

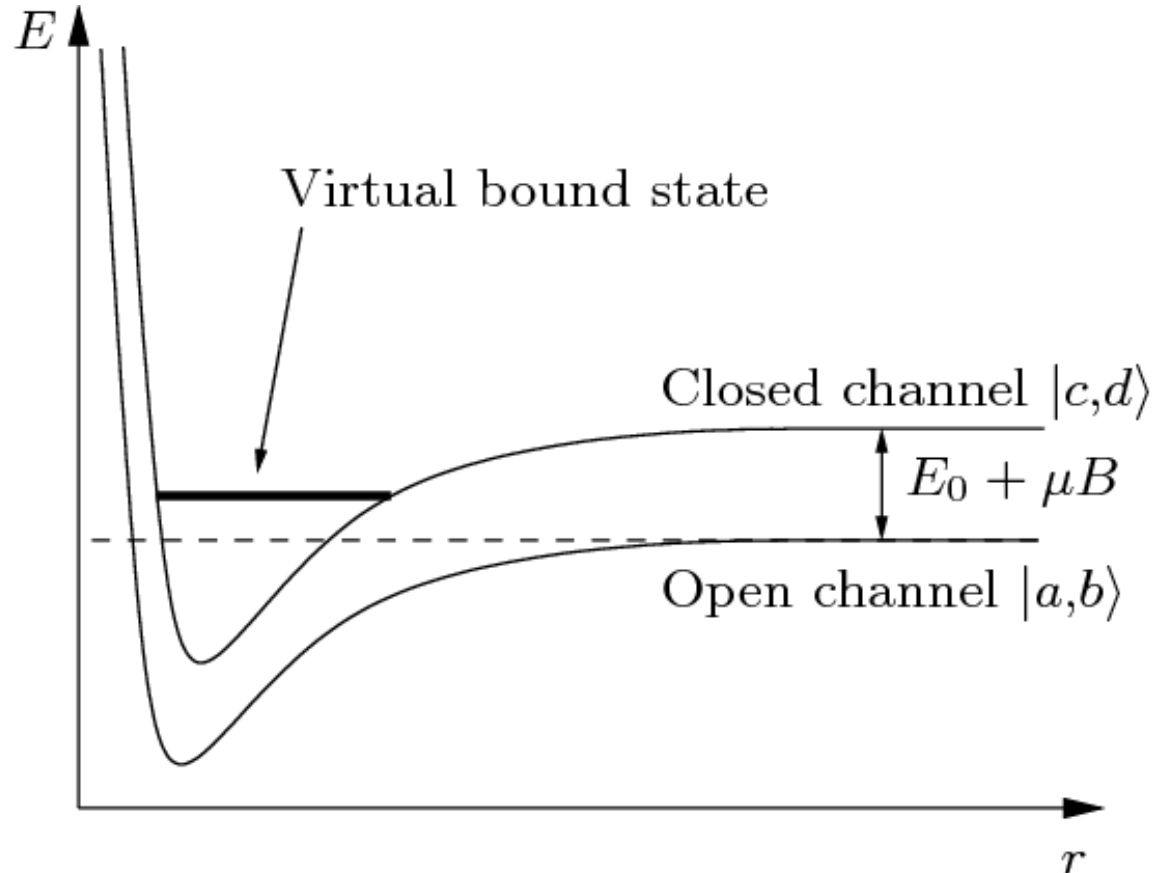
Talk outline

- ◆ Cold atom systems
- ◆ The Fulde-Ferrel-Larkin-Ovchinnikov (FFLO) instability
- ◆ Analytical approaches followed
- ◆ Results for uniform and trapped systems
- ◆ Conclusions

Cold atom gases (I)

