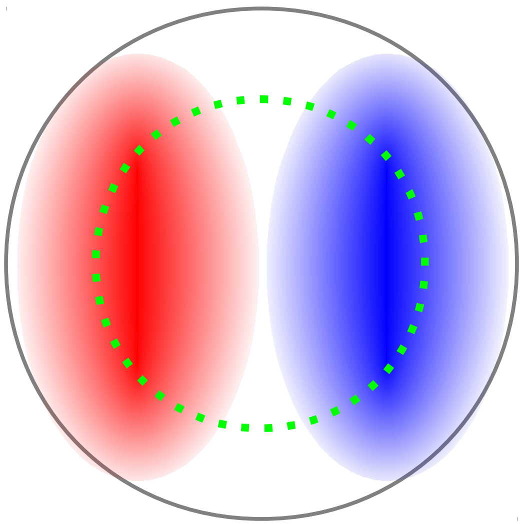


3 atoms in a spherical trap





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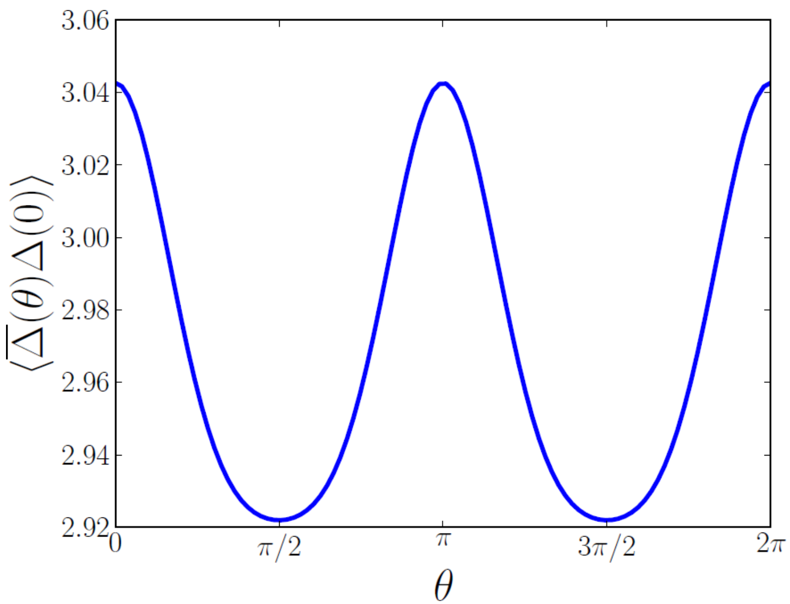
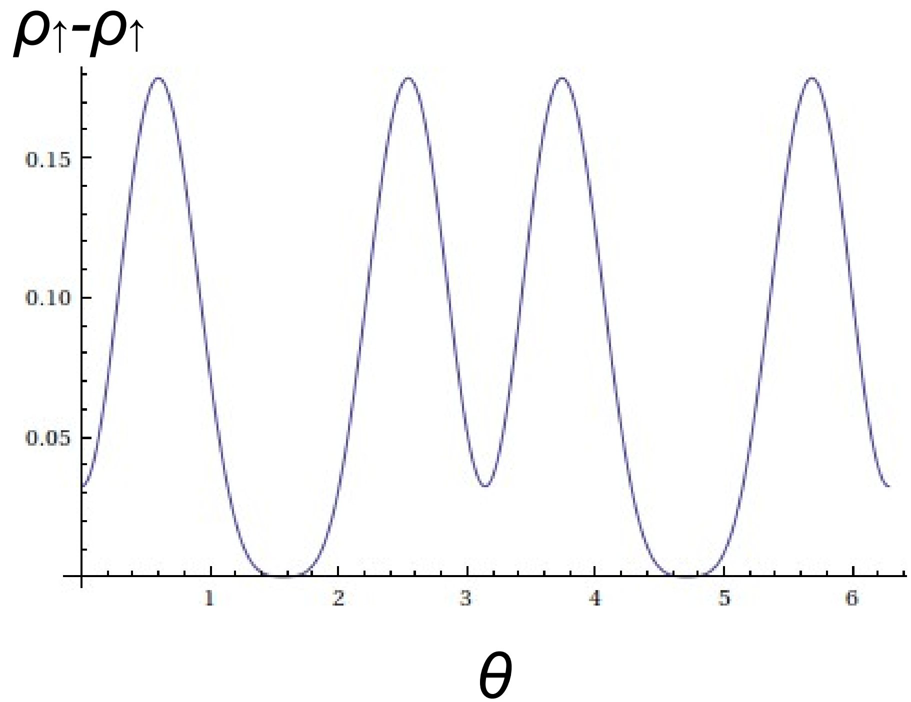
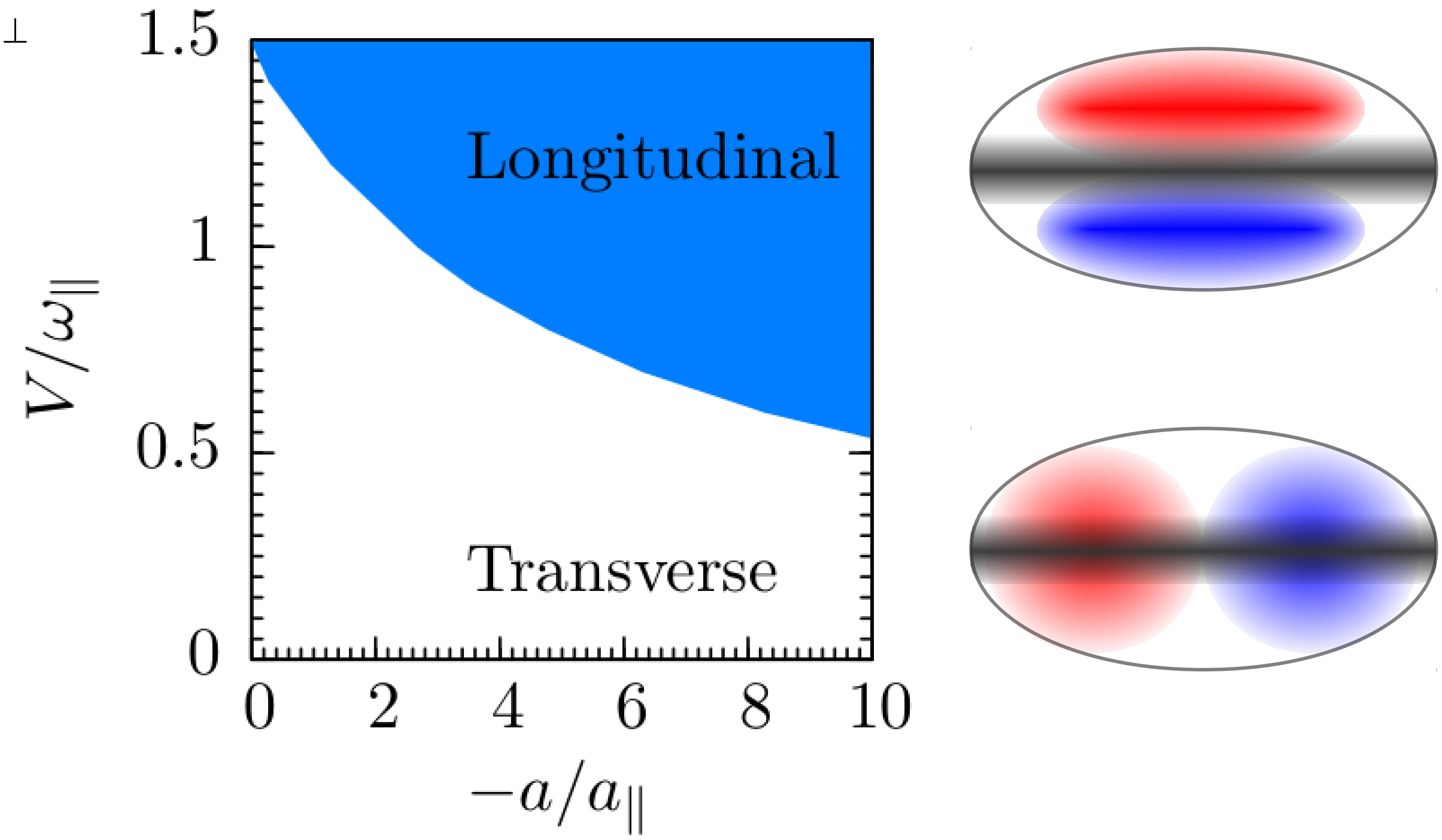


