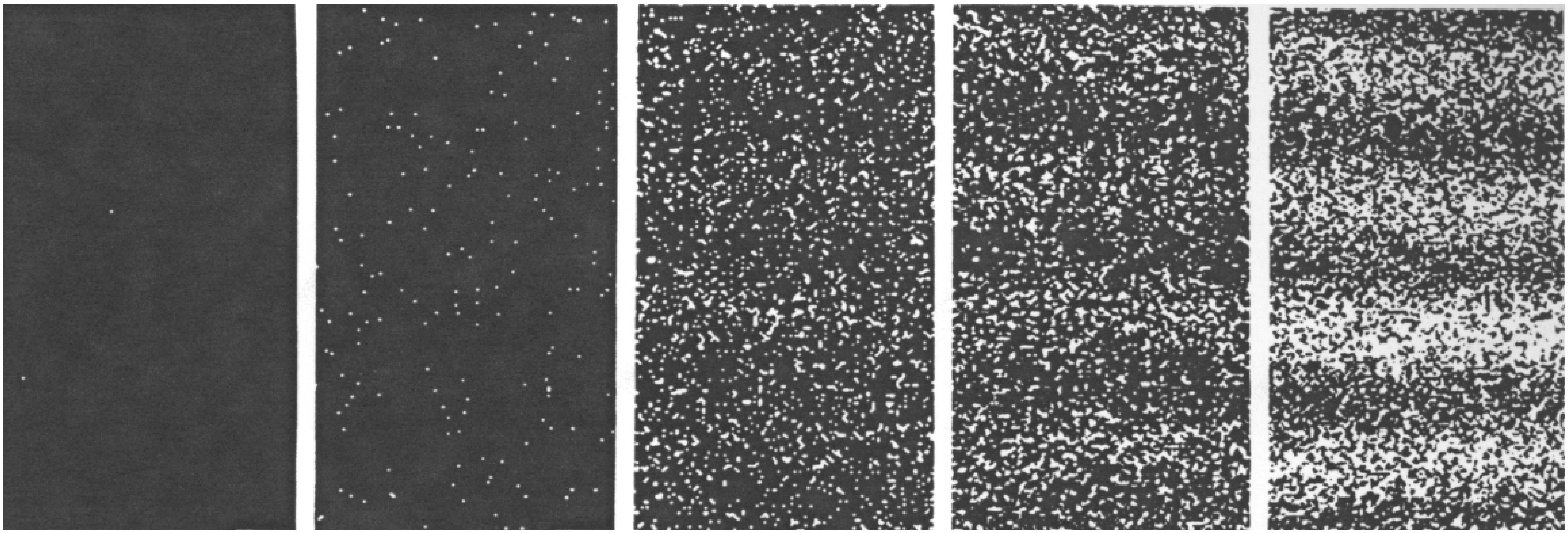
The return of pilot waves

Or, why Bohr, Heisenberg, Pauli, Born, Schrödinger, Oppenheimer, Feynman, Wheeler, von Neumann and Einstein were all wrong about quantum mechanics.

Cambridge University Physical Society, 21st October 2009



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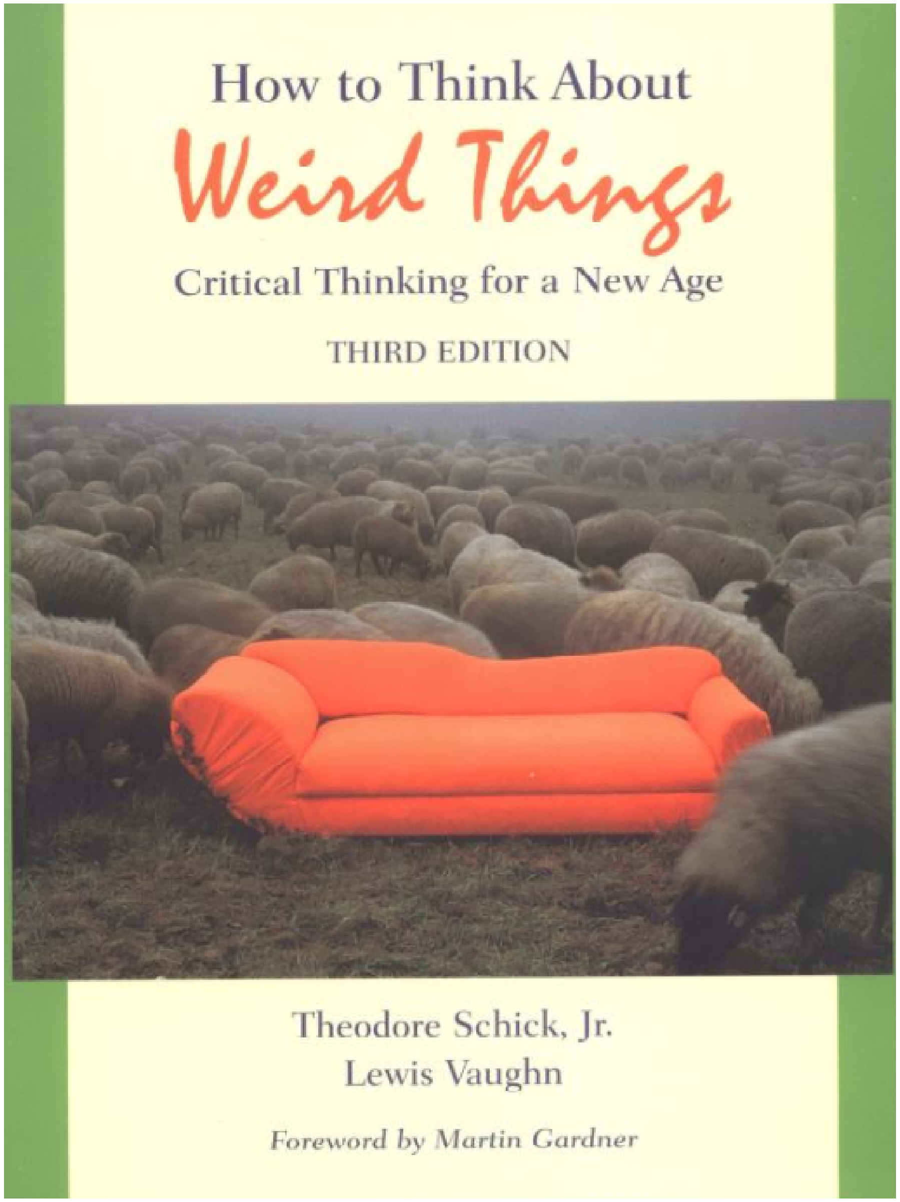
Quantum mechanics is not only weirder than you think..
..it's weirder than you can possibly imagine!

Or so they say. The number of exclamation marks varies.

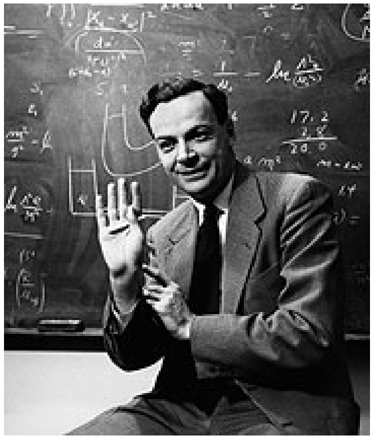


For example, in this book one reads:

"Something strange is going on in physics... This weirdness is taking place in the branch of physics known as quantum mechanics... The notorious weirdness is this: In the quantum realm, particles don't acquire some of their characteristics ***until they're observed by someone****. They seem not to exist in a definite form until scientists measure them.. It has caused some people to speculate that reality is subjective... that the universe is a product of our imagination."*

How many of you believe this?

Two-slit experiment with electrons



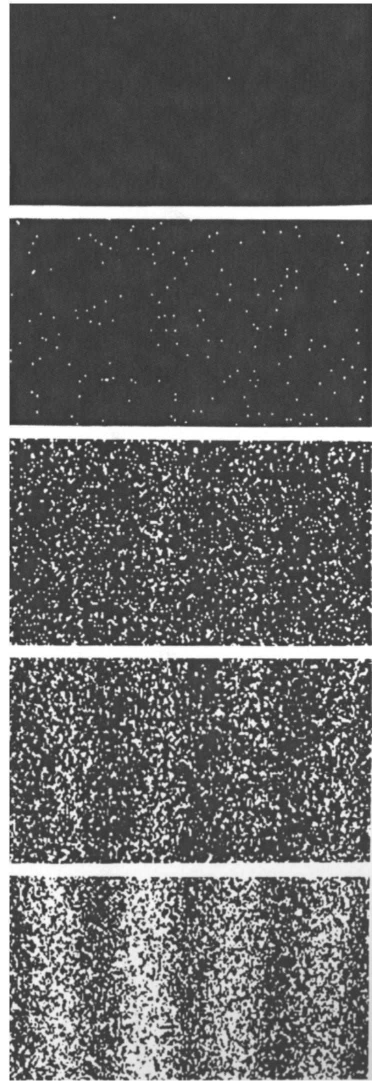
- *"A phenomenon which is impossible, absolutely impossible, to explain in any classical way, and which has in it the heart of quantum mechanics. In reality it contains the only mystery."*

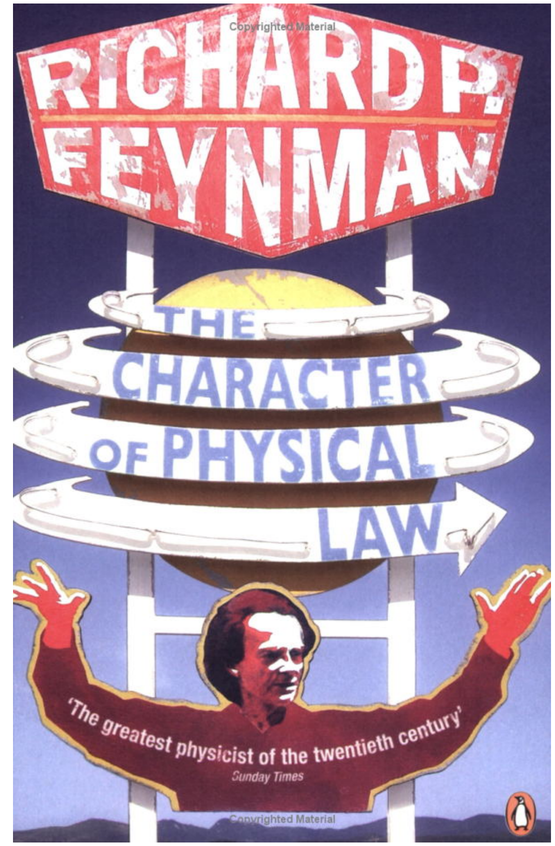
- *"Do not keep saying to yourself, if you can possibly avoid it, 'But how can it be like that?' because you will get 'down the drain,' into a blind alley from which nobody has yet escaped. Nobody knows how it can be like that."*

- *"Many ideas have been concocted to try to explain the curve for* P_{12} [that is, the interference pattern] *in terms of individual electrons going around in complicated ways through the holes. None of them has succeeded."*

- This experiment *"has been designed to contain all of the mystery of quantum mechanics, to put you up against the paradoxes and mysteries and peculiarities of nature one hundred per cent."*

- *"How does it really work? What machinery is actually producing this thing? Nobody knows any machinery. Nobody can give you a deeper explanation of this phenomenon than I have given; that is, a description of it."*





Feynman spends 19 pages of *The Character of Physical Law* discussing this one experiment. His conclusions (see opposite) are startling. You have just seen a video of the gradual build-up of the pattern, electron by electron. What do you think?

What is quantum theory? The instrumentalist view

Quantum mechanics (QM) is the theory we use to study matter at the atomic level. It takes the form of a probability calculus for calculating the results of experiments. It is essentially an *instrumentalist* theory, i.e. a theoretical 'instrument' or 'tool' for making predictions of the possible results of experiments on quantum systems.

To a *pragmatist*, who wishes to make money out of building quantum devices and flogging them, this is quite sufficient. However, out of basic curiousity, many people wish to *understand and explain* the behaviour of the universe. One way to approach this problem is to 'interpret' QM. How or why does it work? What, if anything, do the mathematical objects in the theory represent?

Definition: *An 'interpretation of quantum mechanics' is a statement which attempts to explain how QM informs our understanding of nature.*

There are a number of contending schools of thought, differing over whether QM can be understood to be deterministic, which elements of QM can be considered 'real', etc. However, until the 1980s, there was only one interpretation that was taken seriously; thinking otherwise could easily damage people's careers (still true today?).

"In recent years the debate on these ideas has reopened, and there are some who question what they call 'the Copenhagen interpretation of quantum mechanics' - as if there existed more than one possible interpretation of quantum mechanics." [Rudolf Peierls, 1979].