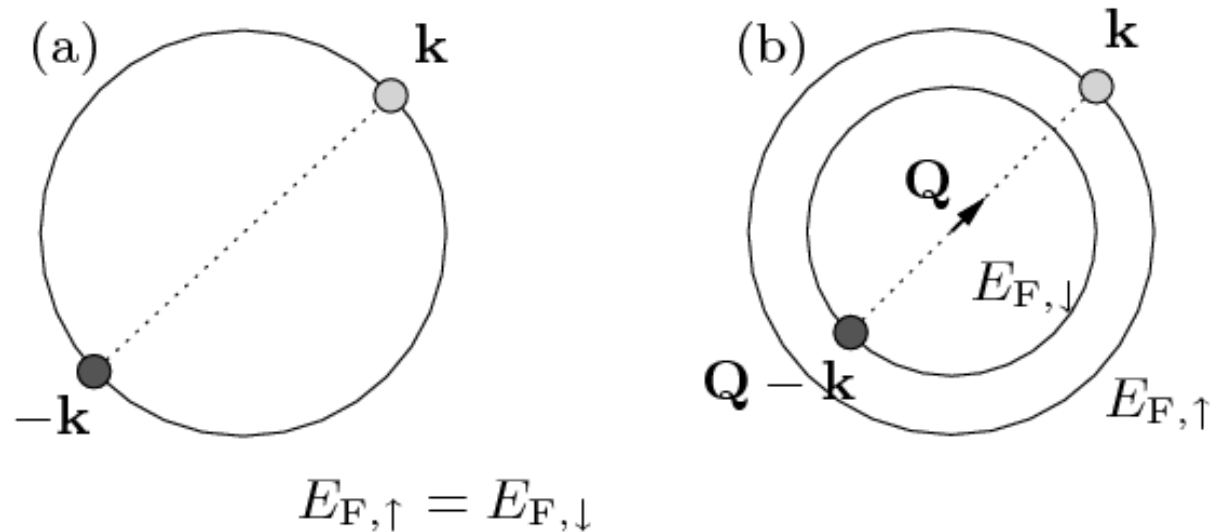
- ◆ Fermionic alkali atoms e.g. ${}^6\text{Li}$ are trapped by lasers and cooled
- ◆ Two contributions to spin: nucleus and valence electron
- ◆ A good quantum number is total projected spin
- ◆ The electron's interaction with the magnetic field dominates
- ◆ In the isolated atom limit get spin-up particles
 $|a\rangle = |m_{fa} = 1/2\rangle \approx |m_s = -1/2, m_l = 1\rangle$, with a bit of $|m_s = 1/2, m_l = 0\rangle$
- ◆ And spin-down particles
 $|b\rangle = |m_{fb} = -1/2\rangle \approx |m_s = -1/2, m_l = 0\rangle$, with a bit of $|m_s = 1/2, m_l = -1\rangle$
- ◆ In the dense limit the scattering operator is not diagonal in the states $|a, b\rangle$ and $|c, d\rangle$ so scattering can occur from $|a, b\rangle$ (open channel) into $|c, d\rangle$ (closed channel)

- ◆ States $|a, b\rangle$ and $|c, d\rangle$ magnetic moments differ by μ so relative energies are shifted by μB
- ◆ In a Feshbach resonance a bound state of the closed channel is brought into resonance with the open channel, affecting particle scattering
- ◆ Can have atoms with different masses
- ◆ Any population imbalance is maintained
- ◆ System is in quasiequilibrium, actual equilibrium is a solid



FFLO instability

- ◆ In strong binding limit have a Bose condensate, weak binding gives Cooper pairs, in the intermediate regime a modulated phase is possible
- ◆ BCS Cooper pairs have no total momentum (a)
- ◆ A population imbalance (or a ratio of masses) means Cooper pairs have a non-zero total momentum (b)
- ◆ This Fulde-Ferrel-Larkin-Ovchinnikov (FFLO) instability results in a textured state
- ◆ Inferred in superconductors with external magnetic field



- ◆ Similarities to electron-hole bilayers, where electron/holes are the normal phase and excitons the superfluid

Ginzburg-Landau approach

- ◆ Expand thermodynamic potential Φ in terms of an order parameter Δ_q to quadratic order over all wave vectors \mathbf{q}

$$\Phi = \sum_{\mathbf{q}} \alpha_{\mathbf{q}} |\Delta_{\mathbf{q}}|^2$$

- ◆ If coefficient $\alpha_{\mathbf{q}}$ is negative, it is favourable for $\Delta_{\mathbf{q}} \neq 0$ -- an FFLO instability
- ◆ Get analytical results for phase boundaries but it cannot pick up first order transitions