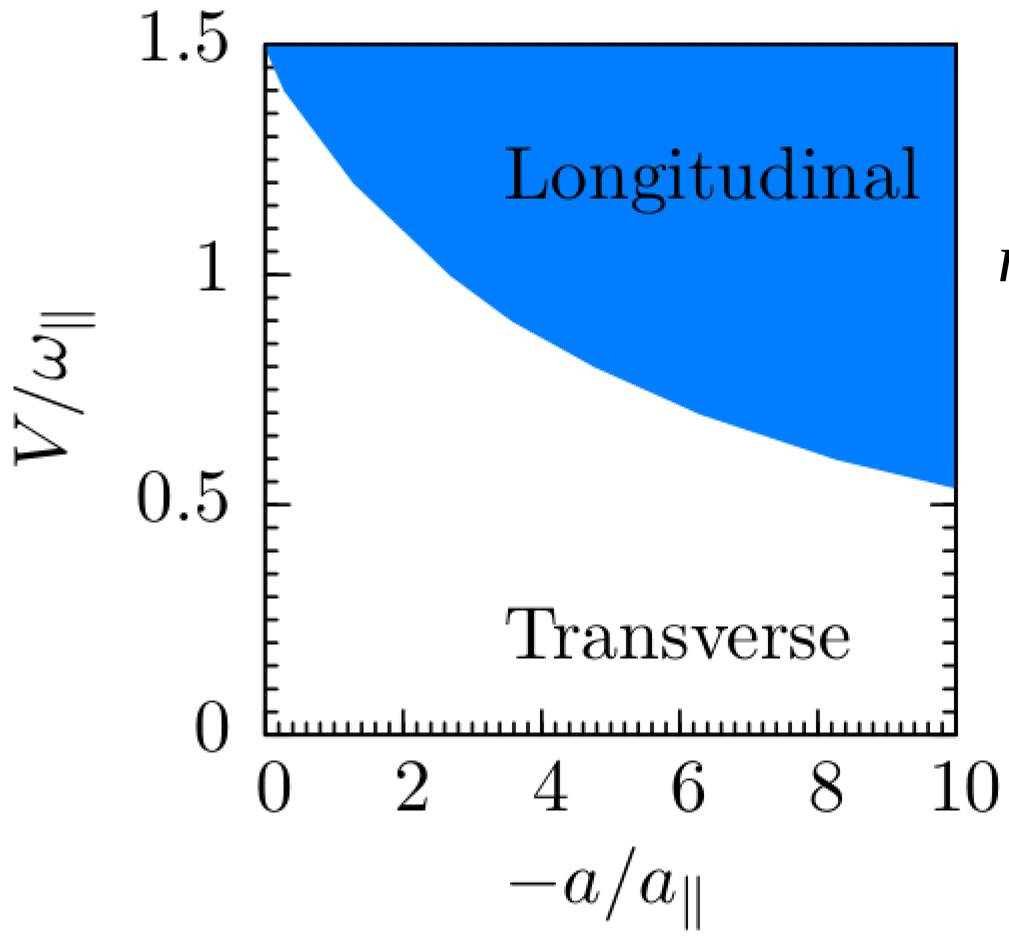


Phase diagram

$\omega_{\parallel} = 2\omega_{\perp}$



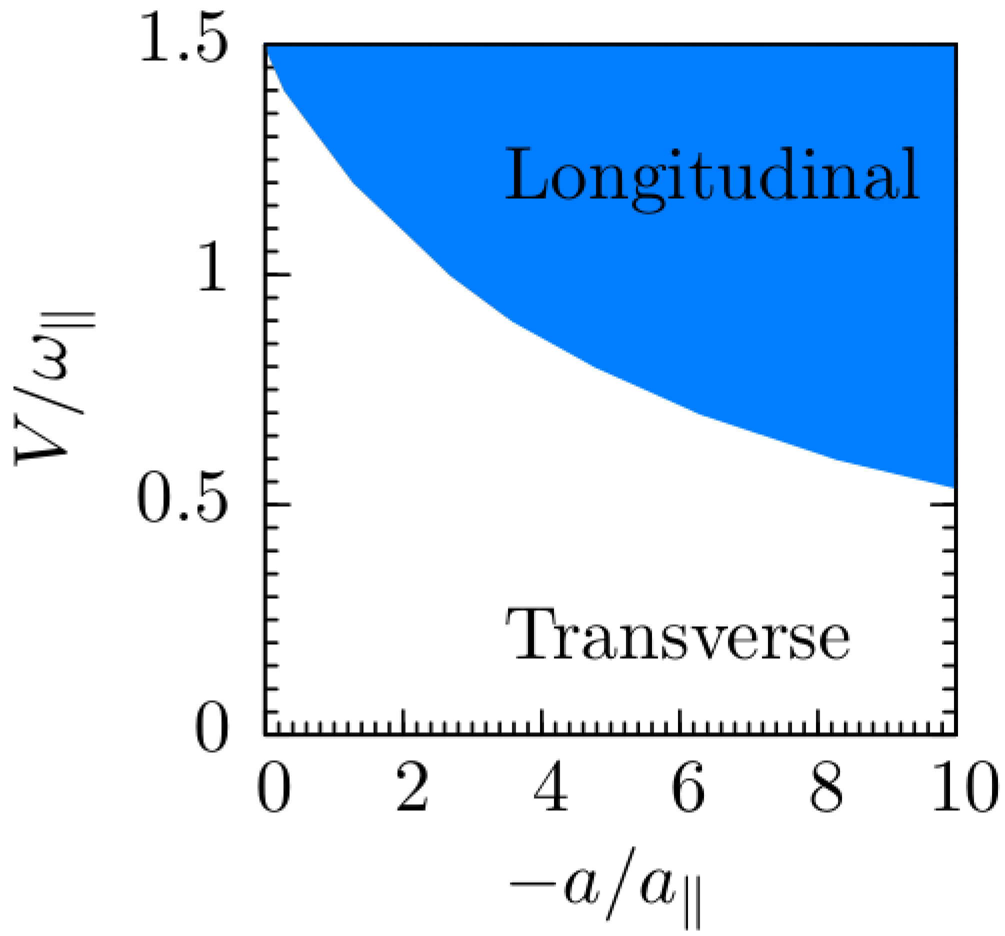
Phase diagram

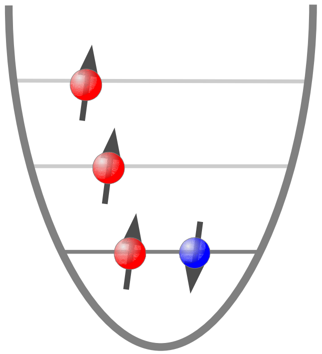


$n_z = 1$

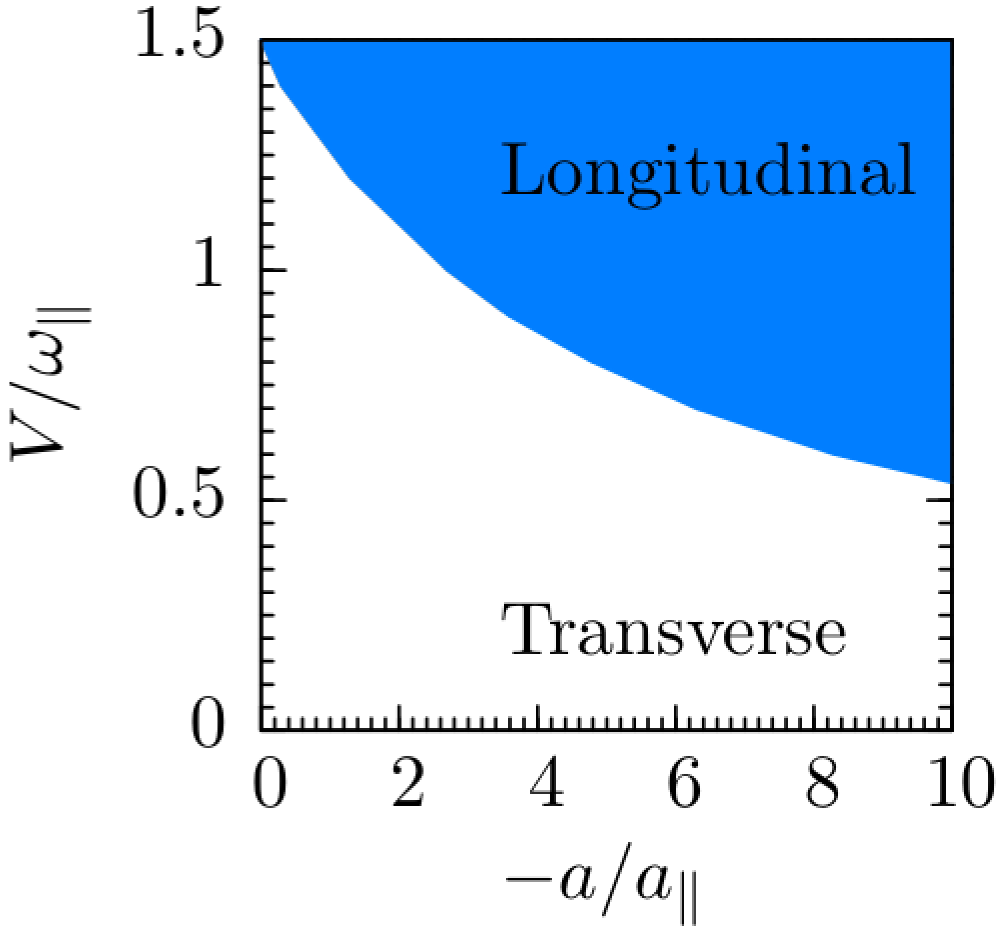
$n_x = 1$ or $n_y = 1$

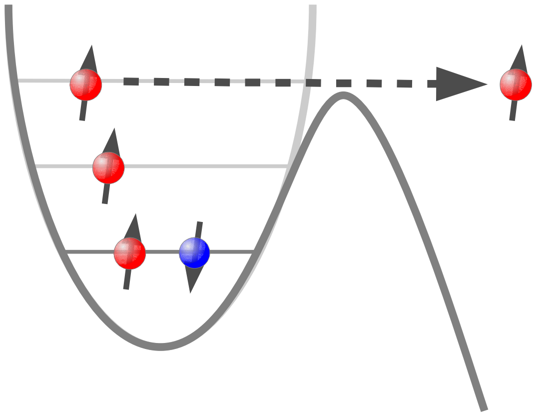
Phase diagram



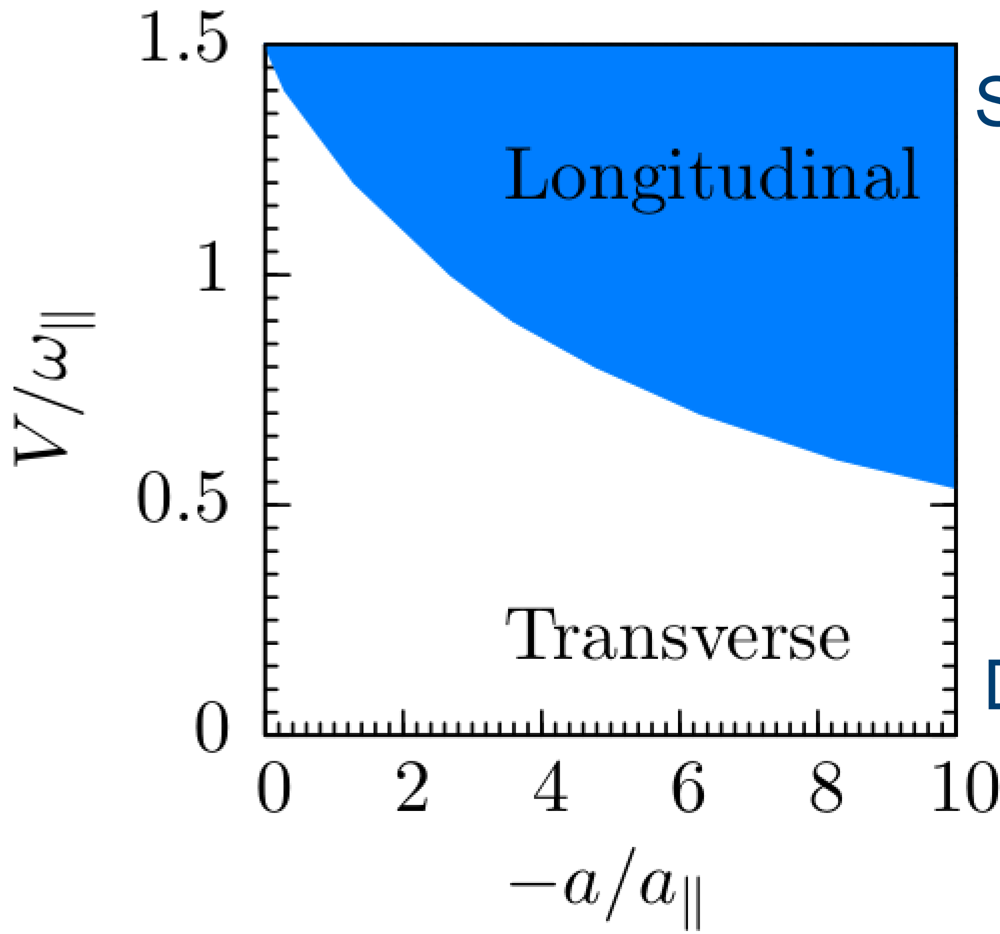


Phase diagram

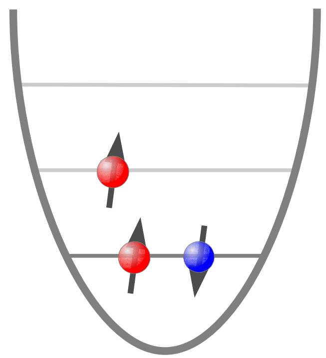