The ‘orthodox interpretation’ of QM : Copenhagen

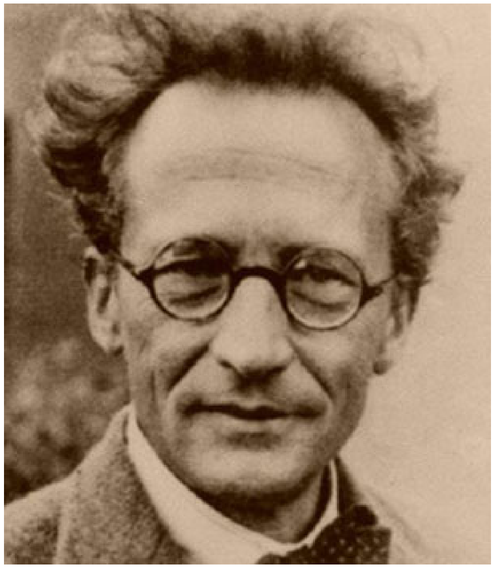
“A philosophical extravaganza dictated by despair” (Schrödinger)

1. System *completely described* by wave function Ψ usually taken to represent observer’s knowledge of it, or ‘potentiality’. Ψ evolves in time according to Schrödinger’s equation, except when it doesn’t.
2. Nature is *fundamentally* probabilistic. Probability of event given by absolute square of Ψ (Born rule). ‘Measurement’ has special status and randomly picks out exactly one of the many possibilities allowed for by the state’s wave function through nonlocal ‘collapse process’. ‘Hidden variables’ distinguishing systems with identical Ψ (and possibly restoring determinism) are *impossible*.
3. Heisenberg’s uncertainty principle: observed fact that it is not possible to know values of all properties of system at same time; those properties not known with precision must be described by probabilities. Conclude properties are *indeterminate* not uncertain.
4. Complementarity principle: there is no logical picture (obeying classical propositional logic) that can simultaneously describe and be used to reason about all properties of a quantum system. Example: matter exhibits a wave-particle duality. An experiment can show the particle-like properties of matter, or wave-like properties, but not both at the same time. (Niels Bohr)
5. Measuring devices are *classical*, and measure classical properties like momentum. QM description of *large* systems must closely approximate classical description (‘correspondence principle’).

Don’t confuse this with instrumentalism/pragmatism.

Now well-known that Copenhagen cannot be reconstructed as a coherent philosophical framework - it is a collection of local, often contradictory, arguments embedded in changing theoretical and sociopolitical circumstances.. ..riddled with vaccillations, about-faces and inconsistencies. [See Mara Beller book ‘*Quantum Dialogue*’]

Schrödinger's equation



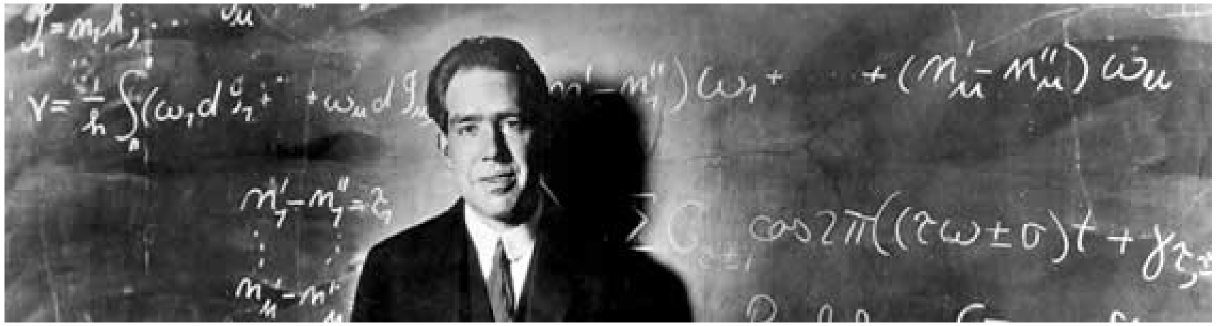
$$\boxed{i\hbar\frac{\partial\Psi(\mathbf{x},t)}{\partial t} = \hat{H}\Psi(\mathbf{x},t) = -\frac{\hbar^2}{2m}\nabla^2\Psi(\mathbf{x},t) + V(\mathbf{x},t)\Psi(\mathbf{x},t)}$$

where $\mathbf{x} = \{\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_N\}$ is a point in the *configuration space* of the system.

Note that $|\Psi(\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_N)|^2$ is generally interpreted as the probability of *finding* particles at positions $\mathbf{x}_1, \ldots, \mathbf{x}_N$ etc. in a 'suitable measurement' (Born rule). Between measurements, the particles 'do not have positions' in the Copenhagen view.

"*It seems a little paradoxical to construct a configuration space with the coordinates of points that do not exist.* [L. de Broglie, Solvay conference, 1927]

An undesired heritage



Physics is about measurement and nothing else.

Measurements can be described as 'the reading of macroscopic apparatus states', a problem clearly concerned with the *classical limit*.

All quantum foundations talks have Bell quotes

"*The problem of measurement and the observer is the problem of where the measurement begins and ends, and where the observer begins and ends. . . . I think that – when you analyse this language that physicists have fallen into, that physics is about the results of observation – you find that on analysis it evaporates, and nothing very clear is being said.*"

J.S. Bell (1986, Interview in Davis and Brown's *The Ghost in the Atom*)

The measurement problem in orthodox QM

- **1**. Assumption of classical background leads to undefinable division between microscopic and macroscopic worlds.

- **2**. Measurement itself apparently not physical process describable in purely quantum-theoretic terms. Observers must be added as extra-physical elements.

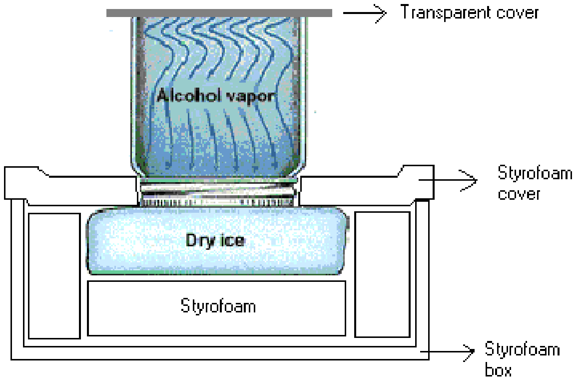
- **3**. Schrödinger evolution of Ψ in time gives linear superposition of all possibilities for ever. When correlated with measuring apparatus, get a *macroscopic* superposition of quantum states, which is not what one sees. And which means what, exactly? (Schrödinger's cat problem).

- **4**. Need to postulate non-local 'collapse' in which time-dependent wave function suddenly stops obeying Schrödinger eqn and does something else when 'observed'.

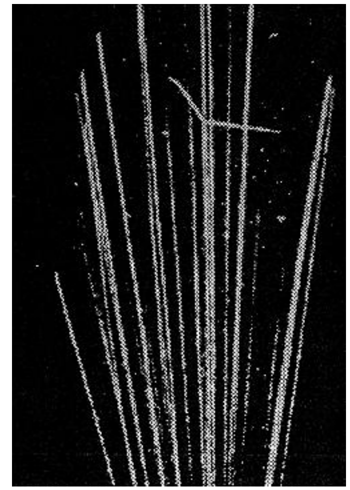
- **5**. Not possible to use standard QM in cosmological problems.

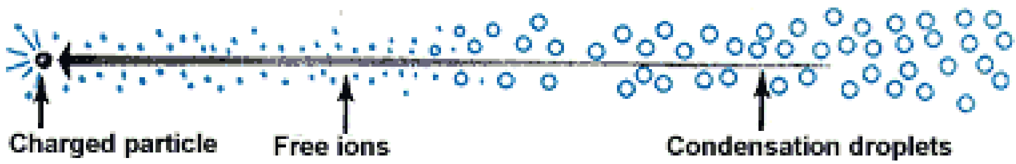
NB: *What exactly is considered a 'problem' depends fundamentally on whether you believe wave function is a real object that is part of structure of individual system, or it represents 'knowledge of the system'(whose?), or it is merely a mathematical tool for calculating and predicting the measured frequencies of outcomes over an ensemble of similar experiments.*

Cloud chamber



Sealed chamber with supersaturated vapour kept near condensation point by regulating T. Ionizing radiation leaves trail of charged ions that serve as condensation centers. Vapour condenses around them. Radiation *path* thus indicated by tracks of tiny liquid droplets in supersaturated vapour.





- If α-particle emission undirected - so emitted Ψ spherical - how account for straight particle track revealed by cloud chamber? Intuitively would think it ionizes atoms at random throughout space.
- If only α-particle quantum (only its coords in Ψ) vapour is 'external measuring equipment'. On producing visible ionization α-particle wave packet 'collapses' then spreads until more visible ionization then collapse occurs again etc. Prob for resulting 'trajectory' concentrated along straight lines. Similar result [Mott, 1929] if consider interaction in configuration space with all atoms.
- So in standard QM trajectories emerge only at macroscopic level and are constructed by successive wave packet collapses. Works only because α-particle largely 'classical' (has billions of eV but requires only a few eV to ionize one atom so preserves its identity). But why can't macroscopic trajectories simply be a consequence of microscopic trajectories?

Schrödinger's Cat

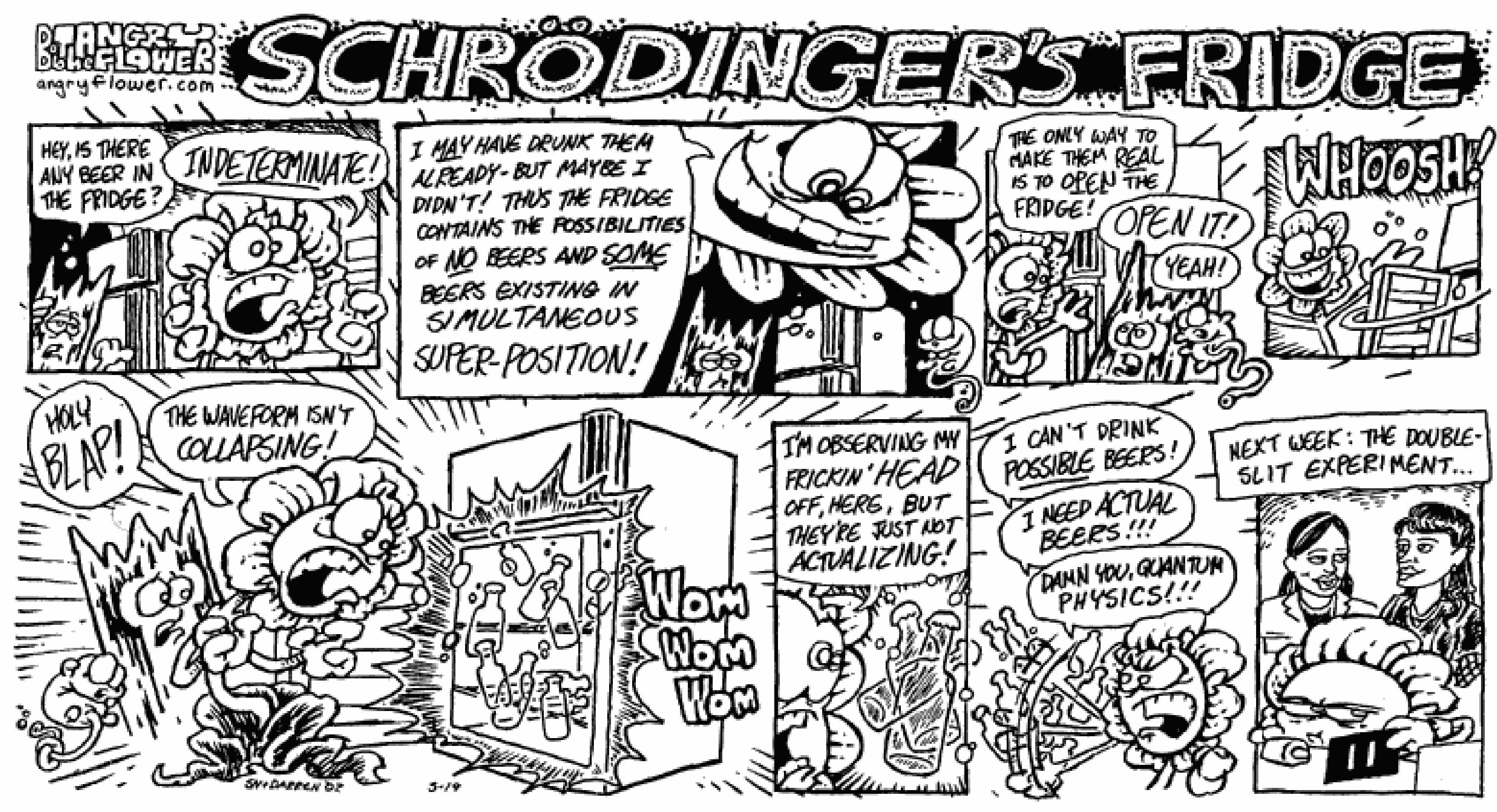
Schrödinger's Cat

Schrödinger's Cat

Schrödinger's Cat

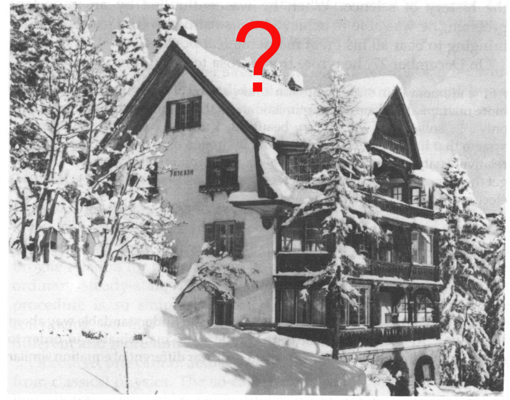
Schrödinger's Cat

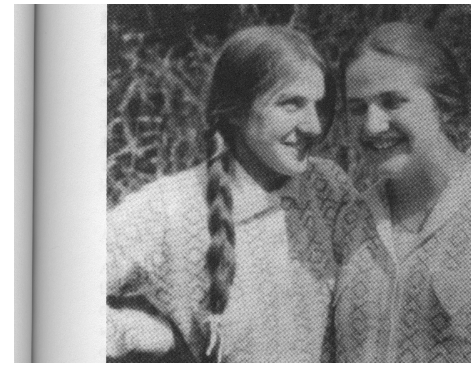
Schrödinger's Fridge



Schrödinger's Woman (and Wigner's Friend)

Choose one from: wife, 'dark lady of Arosa', teenage nymphette twins Ithi and Withi.