Single Fourier component approach

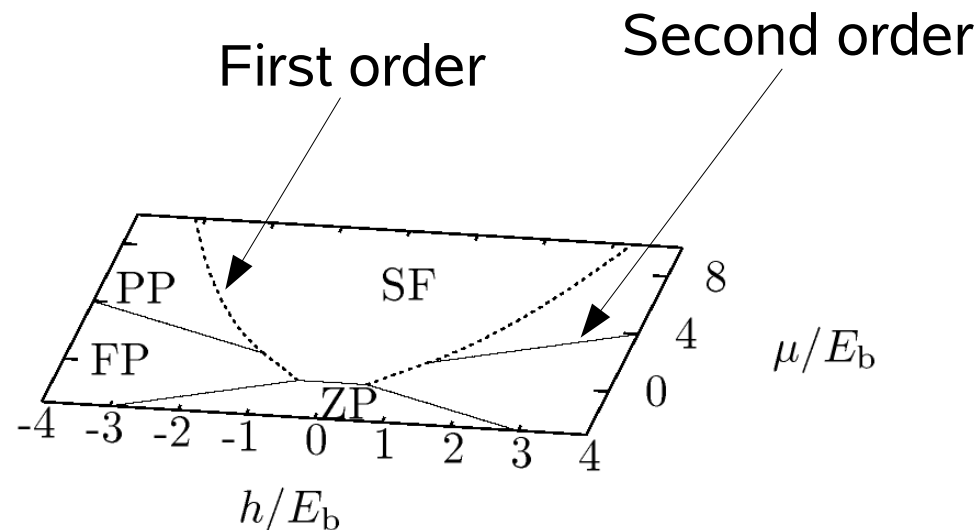
- ◆ Consider just a single wave vector \mathbf{Q} and minimise exact thermodynamic potential with respect to that wave vector and the order parameter $\Delta_{\mathbf{Q}}$

$$\Phi(\Delta_{\mathbf{Q}})$$

- ◆ Distinguishes between first and second order transitions but results are numerical
- ◆ The $\mathbf{Q}=\mathbf{0}$ state can be evaluated analytically

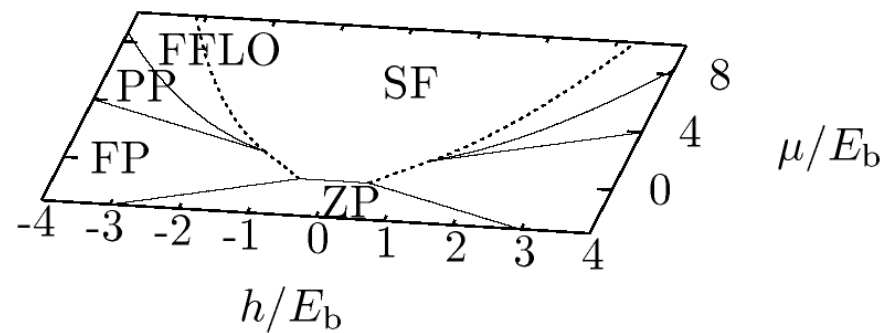
Free system: $Q=0$

- ◆ Equal masses $m_{\uparrow}=m_{\downarrow}$, population imbalanced system with $\mu_{\uparrow}=\mu+h$ and $\mu_{\downarrow}=\mu-h$
- ◆ The superfluid (SF), partially polarised normal (PP), fully polarised normal (FP), phases and the system containing no particles (ZP) are shown
- ◆ There is a first order phase transition from the normal into the superfluid phase (dotted line)
- ◆ The transitions between normal phases are second order and are straight lines (solid) with $\mu=\pm h$
- ◆ In constant n system would see phase separated SF and PP normal state