Phase diagram



Singly degenerate



Doubly degenerate

Summary

Few trapped fermions offers chance to observe spatially modulated pairing

Trap ellipticity and central barrier are experimental probes of the pairing state

Appendix: System Geometry

$$g(n) = (n+1)(n+2)/2$$

$$N \sim \int_0^{n_{max}} dn \; g(n) \simeq n_{max}^3/6$$

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Hamiltonian for outermost electron

$$\langle \hat{H}_{n_{max}} \rangle \simeq \frac{1}{2} m\omega^2 \langle \hat{r}^2 \rangle$$

$$\langle \hat{H}_{n_{max}} \rangle = \hbar\omega \left(n_{max} + \frac{3}{2} \right)$$

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$$\sqrt{\langle \hat{r}^2 \rangle} \sim N^{1/6}$$

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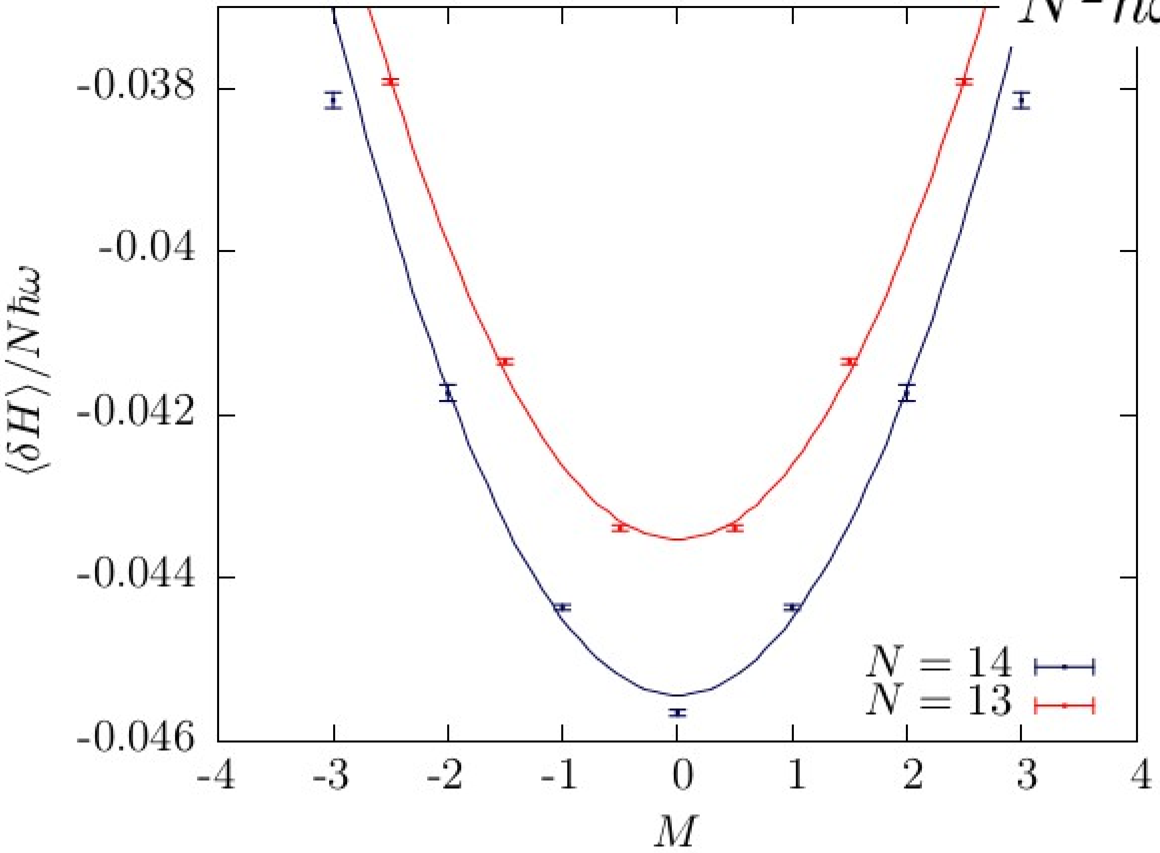
$$\frac{E_{\text{int}}}{N^2} = -\frac{V_0}{4L^3}$$