So put - say - Ithi in a box, with an experiment to do involving microscopic system initially in superposition of energy states $\psi = \frac{1}{\sqrt{2}}(|E_1\rangle + |E_2\rangle)$. Schrödinger (unlucky!) is outside the box. Ithi does experiment and presumably finds one outcome E_1 or E_2 but from Schrödinger's view she is in a superposition of *girl found* E_1 and *girl found* E_2. Paradox is contradiction between following statements (referring to ensembles) concerning physical state of Ithi just before Schrödinger decides what to do next:

I: There is no definite state of Ithi, because Schrödinger can if he wishes perform measurements showing the presence of interference between different states.
II: There is a definite state of Ithi, because Ithi is human, and instead of testing for interference Schrödinger can simply open the box and ask Ithi what she saw.

Orthodox QM leads one to suppose that Ithi "was in state of suspended animation before she answered the question" (Wigner) since assume reality of other minds (no solipsism!) and conscious beings ought always to have definite state of consciousness.

Basic problem - what is the wave function?

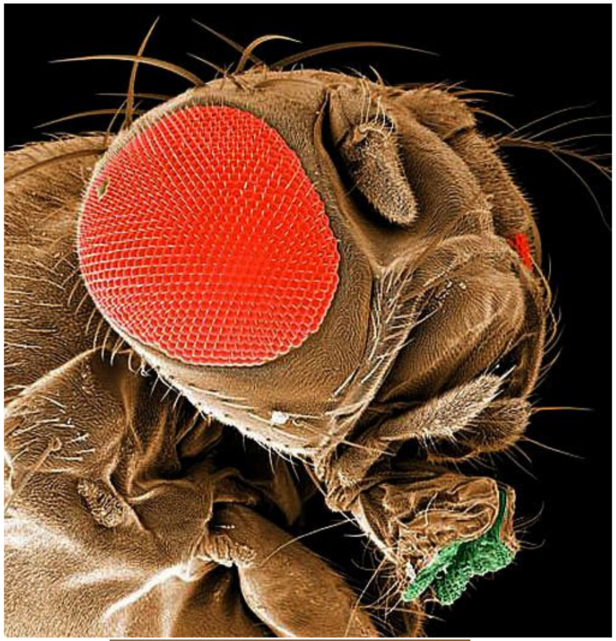
What is Ψ? The most puzzling object in quantum mechanics!

- Is it subjective or objective?
- Does it represent *information/knowledge* (whose?) or an observer-independent real *wave field?*
- If it is objective, does it represent a concrete material sort of reality, or does it somehow have an entirely different and perhaps novel nature?
- What's going on with the collapse?

Very little agreement about the answers to these questions. *However, the idea that Ψ represents information, and does not describe an objective state of affairs, raises many questions and problems*:

- Information about what? Ψ somehow captures or contains information about, but does not directly describe, whatever is physically real. But then it should be possible to formulate the theory without even *mentioning* Ψ - which, after all, according to the theory, doesn't actually *exist*.
- Quantum interference? How can terms of a quantum superposition interfere with each other, producing observable interference pattern, if such superpositions just expression of our ignorance?
- Problem of vagueness: QM supposed to be fundamental physical theory. As such it should be precise. But if fundamentally about information, then presumably concerned directly either with mental events or with behaviour of macroscopic variables. But 'macroscopic' intrinsically vague.
- Simple physical laws to be expected, if at all, at the most fundamental level of the basic microscopic entities. Messy complications should arise at level of larger complex systems. Only at this level should talk of *information*, as opposed to microscopic reality, become appropriate.
- The very form of the Hamiltonian and wave function strongly points to a microscopic level of description. Why else $\Psi(\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_N)$?

The killer experiment: How to cripple a fruit fly



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Differential Toxicity of Carbon Nanomaterials in *Drosophila*: Larval Dietary Uptake Is Benign, but Adult Exposure Causes Locomotor Impairment and Mortality

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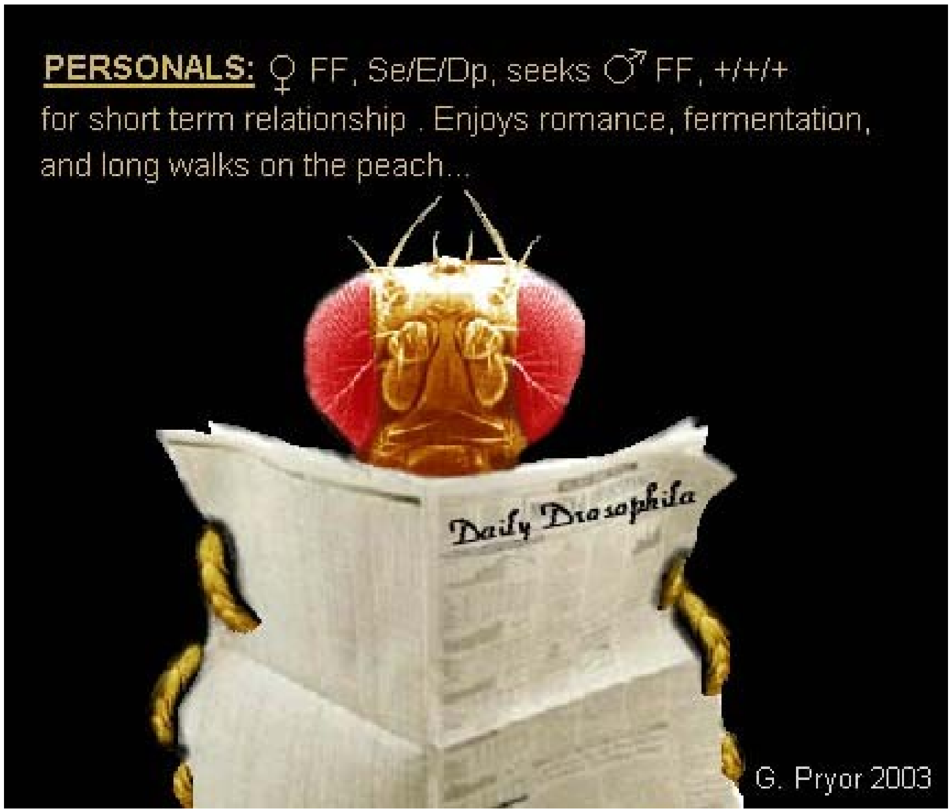
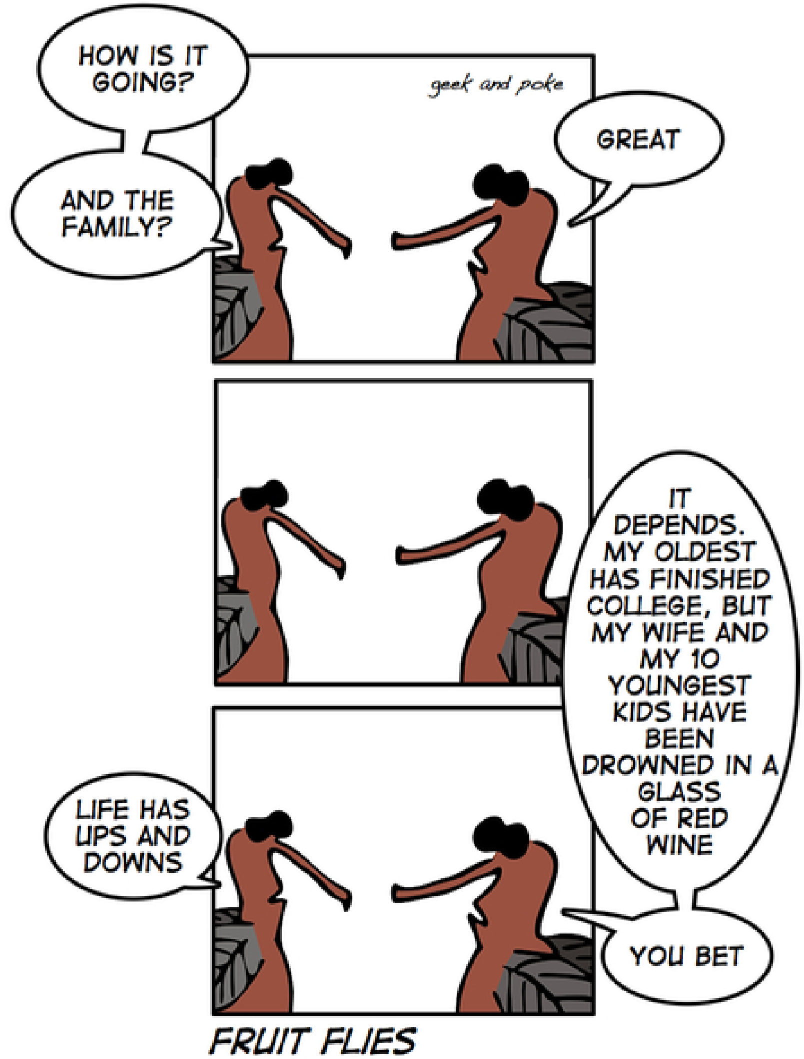
Put fullerene molecules on their feet, and they lose their ability to climb walls!

But fullerenes are quantum particles which can be diffracted in single-molecule interferences experiments, just like electrons and single atoms [see e.g. Nairz, Arndt, Zeilinger, *Am. J. Phys.* **71**, 319 (2003)]. So it seems that if only the flies had the good sense to avoid looking at their own feet, then this wouldn't be a problem.

Some fruit fly jokes

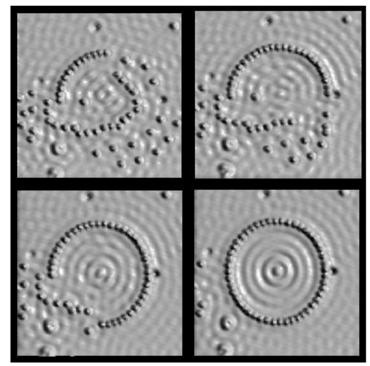




1927 was a long time ago

Orthodox Copenhagen QM is both an algorithm for obtaining statistical predictions for the results of experiments and a prescription for *avoiding fundamental questions*. Bohr *et al.* designed it that way because in 1927 quantum entities were unobservable and thus [non sequitur] not real: "*. . . the idea of an objective real world whose smallest parts exist objectively in the same sense as stones or trees exist, independently of whether or not we observe them . . . is impossible.* " [Heisenberg, 1958]

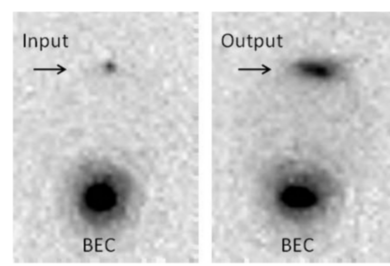
However, modern technological progress shows essentially without doubt that quantum entities exist whether we 'observe' or do experiments with them, or not.



Single atoms and even electrons can be isolated and trapped in containment vessels for long periods. Can repeat examination many times and get same data. Individual atoms can be 'pushed around' and arranged into patterns (which can also be imaged). These experiments all yield *consistent* results and information about quantum entities using a variety of techniques and under different conditions.

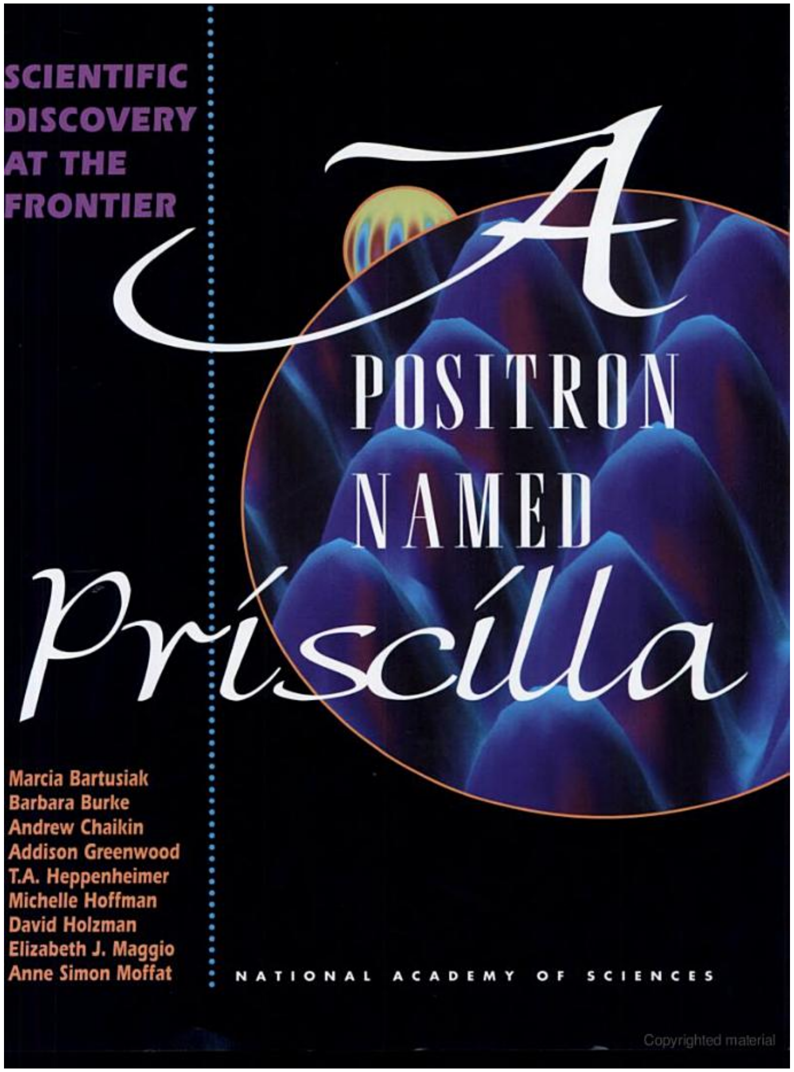
"*Perhaps the most convincing proof of the reality of the quantum world would be to capture some of its creatures and hold them in place for all to see. This has become feasible.*" [Ho-Kim *et al.*, 2004]

Clear evidence for **wave field existence** from *matter wave optics*. Ultracold atomic gases have dominant wave behaviour. Can manipulate by 'optical devices'. Significant quantities of matter diffracted, focussed, reflected etc. Also 'matter wave amplification' experiments: production of output of atoms with particular properties from BEC reservoir of atoms in a trap using process similar to stimulated emission of light in a laser. If matter wave can be subject to and utilized in such a process, it logically follows matter wave must exist in order to act and be acted upon.