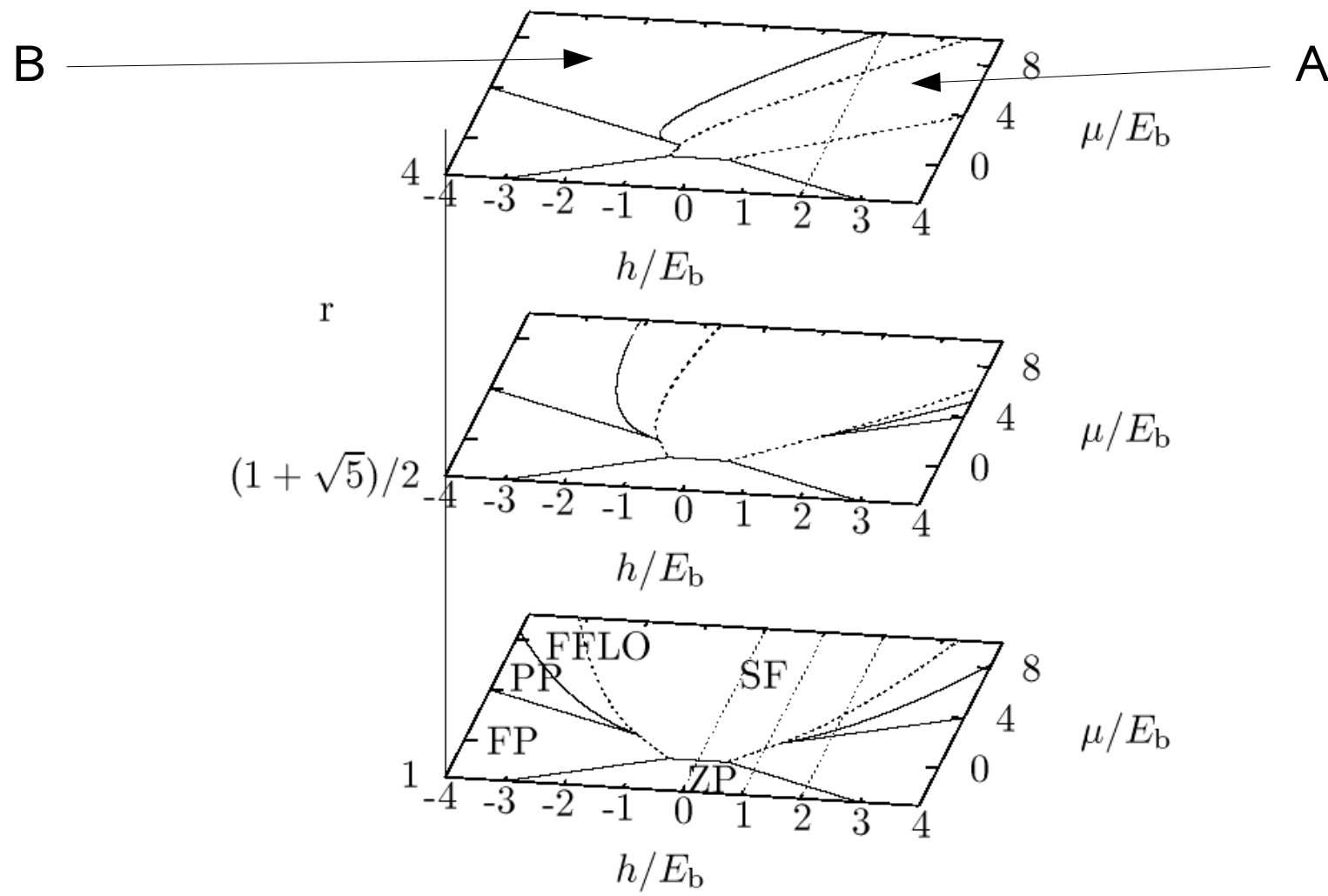
Free system: $Q \neq 0$

- ◆ The FFLO instability encroaches into the partially polarised normal state (PP) but not the superfluid
- ◆ Second order transition from partially polarised normal state (PP) into FFLO instability
- ◆ First order transition from FFLO instability into superfluid (SF) state
- ◆ No FFLO instability in fully polarised state (FP) as there are no minority spin particles
- ◆ In constraint n system see FFLO rather than a phase separated region



Free system: changing mass ratio r

- ◆ Generalise to allow different particle masses where $r = m_{\downarrow}/m_{\uparrow}$ and chemical potentials $\mu_{\uparrow} = \mu + h$ and $\mu_{\downarrow} = \mu - h$
- ◆ Superfluidity is favoured if the light species is in excess
- ◆ A trap has a varying effective chemical potential $\mu(\mathbf{r}) = \mu_0 - V(\mathbf{r})$, corresponding to straight line (dotted) trajectories



- ◆ Ginzburg-Landau and single Fourier component approaches were used to derive analytic expressions for phase boundaries in a 2D fermionic atomic gas
- ◆ Superfluidity is favoured if the light species is in excess and the FFLO instability was seen
- ◆ In a trapped system a superfluid could be bordered on one or both sides by normal phases
- ◆ Thanks to Ben Simons and Peter Conlon