$$\frac{E_{\text{int}}}{N^2} \sim -V_0 N^{-1/2}$$

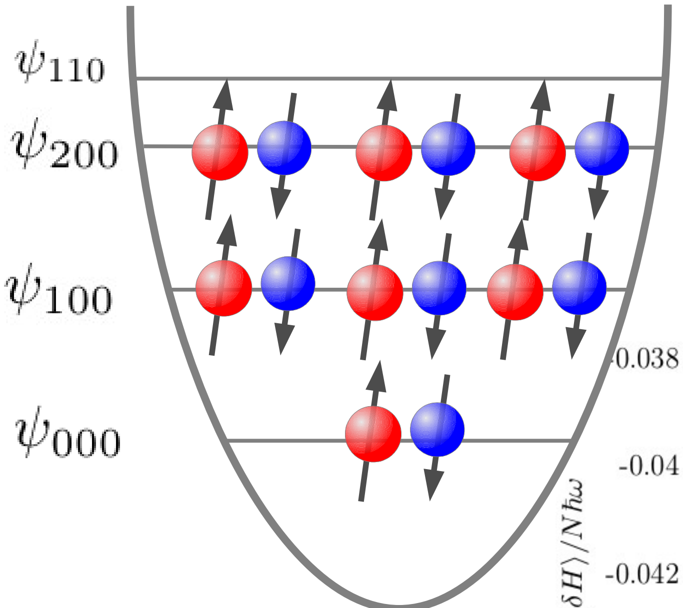
Appendix: Magnetised Fermionic Gases

$$\frac{E_{\mathrm{int}}}{N^2\hbar\omega} \simeq -\frac{V_0}{4L^3}\left(1-4\frac{M^2}{N^2}\right)$$