Many older physicists get really angry about this and clearly will never overcome the dominant thought patterns of the prevailing paradigm of orthodox quantum theory (such as the denial of an independently existing quantum realm). Sadly, it will almost certainly be necessary to wait for them all to die.

The impossible observed

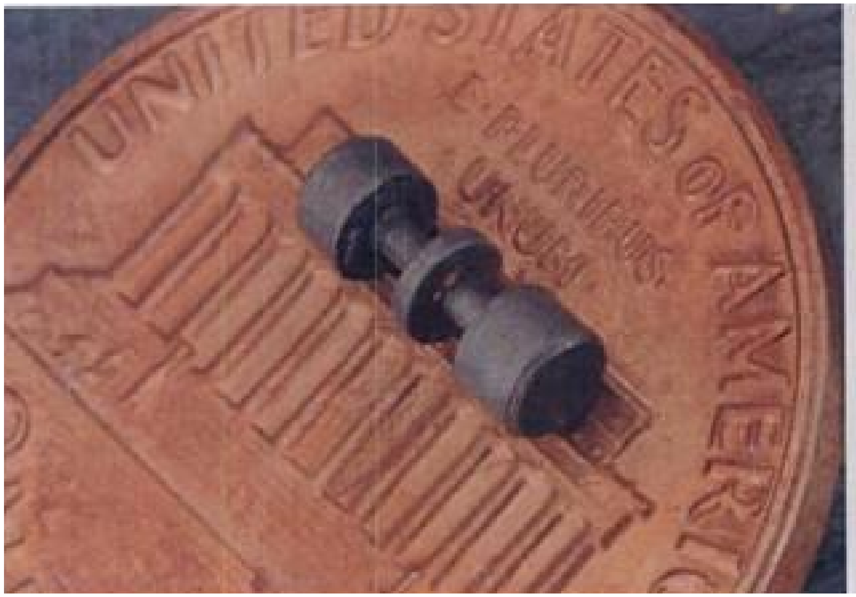


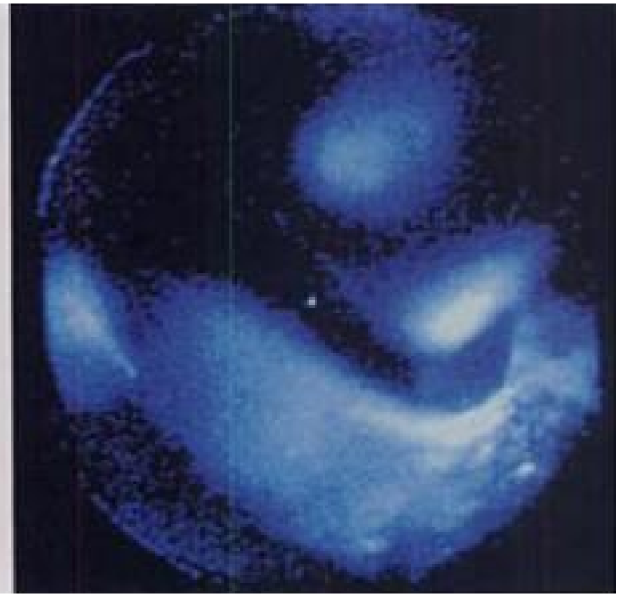
Hans Dehmelt has carried out exquisitely precise studies of *individual electrons and positrons* - including measuring their magnetic moments to twelve decimal places - by capturing them in electromagnetic traps. Because positrons do not exist naturally on Earth, he showed that the particular positron under study had no opportunity to swap with a different one. He has held that particle in place for as long as three months. He writes "*The well-defined identity of this elementary particle is something fundamentally new, which deserves to be recognized by being given a name, just as pets are given names of persons.*" So he called her **Priscilla**, and won the Nobel prize for it in 1989.

Bohr, Pauli and other Copenhagenists had conclusively proved in 1928 that the magnetic moment of a free electron could never be observed (an argument still being defended up to 1985). '*From these arguments we must conclude that it is meaningless to assign to the free electron a magnetic moment*'. Today this quantity may be the best measured number in all of science.

Moral: Be modest about the implications of your theories and never underestimate the cleverness of experimentalists.

Astrid the atom





Dehmelt also trapped a single barium atom that he named Astrid, and kept it floating like a pixie in a tiny ion-trap vacuum chamber for ten months. Under suitable conditions, she turned out to be visible to the naked eye..

It used to be claimed that no-one could ever see an atom with their naked eyes. The mistake here is assuming that *smallness* is the important issue; actually *brightness* and *isolation from other atoms* are what matters. A laser-stimulated barium atom produces 10^8 photons per second; your eyes can collect several thousand. The normal retina is sensitive to even a few photons, so you can see the atom, just as you would a distant star or any other bright, isolated object.

An alternative view: hidden variables

Do experiment on ensemble of systems with 'identical state preparation' (i.e. each system prepared with same Ψ). Initially, nothing to distinguish any one system from any other. Nevertheless, results are (say) particle positions randomly distributed as Ψ^2 over the ensemble. Copenhagenists conclude Nature 'inherently probabilistic'. However there is another way: imagine QM '*incomplete*' (as Einstein repeatedly insisted): then there is some 'hidden variable' making each system different from the outset.

For example, say electrons are particles with definite position at all times (hardly revolutionary!). Then ψ^2 represents distribution of particles with imperfectly known positions (the 'hidden variables'). 'Identical state preparation' then means 'choosing starting positions from a fixed probability distribution' with consequent randomness on hitting the screen. If it were possible to derive such a theory, suggests much of apparent peculiarity of QM arose from *mistaking an incomplete description for a complete one*. De Broglie and Bohm ('pilot wave theory') worked along these lines - as we shall see..

Recall that 100+ years ago, an important step took place (Boltzmann, Maxwell, Gibbs, Einstein) when classical thermodynamics was *derived* from microscopic physics, from the behaviour of the constituents of the macroscopic systems (very controversial at the time! Mach etc.). A hidden variable derivation of QM would be exactly equivalent to that. Is this possible?

Footnote: Even if they don't admit it, one would think that almost all quantum physicists ought to believe in hidden variables. Framed in QM terms, there are only two other alternatives:

(1) Ψ and only Ψ exists (and thus doesn't just represent 'knowledge' or whatever).
(2) solipsism (only your mental processes exist).

If you believe **neither** of these things, then you believe in hidden variables. Don't you? However..

Hidden variables impossible since lots of famous people say so..

"*The idea of an objective real world whose smallest parts exist objectively in the same sense as stones or trees exist, independently of whether or not we observe them... is impossible.*" [Heisenberg, 1958]

"*Every attempt, theoretical or observational, to defend such a hypothesis (the notion of hidden variables supplementing the wave function description) has been struck down.*" [J.A. Wheeler (1983)]

"*It is clear that* [the double slit experiment] *can in no way be reconciled with the idea that electrons move in paths. In quantum mechanics there is no such concept as the path of a particle.*" [Landau and Lifshitz, *Quantum Mechanics* textbook, 1977].

"[The quantum postulate] *implies a renunciation of the causal space-time coordinates.*" [Bohr, 1934]

"*No concealed parameters can be introduced with the help of which the indeterministic description could be transformed into a deterministic one. Hence if a future theory should be deterministic, it cannot be a modification of the present one but must be essentially different. How this could be possible without sacrificing a whole treasure of well-established results I leave to the determinists to worry about.*" [Born, 1949]

"*How does it really work? What machinery is actually producing this thing? Nobody knows any machinery. Nobody can give you a deeper explanation of this phenomenon than I have given; that is, a description of it.*" [Feynman, 1965]

There's more..

"*In contrast to ordinary mechanics, the new quantum mechanics does not deal with a space-time description of the motion of atomic particles... The difficulties* $\cdots$ *seem to require just that renunciation of mechanical models in space and time which is so characteristic a feature in the new quantum mechanics.*" [Bohr 1934]

"*..We consider it juvenile deviationism .. we don't waste our time ...* [by] *actually read*[ing] *the paper. If we cannot disprove Bohm, then we must agree to ignore him.*" [Oppenheimer, 1953. Abraham Pais also referred to 'juvenile deviationism'.]

"..[Bohm] *is a public nuisance.. a Trotskyite and a traitor*" [Princeton Institute, 1953]

"..[Bohm's work is] *a short-lived decay product of the mechanistic philosophy of the nineteenth century*" [Rosenfeld]

"*[It] is understandable that the pioneer who advances in unknown territory does not find the best way at the outset; it is less understandable that a tourist loses his way again after this territory has been drawn and mapped in the twentieth century.*" [Rosenfeld, 1952, referring to Bohm who had just been exiled to Brazil.]

"*To hope for hidden variables is as ridiculous as hoping that* $2 \times 2 = 5$.*"* [Heisenberg]

To cap it all, much to everyone's delight, von Neumann gave a *mathematical proof* that hidden variables are *impossible*.

Laying the boot in

Bohrian rhetoric of finality and inevitability: '*We see that it cannot be otherwise*', '*This is something there is no way round*', '*The situation is an unavoidable one*', [complementarity] is '*most direct expression of a fact..as the only rational interpretation of quantum mechanics*', '*obvious*', '*evident*', '*clear from the outset*', '*a simple logical demand*', '*we must recognize*', '*it is imperative to realize*'. **(Circular) demonstrations of consistency disguised as compelling arguments of inevitability**. [Those who do not agree are] '*unable to face the facts*' and disagreeing with the masters of the universe thus becomes bad for your career..

The resistance: last men standing

"*Bohr's approach to atomic problems is really remarkable. He is completely convinced that any understanding in the usual sense of the word is impossible. Therefore the conversation is almost immediately driven into philosophical questions, and soon you no longer know whether you really take the position he is attacking, or whether you really must attack the position he is defending.*" [Schrödinger, letter to Wien]

"[Complementarity] *is a thoughtless slogan. ... If I were not thoroughly convinced that the man* [Bohr] *is honest and really believes in the relevance of his - I do not say theory but - sounding word, I would call it intellectually wicked.*" [Schrödinger]

"*I am, in fact, rather firmly convinced that the essentially statistical character of contemporary quantum theory is solely to be ascribed to the fact that this theory operates with an incomplete description of physical systems.*" [Einstein]

The amazing thing is..

..all these famous professors (apart from the last two) were completely wrong.

"*Bohr brainwashed a whole generation of physicists into believing that the problem had been solved.*" [Murray Gell-Mann]

"*In 1952 I saw the impossible done. It was in papers by David Bohm. Bohm showed explicitly how parameters could indeed be introduced, into nonrelativistic wave mechanics, with the help of which the indeterministic description could be transformed into a deterministic one. More importantly, in my opinion, the subjectivity of the orthodox version, the necessary reference to the "observer," could be eliminated. ... But why then had Born not told me of this "pilot wave"? If only to point out what was wrong with it? ... Why is the pilot wave picture ignored in text books? Should it not be taught, not as the only way, but as an antidote to the prevailing complacency? To show us that vagueness, subjectivity, and indeterminism, are not forced on us by experimental facts, but by deliberate theoretical choice?*" [John Bell, Speakable and Unspeakable in Quantum Mechanics, 1987]

"*As for Pais and the rest of the 'Princetitute' what those little farts think is of no consequence to me. In the past six years, almost no work at all has come out of that place.. I am convinced that I am on the right track.*" [Bohm, 1953]

So why can't electrons have trajectories?

Because von Neumann proved hidden variables were impossible. However, '*such was the momentum of the Copenhagen interpretation and von Neumann's reputation that when Grete Hermann pointed out in 1935 that the supposed proof contained a blatant and devastating fallacy, she was simply ignored, and the Copenhagen interpretation remained the almost unquestioned accepted interpretation for decades.*' It was left to Bell to rediscover the flaw in 1966.

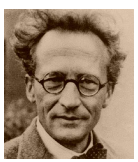
The eigenvalue-eigenstate link: position and momentum represented by operators that do not commute (and therefore cannot have complete set of common eigenstates). If accept quantity has definite value only if system is in eigenstate of corresponding observable, then particle cannot have both a well-defined position and well-defined momentum ever $\Longrightarrow$ no trajectory. Completely ambiguous! The numbers refer only to the *measured* values. Dirac said, '*measurement always causes the system to jump into an eigenstate of the dynamical variable that is being measured*', but jumped from where? Any serious analysis of a hidden variable theory shows us - sadly - that '*most of what can be measured is not real and most of what is real cannot be measured*'.

The Uncertainty Principle $\Delta x \Delta p \geq \frac{\hbar}{2}$ implies that particles cannot have simultaneously well-defined x and p $\Longrightarrow$ no trajectory. Nonsense! Now understood that Heisenberg's principle doesn't relate to measurements on individual systems. Uncertainty in the value of a dynamical variable refers to the statistical spread over the measured values for the various identical members of an *ensemble* of systems.

Atoms with *Newtonian* trajectories: a surprising observation

Classical atoms are small and we cannot know their position with certainty, so we deal with a statistical ensemble in which only the *probability density* $\rho(\mathbf{x},t)$ is known.

- Probability must be conserved, i.e. $\int \rho d^3x = 1$ for each t. Therefore must satisfy continuity equation $\partial\rho/\partial t = -\nabla\cdot(\rho\mathbf{v})$ where $\mathbf{v}(\mathbf{x},t)$ is the velocity of the particle.
- Classical mechanics has various equivalent formulations. Choose the less well-known *Hamilton-Jacobi* version, where velocity $\mathbf{v}(\mathbf{x},t) = \frac{\nabla S(\mathbf{x},t)}{m}$ and $S(\mathbf{x},t)$ - related to the 'action' - is a solution of the Hamilton-Jacobi equation, $-\frac{\partial S}{\partial t} = \frac{(\nabla S)^2}{2m} + V$.
- Can write the two green *real* equations more elegantly as single *complex* equation. Introduce a general complex function $\Psi = re^{i\theta} = \sqrt{\rho}e^{\frac{iS}{\hbar}}$ with $\hbar$ arbitrary constant giving dimensionless exponent. The two equations may then be rewritten as:



$$i\hbar\frac{\partial\Psi}{\partial t} = \left(-\frac{\hbar^2}{2m}\nabla^2 + V - Q\right)\Psi \qquad \text{with} \qquad Q = -\frac{\hbar^2}{2m}\frac{\nabla^2\sqrt{\rho}}{\sqrt{\rho}}.$$

This is the time-dependent Schrödinger equation (!) with an extra term Q. Note $|\psi(\mathbf{x},t)|^2$ has same interpretation as in QM: a probability density of particle positions. *So to recover classical mechanics from quantum mechanics we simply have to subtract out something that behaves exactly like a potential, thus implying that* **QM is just like classical statistical mechanics with a non-classical dynamics** *(due to an 'extra force' $-\nabla Q$ over and above the classical $-\nabla V$).* Er, what??

How to find a valid hidden variables theory?

We wish to change QM from a *statistical theory of observation* into a *dynamical theory of particle trajectories* (that looks like classical statistical mechanics with an extra force?), without altering the predictions that the theory makes. What changes are required?

We simply drop the usual positivistic philosophical baggage and look at the equations: in particular we stop pretending that particles don't exist when no-one is looking at them. To do this, change the meaning of one word: **probability**. $|\Psi^2(\mathbf{x},t)|^2$ is now the probability of the particle (or configuration) *being* at $\mathbf{x}$ at time t, rather than the probability of *being found there in a suitable measurement*. Has implication that particles exist continuously and have *trajectories*, independently of being observed.

In doing this we are making a *metaphysical commitment* (defining an 'ontology'). Metaphysics is not a term of abuse - it means the study of reality; you specify 'what exists' - what is QM actually about?

Because we are being honest, we must also say what we think the wave function Ψ means. We have seen that it is often claimed to represent '*information*' or 'knowledge', but it seems clear from the fact that it can be directly manipulated by essentially optical instruments that it represents something real. 'Something' wave-like passes along the different paths in an interference experiment; to refuse to call it 'real' is merely to play with words. We therefore say a *wave field* exists and this is represented mathematically by the usual QM wave function evolving according to Schrödinger's equation.

Wave-particle duality : both particles **and** wave exist!

These are the **only** *changes required to orthodox QM to get an apparently completely-valid hidden variables theory - the 'pilot-wave theory' of de Broglie and Bohm. All equations and results follow directly from the established formalism of orthodox QM.*

Particle trajectories regained: pilot wave theory

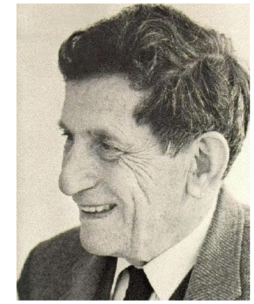
Wave field evolution from Schrödinger equation $i\hbar\frac{\partial\Psi}{\partial t} = \sum_{i=1}^{N} -\frac{\hbar^2}{2m_i}\nabla_i^2\Psi + V\Psi$. Evolving quantum system behaves like 'probability fluid' of density $|\Psi|^2 = \Psi\Psi^*$ with an associated time-dependent quantum *probability current* $\mathbf{j} = \frac{\hbar}{m}\mathrm{Im}(\Psi^*\nabla\Psi)$. Suspect particle trajectories follow streamlines of current: velocity $\mathbf{v} = \frac{\hbar}{m}\mathrm{Im}\nabla\ln\Psi$ (current/density). Using complex polar form $\Psi = |\Psi|\exp[iS/\hbar]$, the wave function *phase* $S(\mathbf{x}_1,\ldots,\mathbf{x}_N,t)$ is given by $S = \hbar\mathrm{Im}\ln\Psi$ (similar to velocity expression). Thus deduce trajectories $\mathbf{x}_i(t)$ given by the *de Broglie guidance equation* for the velocity:

$$\mathbf{v}_i = \frac{d\mathbf{x}_i}{dt} = \frac{\nabla_i S}{m_i}$$

- Can write in 2nd-order '$F = ma$' form by taking time derivative, i.e. $m_i\ddot{\mathbf{x}}_i = -\nabla_i(V+Q)$, where $Q = -\sum_i \frac{\hbar^2}{2m_i}\frac{\nabla_i^2|\Psi|}{|\Psi|}$ (*quantum potential*). Extra 'quantum' force is $-\nabla_i Q$ (big where large curvature in wave field). *Non-classical* dynamics since particles 'pushed along' by wave along trajectories perpendicular to surfaces of constant phase, as well as by 'classical force' from other particles. Particles evolving in this way naturally become distributed as $|\Psi|^2$ (dust in a hurricane).
- Guidance equation *identical* to trajectory equation in Hamilton-Jacobi theory (a standard form of classical mechanics like Hamiltonian or Lagrangian dynamics). There S is indefinite integral of classical Lagrangian with respect to t (note the 'action' is the *definite* integral with fixed endpoints). Suggests immediately how to obtain the classical limit (impossible in orthodox QM!).
- Can also guess guidance equation from de Broglie relation $\mathbf{p} = \hbar\mathbf{k}$ (connects particle and wave properties). Wave vector $\mathbf{k}$ defined only for plane wave. For general wave, obvious generalization of $\mathbf{k}$ is local wave vector $\nabla S(\mathbf{x})/\hbar$. Hence $\mathbf{v} = \nabla S/m$.

Pilot wave theory

(a.k.a. Bohmian mechanics, de Broglie-Bohm theory, Bohm interpretation, causal interpretation..)



What is pilot-wave theory?

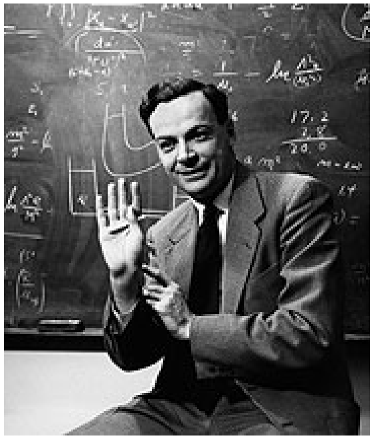
- It is the *original* interpretation of QM developed by de Broglie from 1923-1927, and rediscovered by David Bohm in 1952.
- It is also a new theory (different axioms, new predictions) and a mathematical reformulation of QM equivalent in status to Feynman's path-integral theory.

Why are people interested in it?

- It shows that QM can simply be interpreted as the statistical mechanics of particles with a non-classical dynamics. QM does not have to be 'weird'.
- It directly resolves all paradoxes of orthodox QM, in particular the measurement problem/wave collapse, all without the usual massively expanded ontology of parallel worlds, shadow universes, multiple intersecting realities etc..
- The *quality of its explanation* is greatly superior to the orthodox theory (which anyway rejects the need for explanations on principle).
- If adopted widely, it would greatly reduce the amount of time spent trying to explain the unexplainable to novices.
- It can be used to do interesting calculations based on 'quantum trajectories' (there is a community of physical chemists who do this).

Abused and ignored throughout its history but currently undergoing a major resurgence.

Two-slit experiment with electrons



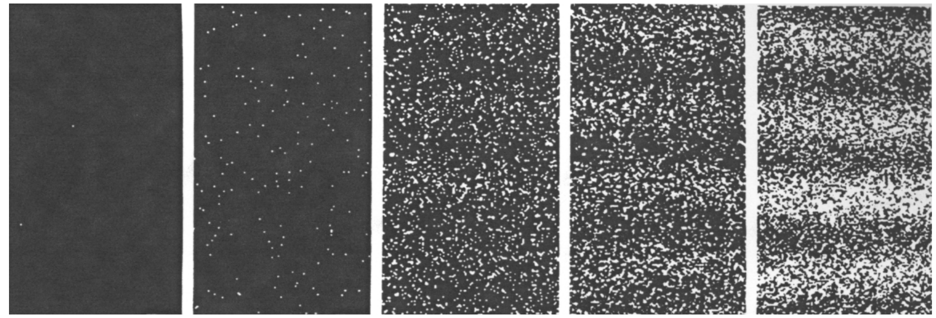
- "*A phenomenon which is impossible, absolutely impossible, to explain in any classical way, and which has in it the heart of quantum mechanics. In reality it contains the only mystery.*"

- "*Do not keep saying to yourself, if you can possibly avoid it, 'But how can it be like that?' because you will get 'down the drain,' into a blind alley from which nobody has yet escaped. Nobody knows how it can be like that.*"

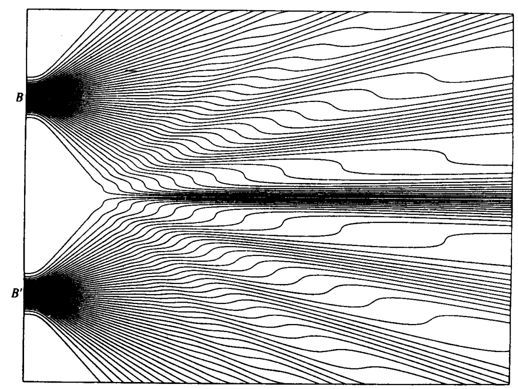
- "*Many ideas have been concocted to try to explain the curve for* P_{12} [that is, the interference pattern] *in terms of individual electrons going around in complicated ways through the holes. None of them has succeeded.*"

- This experiment "*has been designed to contain all of the mystery of quantum mechanics, to put you up against the paradoxes and mysteries and peculiarities of nature one hundred per cent.*"

- "*How does it really work? What machinery is actually producing this thing? Nobody knows any machinery. Nobody can give you a deeper explanation of this phenomenon than I have given; that is, a description of it.*"



Pilot-wave theory: while each particle track passes through just one slit, the wave passes through both; the interference profile that consequently develops in the wave generates similar pattern in the trajectories guided by the wave.



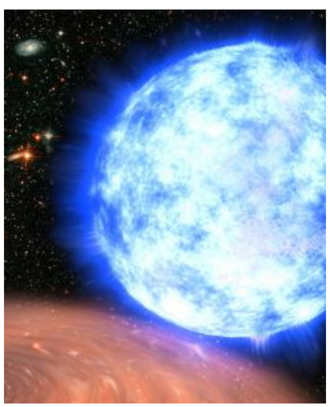
Compare Feynman commentary with that of John Bell:

"*Is it not clear from the smallness of the scintillation on the screen that we have to do with a particle? And is it not clear, from the diffraction and interference patterns, that the motion of the particle is directed by a wave? De Broglie showed in detail how the motion of a particle, passing through just one of two holes in the screen, could be influenced by waves propagating through both holes. And so influenced that the particle does not go where the waves cancel out, but is attracted to where they cooperate. This idea seems to me so natural and simple, to resolve the wave-particle dilemma in such a clear and ordinary way, that it is a great mystery to me that it was so generally ignored.*"

The quantum force

Standard textbooks tell us there are four 'fundamental forces of nature': electromagnetic, gravity, weak and strong nuclear. 'Quantum force' or *quantum-mechanical force* $\mathbf{F} = -\nabla Q$ seems to be a fifth one! Implies pilot-wave theory must be work of an insane crackpot nutter since surely somebody would have noticed. Maybe - it is true that the existence of the quantum force is not generally recognized; indeed - the possibility is *not even known* to most physicists! However there are many circumstances in which the quantum force is present but curiously unacknowledged. Three typical circumstances:

- QM force acts but phenomenon merely described as 'quantum effect' with no classical analogue (e.g. Aharanov-Bohm effect - QM force on particles non-zero even when other forces absent).
- Another force postulated which is QM force with different name (e.g. 'Pauli force/Pauli repulsion').
- QM force acts with accepted fundamental forces but is not recognized (e.g. covalent bonding).



Example: *when typical star runs out of fuel it collapses in on itself and eventually becomes a* white dwarf. *The material no longer undergoes fusion reactions, so the star has no source of energy, nor is it supported against gravitational collapse by the heat generated by fusion. It is supported only by* electron degeneracy pressure. *This is a force so large that it can stop a star from collapsing, yet no-one seems to know what it is.. Ask yourself: which of the four fundamental forces is responsible for it? Answer - none of them. It is in fact the* quantum force.