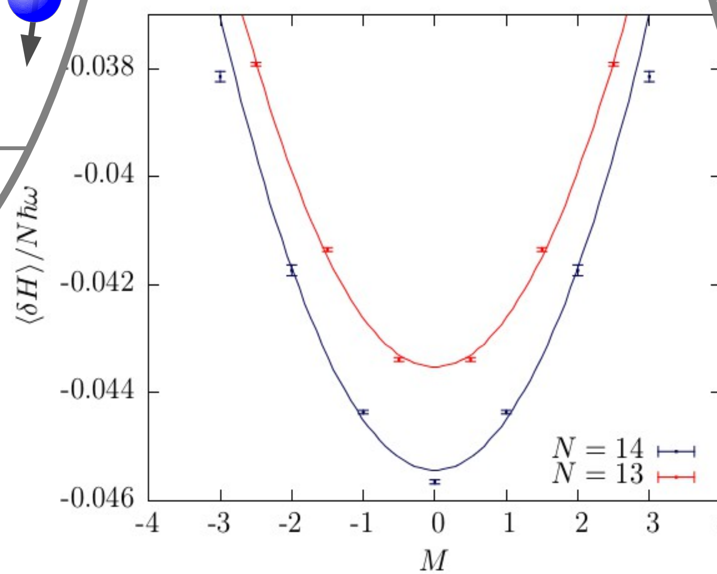


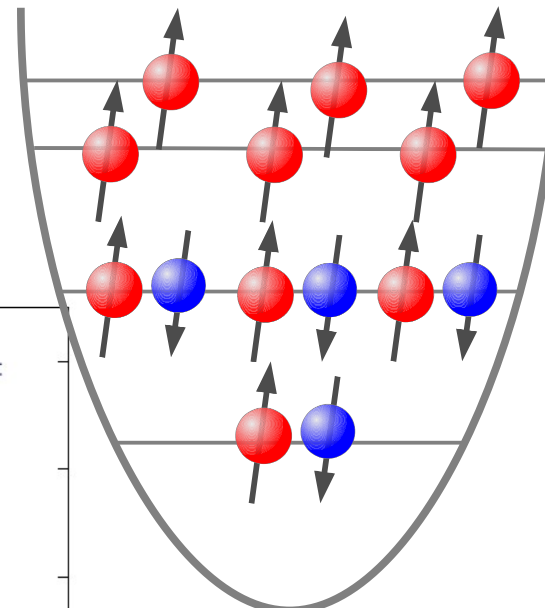
Appendix: Magnestised fermionic gases

- N = 14



M = 0

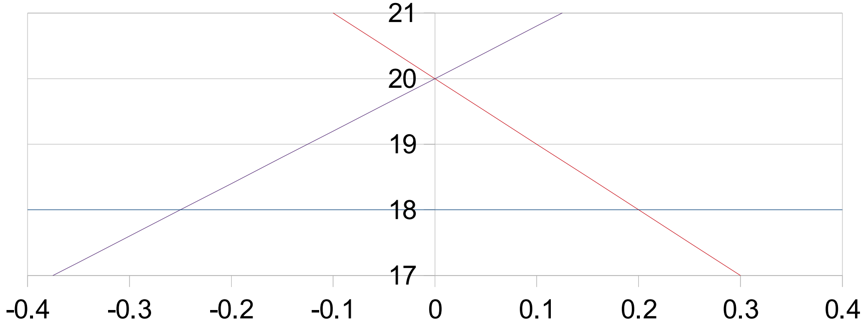




M = 6/2 =3

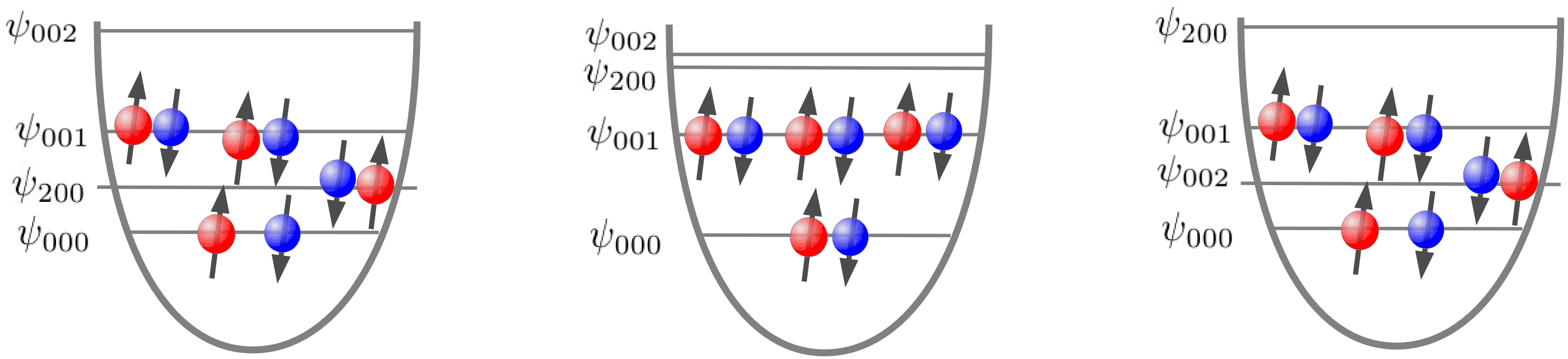
Appendix: Asymmetric Trap

$$\hat{H}^{(0)} = \frac{-\hbar^2}{2m}\nabla^2 + \frac{1}{2}m(\omega_\perp^2 x^2 + \omega_\perp^2 y^2 + \omega_{||}^2 z^2)$$

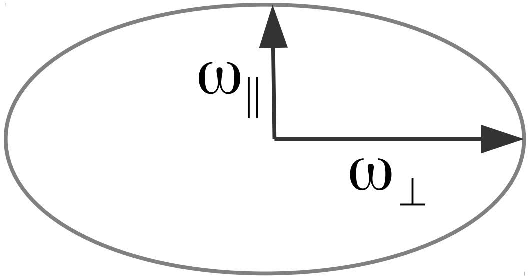


$$\omega_{||}(s) = \omega_0(1-2s)$$

$$\omega_\perp(s) = \omega_0(1+s)$$

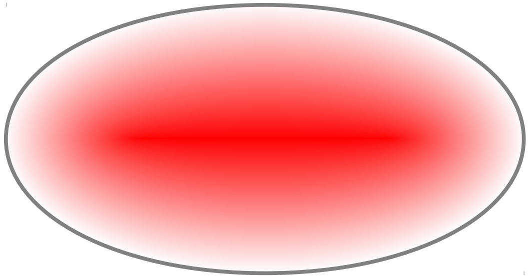


Trapping potential



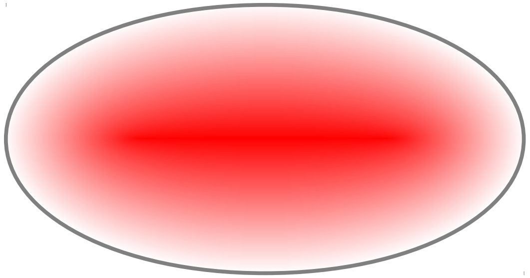
$$V = \frac{1}{2}\left[\omega_{\perp}^{2}\left(x^{2}+y^{2}\right)+\omega_{\parallel}^{2} z^{2}\right]$$