The wave field (mathematically represented by the wave function) is thus a new type of force field joining those of classical physics (electric/magnetic/gravitational). Suggests it is a repository of *energy*.

Curious non-classical properties: strength need not decrease with distance ('nonlocality'?). Although unexpected from classical perspective, not only example in quantum realm (e.g. 'inter-quark force' increases with distance). Antony Valentini describes it as 'a new type of causal agent'.

Energy and diffusion

What is energy? Little or no attention paid in physics to general definition. In classical mechanics, energy defined as *capacity of physical system to perform work*, and work defined in terms of *forces*. This definition has severe limitations and is actually useless in many cases (consider cylinder with hot and cold gases in compartments separated by a heat conducting piston). In fact energy is real attribute of physical systems with following characteristics: it is conserved, can be stored, exists in different interconvertible forms, and can be transferred through space or from one material body to another. A *field* in our sense can be considered to be a spatial distribution of energy which varies with time.

Problem with usual definition of *potential energy*: often stated to be property of *particles* - cannot be correct. In fact it represents a (position-dependent) amount of field energy available to a particle situated within the field (e.g. qEy for particle between charged plates). Different to *total energy stored in field* (i.e. $\frac{1}{2}\epsilon AE^2 d$ in that example).

- Why does time-dependent Schrödinger equation describe propagation of the wave field?
- Why is its mathematical form similar to the *diffusion* or *heat equation* of classical physics?

$$i\hbar\frac{\partial\Psi(\mathbf{x},t)}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\Psi(\mathbf{x},t) \qquad\qquad \frac{\partial u(\mathbf{x},t)}{\partial t} = D\nabla^2 u(\mathbf{x},t)$$

Main difference is imaginary i - ultimately traceable to use of *complex* wave functions (just mathematical convenience as Schrödinger can be rewritten as two coupled DEs involving two *real* functions).

- Standard heat equation derivation with no sources proceeds by specifying energy ‘flux’ from one region to another due to temp variation. Resulting equation describes transfer of heat energy.
- Schrödinger equation describes propagation through space of physical field with no sources (in classical sense) and finite energy content. Wave field time development conserves energy - thus describes energy transfer between spatial regions. The two equations therefore have the same form..

A new language

'Wave field' exists objectively in ordinary 3d space. Mathematically represented by usual Schrödinger wave function in *configuration space* (to account for forces or 'entanglement' between particles). Wave field is repository of potential energy in a quantum sytem, and acts on quantum particle similar to an external field and receives or imparts energy and momentum to the particle. It exhibits the usual wave properties (e.g. reflection, tranmission, diffraction, interference etc.) and it obeys the principle of linear superposition.

'Quantum potential' $Q = -\sum_i \frac{\hbar^2}{2m_i}\frac{\nabla_i^2|\Psi|}{|\Psi|}$ is the potential energy function of the wave field. It represents a portion of the energy contained in the wave field and is the amount of energy available to the particle at its specific position in the field. Q independent of intensity of wave field. Facilitates nonlocal connections in many-particle system.

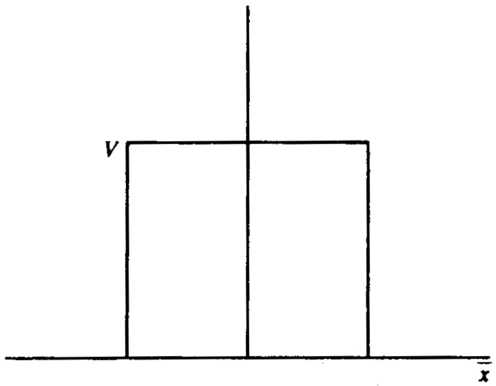
'Quantum equilibrium'. Particle distribution ρ logically distinct from $|\Psi|^2$. But can show if particles not initially distributed as $|\Psi|^2$ then become so under Schrödinger evolution and thereafter remain so distributed (in '*quantum equilibrium*'). Hence can *derive* Born rule - a *postulate* in orthodox QM. See numerical simulations later. [Can also use 'typicality' arguments to show particles expected to be distributed this way in 'typical universe'.]

Operators on Hilbert space and all that play no *fundamental* role, but are exactly right mathematical objects to provide compact representation of the statistics in a de Broglie universe.

Better explanations

- Measurement problem, Schrödinger's cat, wave collapse: Wave doesn't collapse (i.e. momentarily stop obeying Schrödinger's equation and change shape at infinite speed when someone looks at it). Things are made of particles. If wave field branches, particles deterministically end up in some branch with appropriate probability (one cat, one universe not bazillions of them).
- Classical limit: Classical limit emerges from the theory rather than having to be postulated. Classical domain is where wave component of matter is passive and exerts no influence on corpuscular component, i.e. state of particle independent of state of field ($Q, \nabla Q = 0$, essentially).
- Tunnelling: Effective 'barrier' encountered by particle not V but $V + Q$ - may be higher or lower than V and may vary outside 'true' barrier. For tunnelling need only $-\partial S/\partial t \geq V + Q$ then particle may enter/cross barrier region. Basically, particles shoved over barrier by 'quantum force'.
- Angular momentum: Due to rotational motion of electron trajectories 'orbiting' the nucleus! If ψ eigenfunction of $\hat{L}_z$, $\hat{\mathbf{L}}^2$ actual values and eigenvalues coincide. Traditionally x- and y-components 'undefined' but in fact well defined e.g. $L_x = -m\hbar \cot\theta \cos\phi, \quad L_y = m\hbar \cot\theta \sin\phi, \quad L_z = m\hbar$. Along trajectory L_x, L_y are *not conserved* (unlike $\hat{L}_z$, $\hat{\mathbf{L}}^2$).
- Quantum jumps: Standard belief: *systems can only possess certain values of physical quantities corresponding to spectra of Hermitian operators*. In pilot-wave theory quantities well-defined and continuously variable for all quantum states - values for subset of eigenstates have no *fundamental* physical significance. One of characteristic features of QM - existence of discrete energy levels - due to *restriction of basically continuous theory to motion associated with subclass of eigenfunctions*. Such states may possess particular physical importance in relation to stability of matter, but particle momentum and energy just as unambiguously defined when wave is superposition of eigenstates. No 'quantum jumps' in sense of process that is instantaneous or beyond analysis.

Tunnelling through a square barrier

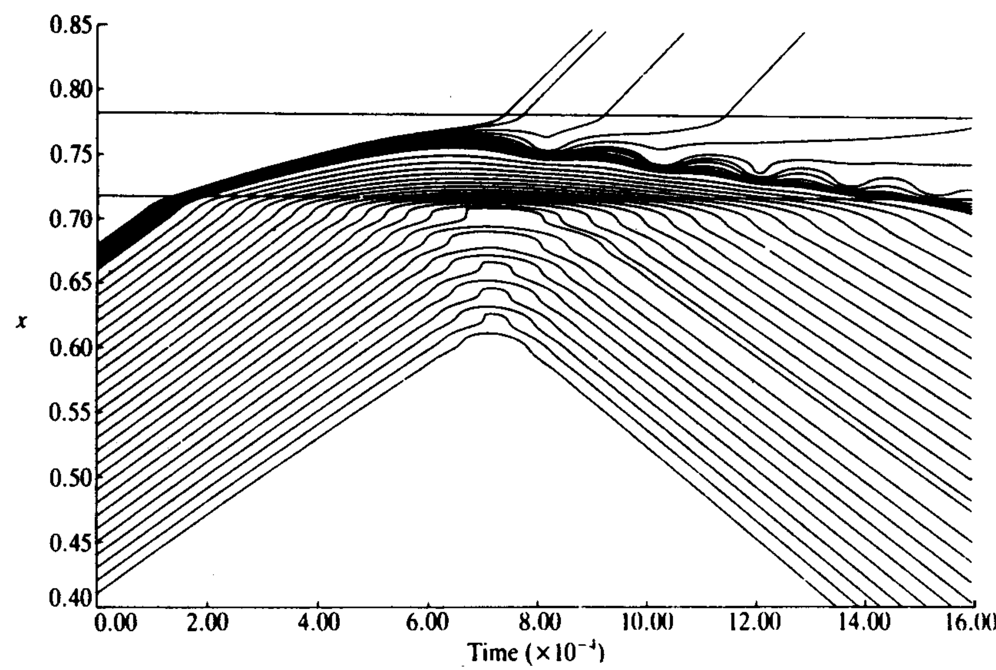


Usual plane wave description incorrect; more realistic 1D scattering of Gaussian wave packet of mean energy E from square barrier $V > 0$. Interaction of packet with barrier leads to formation of reflected and transmitted packets of diminished amplitude, perhaps together with small packet persisting inside barrier. Particle ends up in one of these.

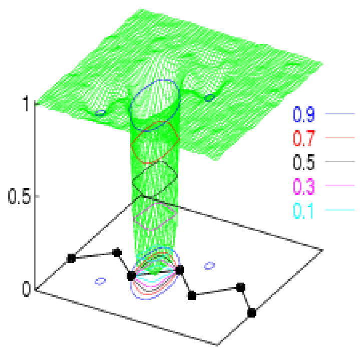
Consider $E < V$. Tunnelling arises from modification of total energy of particle (initially $\approx E$) due to rapid spacetime fluctuation of ψ-wave in vicinity of barrier. Total particle energy $-\partial S/\partial t = (1/2m)(\partial S/\partial x)^2 + V + Q$ evaluated along trajectory.

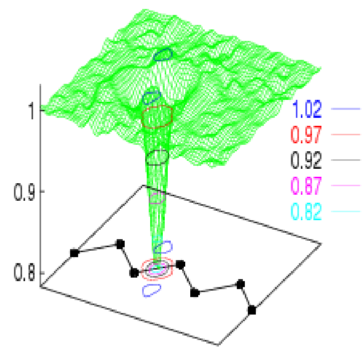
Effective 'barrier' encountered by particle is not V but $V + Q$ - may be higher or lower than V and may vary outside 'true' barrier. For tunnelling require only that $-\partial S/\partial t \geq V + Q$ then particle may enter or cross barrier region.

Impossible to explain this effect consistently using interpretation involving wave function alone.



Quantum-mechanical spin in pilot-wave theory

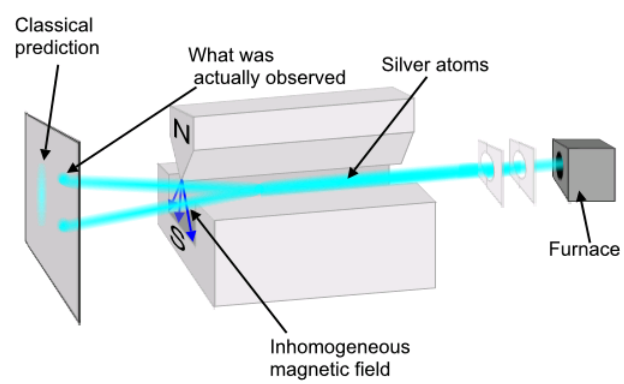




Remember *no such thing as spin* in non-relativistic Schrödinger theory. Artificially incorporate it by imposing *antisymmetry* on unknown Ψ and building it from 'spin orbitals' obeying imposed orthogonality relations etc. Get big 'exchange holes' for parallel and small 'correlation holes' for antiparallel spins. Spin emerges naturally only in relativistic theory, e.g. Dirac equation.

- In pilot-wave theory, guidance equation $m\mathbf{v} = \nabla S$ in fact not unique ('gauge freedom': can add any divergence-free vector field to current and get same density ρ). If take non-relativistic limit of Dirac equation get *Pauli equation* with *unique* $m\mathbf{v} = \nabla S + \nabla(\log\rho) \times \mathbf{s}$ (spin term). Thus the quantum potential (which represents a portion of the wave field's energy) depends on spin, and so **spin must be a property of the wave field** and *not* of the particles.
- In the usual Pauli theory (which gives no clue as to what spin *is*) wave field represented by *scalar* wave function with associated two-component spinors. Functionally equivalent to *vector* wave field; clearly this can have states of polarization like in EM theory. In this view QM spin $\mathbf{s}$ is thus just the *polarization-dependent part of the wave field's angular momentum* . Possibility not generally known, probably due to insistence that Ψ represents 'knowledge'.

Stern-Gerlach experiment: Magnets designed/oriented so that incident wave packet will, due to Schrödinger evolution, separate into distinct packets - corresponding to spin components of Ψ - moving in discrete set of directions. Particle, depending on initial trajectory, randomly ends up in *one* of these packets. Prob. distribution conveniently expressed in terms of QM spin operators (for a spin-$\frac{1}{2}$ particle given by the Pauli spin matrices). Note you are not actually 'measuring' anything whatsoever.



A fundamental question: why $P = |\Psi|^2$?

Pauli objection: Taking a particular particle distribution $P = |\Psi|^2$ as an initial condition is unjustified in a fundamentally deterministic theory, therefore de Broglie's 'theory' is rubbish. [in *Louis de Broglie: physicien et penseur*, 1953. He just can't leave poor Louis alone, even in his Festschrift.]

However, Pauli is right: this should be *derived* from the dynamics, for QM truly to emerge as the statistical mechanics of an underlying deterministic theory.

Easy to show if $P(\mathbf{x}, t) = |\Psi(\mathbf{x}, t)|^2$ at any t it will always remain so under Schrödinger time evolution ('*equivariance*'). Can also show $|\Psi(\mathbf{x}, t)|^2$ is *only* distribution with this property i.e. 'quantum equilibrium' is unique [Goldstein, Struyve 2007]. It is analagous to thermal equilibrium $P = \frac{\exp(-H/kT)}{Z}$.