One trapped atom



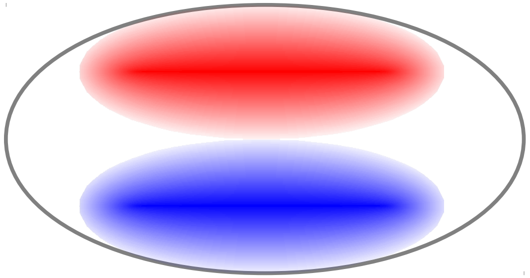
$$E = \frac{1}{2}\omega_{\|} + \omega_{\perp}$$

Two trapped atoms

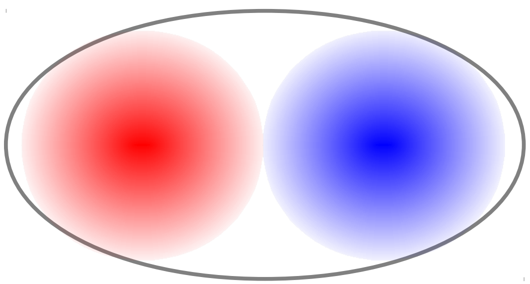


$$E = \omega_{\parallel} + 2\omega_{\perp}$$

Three trapped atoms

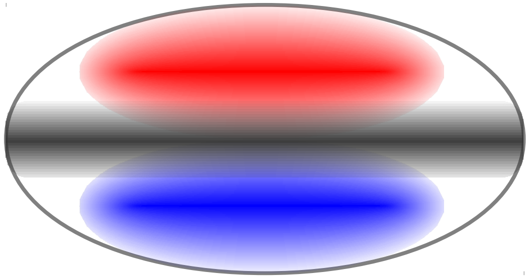


$$E = \frac{5}{2}\omega_{\parallel} + 3\omega_{\perp}$$

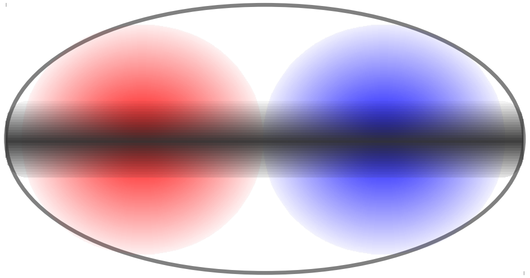


$$E = \frac{3}{2}\omega_{\parallel} + 4\omega_{\perp}$$

Three trapped atoms

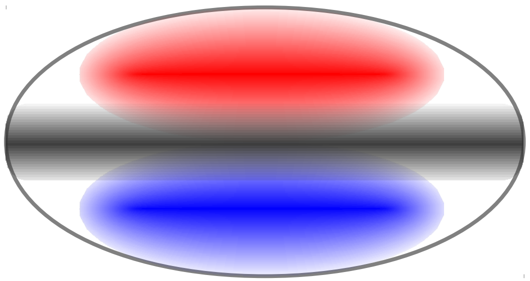


$$E = \frac{5}{2}\omega_{\|} + 3\omega_{\perp} + \sqrt{\frac{\omega_{\|}}{\omega_{\|}+\omega_{B}}} V_{B}\left(2 + \frac{\omega_{\|}}{\omega_{\|}+\omega_{B}}\right)$$

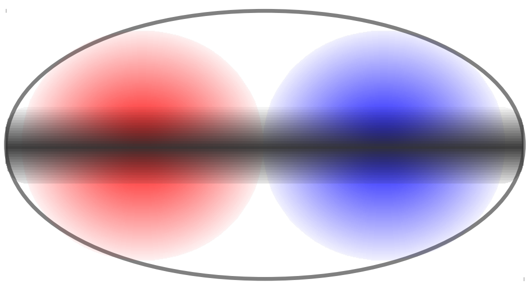


$$E = \frac{3}{2}\omega_{\|} + 4\omega_{\perp} + 3\sqrt{\frac{\omega_{\|}}{\omega_{\|}+\omega_{B}}} V_{B}$$

Three trapped atoms



$$E=\frac{5}{2}\omega_{\parallel}+3\omega_{\perp}+\sqrt{\frac{\omega_{\parallel}}{\omega_{\parallel}+\omega_{\mathrm{B}}}}V_{\mathrm{B}}\left(2+\frac{\omega_{\parallel}}{\omega_{\parallel}+\omega_{\mathrm{B}}}\right)+\frac{a}{a_{\parallel}}\omega_{\perp}\sqrt{\frac{2}{\pi}}\left(\frac{3}{2}-\frac{4\sqrt{2}}{\pi}\frac{V_{\mathrm{B}}}{\omega_{\parallel}+\omega_{\mathrm{B}}}\right)$$



$$E=\frac{3}{2}\omega_{\parallel}+4\omega_{\perp}+3\sqrt{\frac{\omega_{\parallel}}{\omega_{\parallel}+\omega_{\mathrm{B}}}}V_{\mathrm{B}}+\frac{a}{a_{\parallel}}\omega_{\perp}\sqrt{\frac{2}{\pi}}\left(\frac{3}{2}-\frac{6\sqrt{2}}{\pi}\frac{V_{\mathrm{B}}}{\omega_{\parallel}+\omega_{\mathrm{B}}}\right)$$