With deterministic hidden-variable theories the Born distribution should not be regarded as an axiom. It should be seen as dynamically generated, in same sense that one usually regards thermal equilibrium as arising from process of relaxation based on some underlying dynamics.

A quite general argument (due to Antony Valentini, 1992) for the relaxation $P \rightarrow |\Psi|^2$ may be framed in terms of an analogy with the classical coarse-graining H-theorem (see my online lecture course). That's too complicated for this talk, so we shall look at some numerical simulations instead.

Quantum equilibrium: dynamical origin of quantum probabilities

Proc. Roy. Soc. A **461**, 253 (2005).

In this paper, Valentini and Westman show using explicit numerical simulations that $\rho \rightarrow |\Psi|^2$ *arises naturally even from a grossly non-equilibrium particle distribution.*

- System is a single particle in a 2D box with configuration $q = (x, y)$ and a (pure state) wave function $\psi(x, y, t)$ satisfying Schrödinger equation ($\hbar = 1$)

$$i\frac{\partial \psi}{\partial t} = -\frac{1}{2}\frac{\partial^2 \psi}{\partial x^2} - \frac{1}{2}\frac{\partial^2 \psi}{\partial y^2} + V\psi.$$

- Have ensemble of independent particles each guided by same ψ, so define density $\rho(x, y, t)$ of actual configurations. Guidance law $\mathrm{d}\mathbf{q}/\mathrm{d}t = \mathrm{Im}\,\nabla \ln \psi = \nabla S$ defines velocity field $(\dot{x}, \dot{y})$ which determines evolution of ρ via continuity equation:

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho \dot{x})}{\partial x} + \frac{\partial(\rho \dot{y})}{\partial y} = 0$$

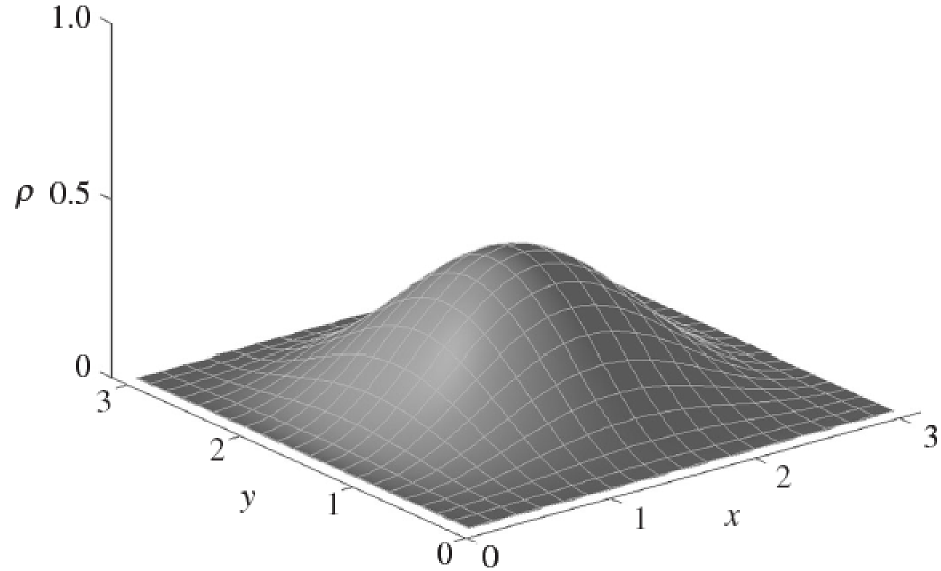
- Box has sides of length π with infinite barriers. The energy eigenfunctions are

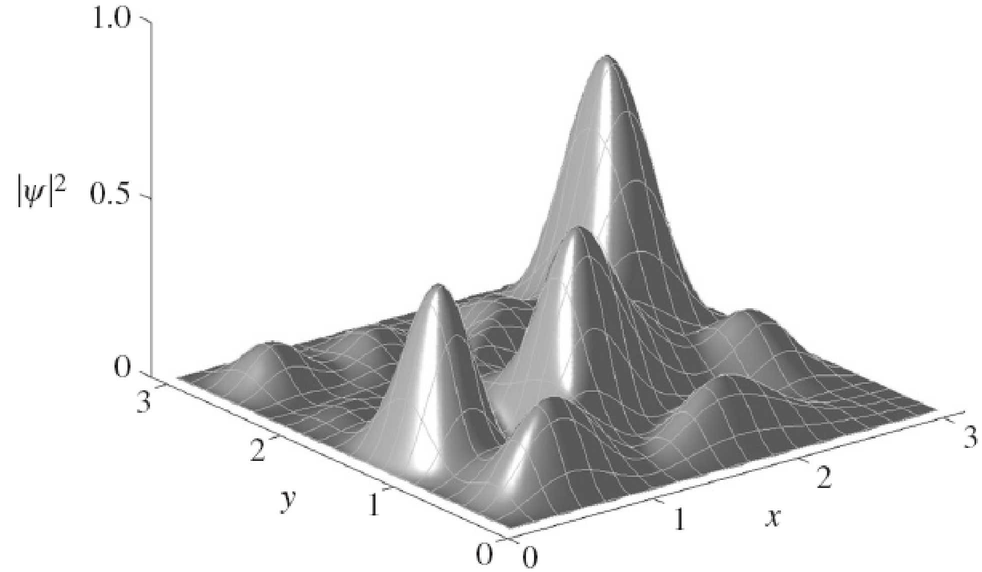
$$\phi_{mn}(x, y) = \frac{2}{\pi}\sin(mx)\sin(ny)$$

with energy eigenvalues $E_{mn} = \frac{1}{2}(m^2 + n^2)$, where $m, n = 1, 2, 3, \ldots$

Dynamical origin of quantum probabilities: numerical simulations

Starting conditions for the simulation: initial $\rho \neq |\Psi|^2$





Want grossly non-equilibrium starting distribution for particles. Choose distribution equal to square of ground-state wave function:

$$\rho(x, y, 0) = |\phi_{11}(x, y)|^2$$

.

Initial ψ is superposition of first 16 modes, $m, n = 1, 2, 3, 4, \ldots$ with equal amplitudes but randomly chosen phases θ_{mn}:

$$\psi(x, y, 0) = \sum_{m,n=1}^{4} \frac{1}{4} \phi_{mn}(x, y) \exp(i\theta_{mn})$$

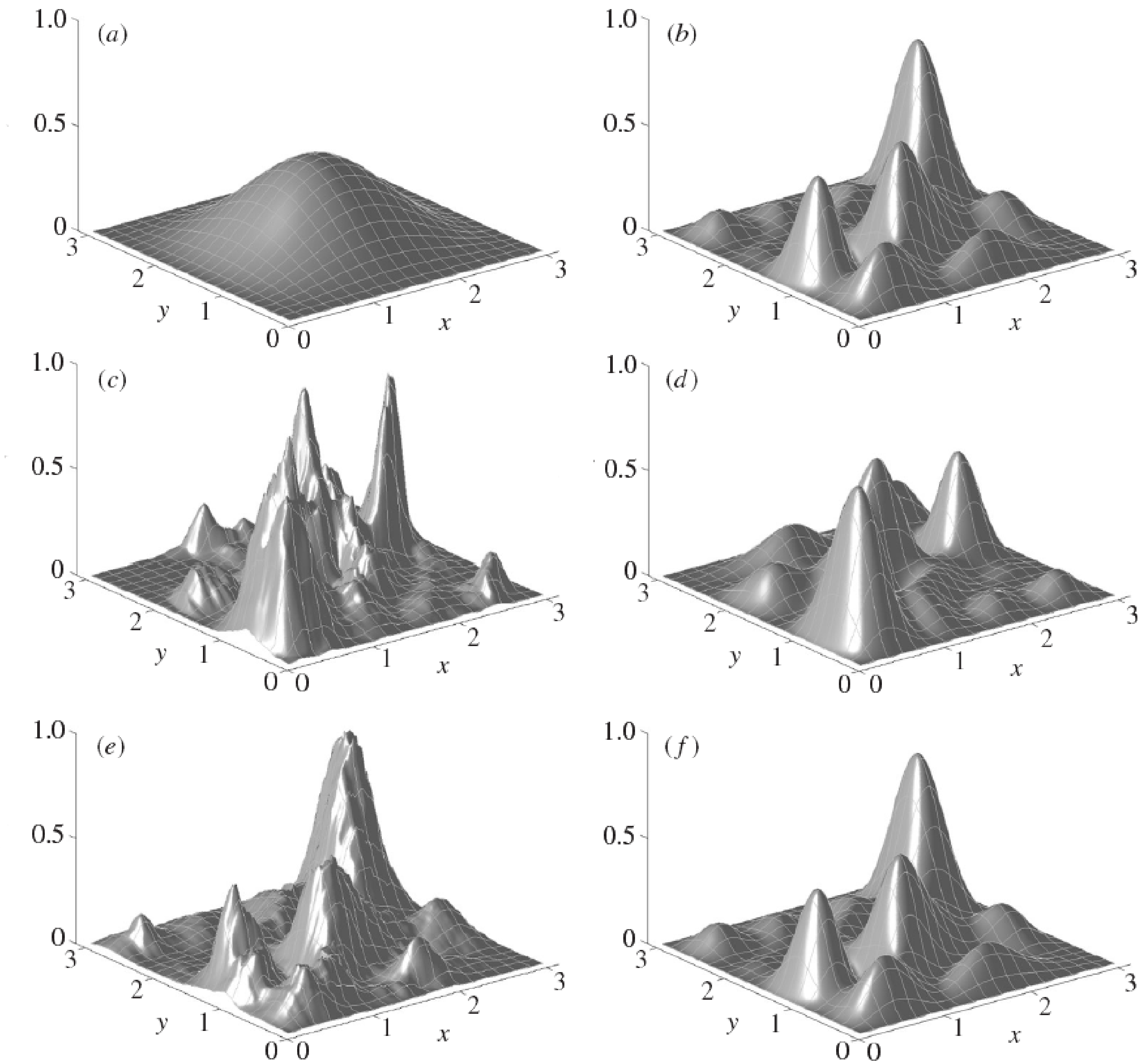
$$\psi(x, y, t) = \sum_{m,n=1}^{4} \frac{1}{4} \phi_{mn}(x, y) \exp i(\theta_{mn} - E_{mn} t)$$

Note ψ periodic in time with period 4π (since $4\pi E_{mn}$ is always an integer multiple of 2π).

Dynamical origin of quantum probabilities: numerical simulations

Results of evolution

ρ



$|\Psi|^2$

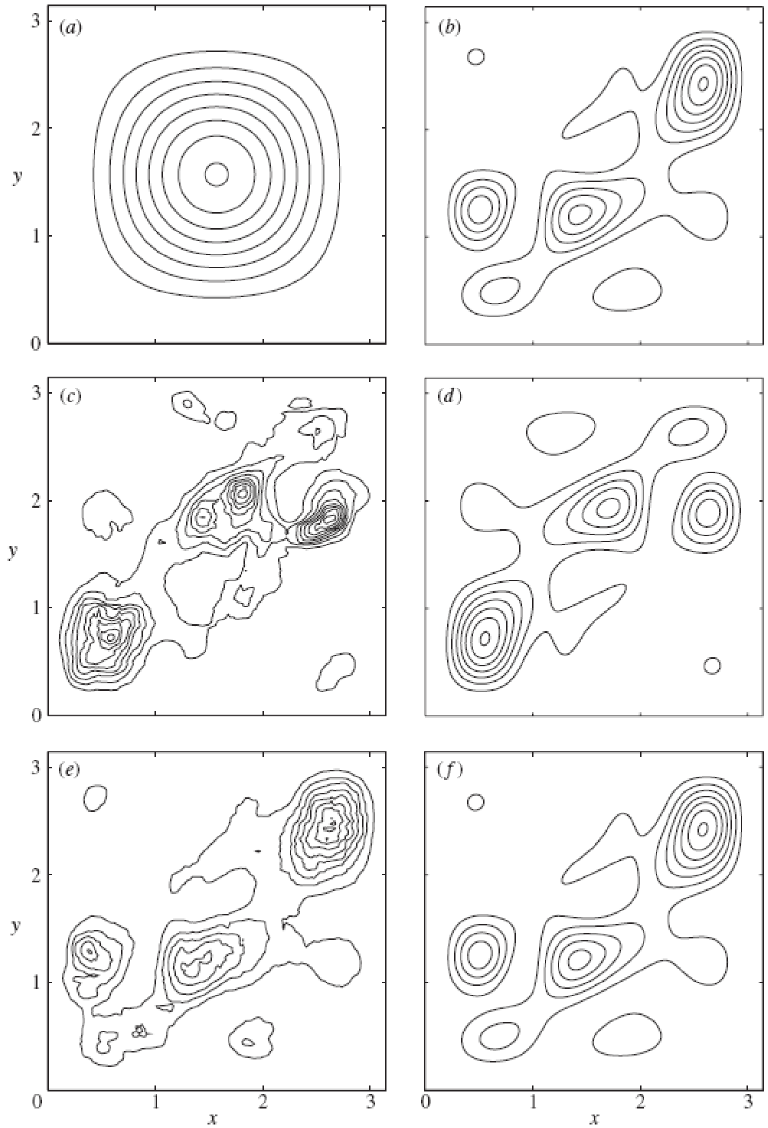
Results for $t = 0$ *(a,b), for* $t = 2\pi$ *(c,d) and for* $t = 4\pi$ *(e,f).*

While $|\Psi|^2$ *recurs to its initial value, the smoothed particle distribution* ρ *shows a remarkable evolution towards quantum equilibrium!*

Dynamical origin of quantum probabilities: numerical simulations

Results of evolution: contour plots

ρ



$|\Psi|^2$

Results for $t = 0$ *(a,b), for* $t = 2\pi$ *(c,d) and for* $t = 4\pi$ *(e,f).*
Same data as previous slide displayed as contour plots.

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Dynamical origin of quantum probabilities: numerical simulations

Character of the trajectories

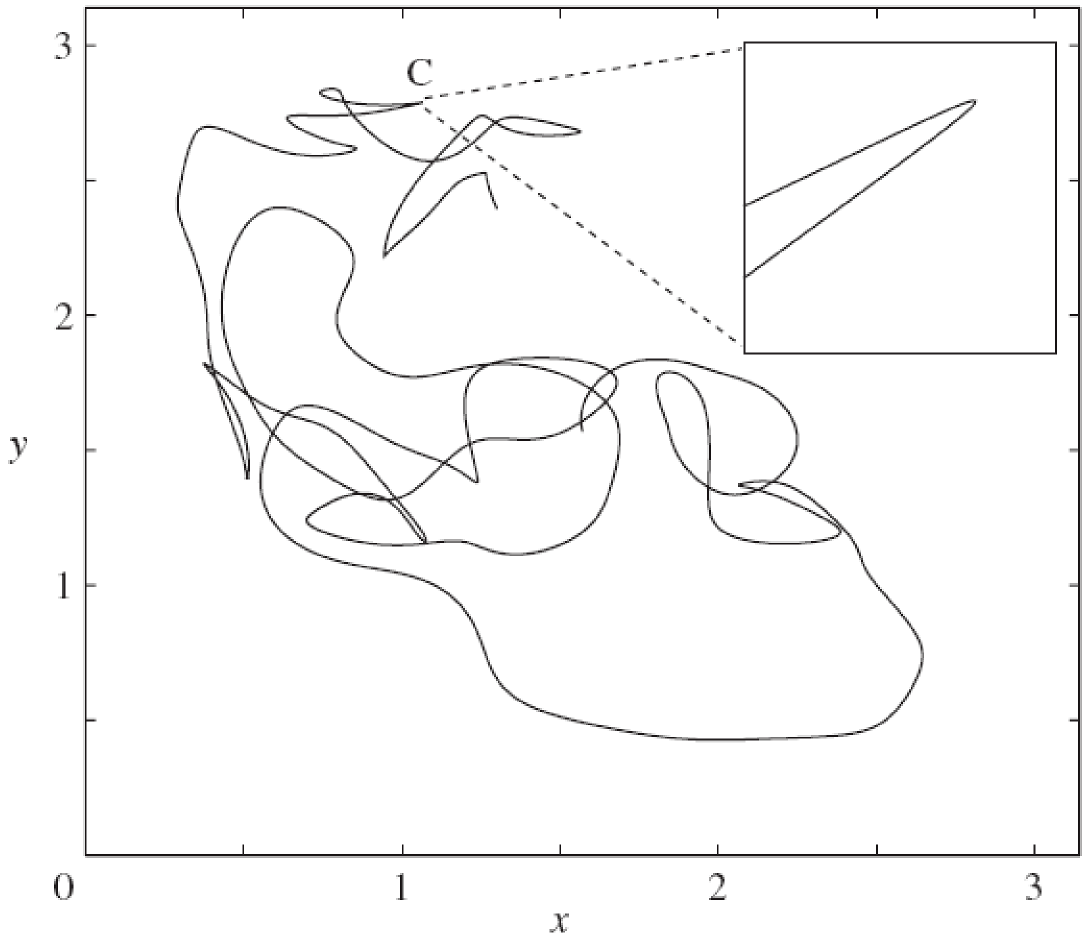
Particle velocity components at t:

$$\frac{\mathrm{d}x}{\mathrm{d}t} = \frac{i}{2|\psi|^2}\left(\psi\frac{\partial\psi^*}{\partial x} - \psi^*\frac{\partial\psi}{\partial x}\right)$$

$$\frac{\mathrm{d}y}{\mathrm{d}t} = \frac{i}{2|\psi|^2}\left(\psi\frac{\partial\psi^*}{\partial y} - \psi^*\frac{\partial\psi}{\partial y}\right)$$

Calculate trajectory $x(t), y(t)$ by numerical integration of above. Typical trajectory shown here - in general they are rather irregular.

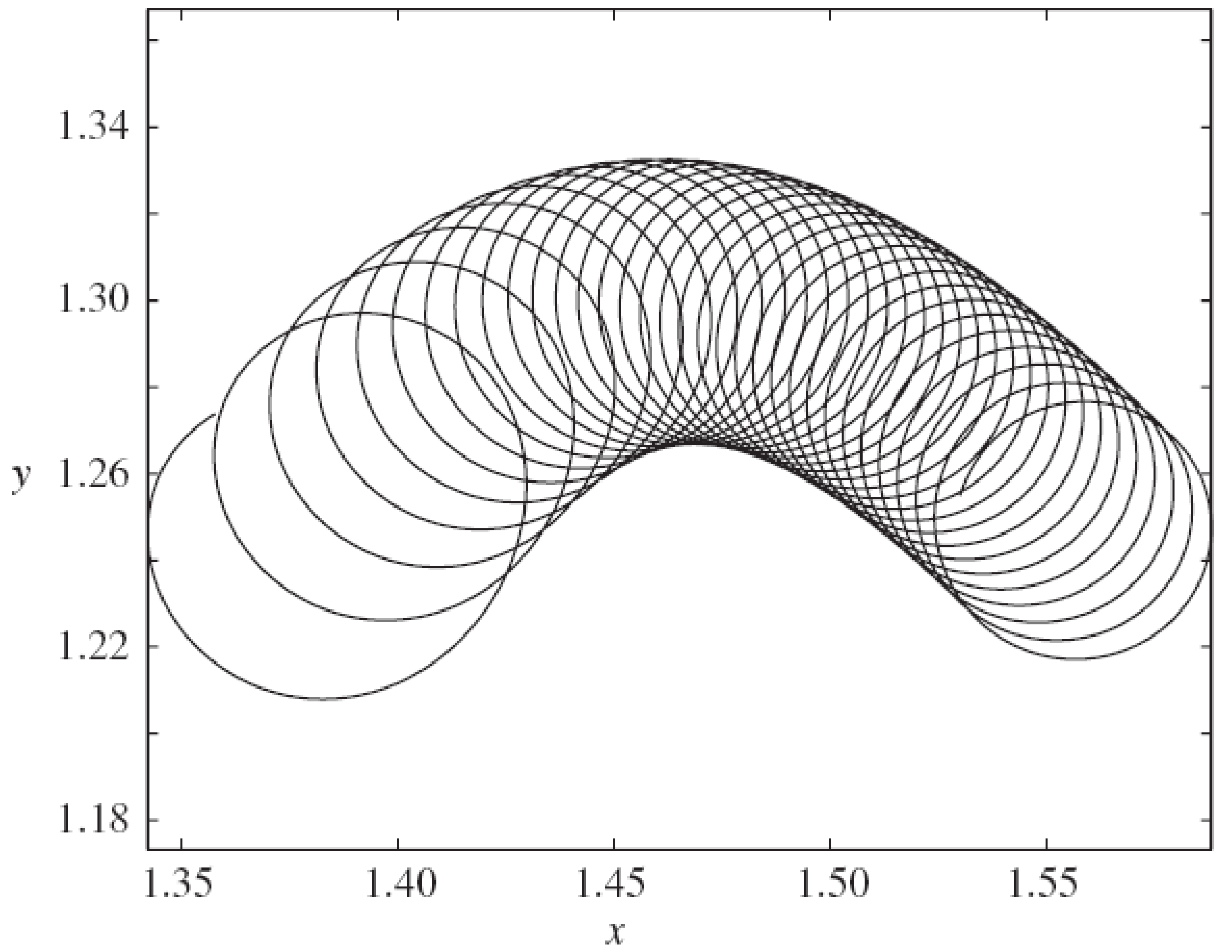
Point C looks like a cusp but tangent not actually discontinuous (particle turning round *slowly*).



Note velocities ill-defined at nodes (where $|\psi| = 0$) and tend to diverge as nodes are approached. This is because - close to a node - small displacements in x and y can generate large changes in phase $S = \mathrm{Im}\ln\psi$ corresponding to large gradient ∇S. Because ψ is smooth, single-valued function, small displacement $(\delta x, \delta y)$ produces small change $\delta\psi$ in complex plane. However, close to a node $\delta\psi$ lies near origin of complex plane and so can correspond to large phase change δS.

Dynamical origin of quantum probabilities: numerical simulations

Close-up of a trajectory near a node



Motion rapid in regions where $|\psi|$ is small. Above close-up of trajectory near nodal or quasi-nodal point where $|\Psi|$ very small (but not known to be strictly zero). Spatial region shown *ca.* 0.3% of whole box. Particle follows rapid circular motion around point moving from right to left - and moving point is a node or quasi-node at which $1/|\psi|$ is highly peaked.

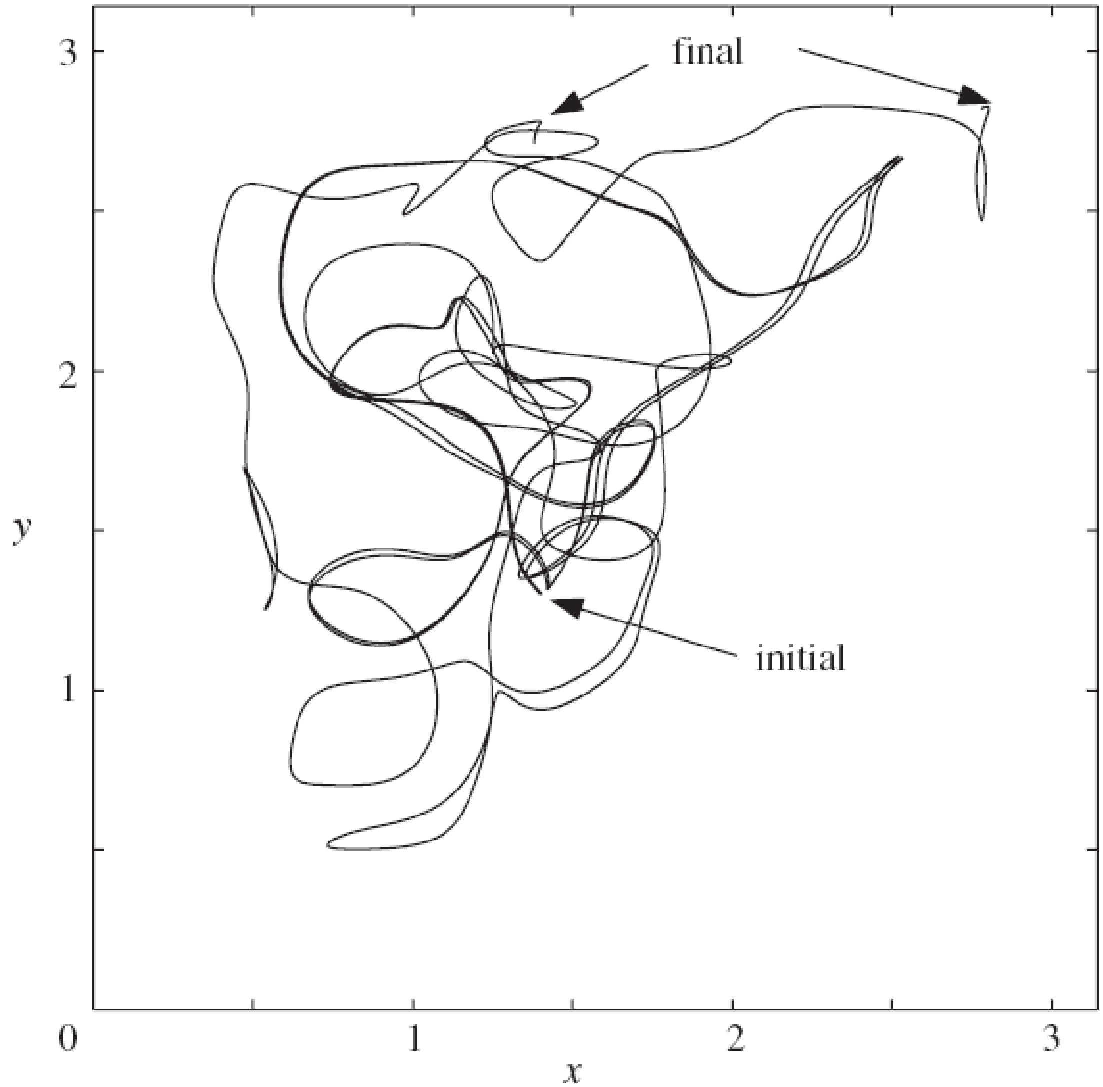
Dynamical origin of quantum probabilities: numerical simulations

Chaotic nature of the trajectories

Two distinct but very close initial positions evolve after $t = 4\pi$ *into widely separated final positions.*

Relevant quantity is *Lyapunov exponent* which characterizes rate of separation of infinitesimally close trajectories. Separation rate depends on orientation of initial separation vector, thus whole spectrum of n Lyapunov exponents - where n is dimensionality of the phase space. Usually use largest one - the *Maximal Lyapunov exponent* (MLE) as it determines predictability of a dynamical system. Positive MLE usually taken as indication that system is *chaotic*.

Difficulty to even *define* 'quantum chaos' in standard QM with no trajectories!

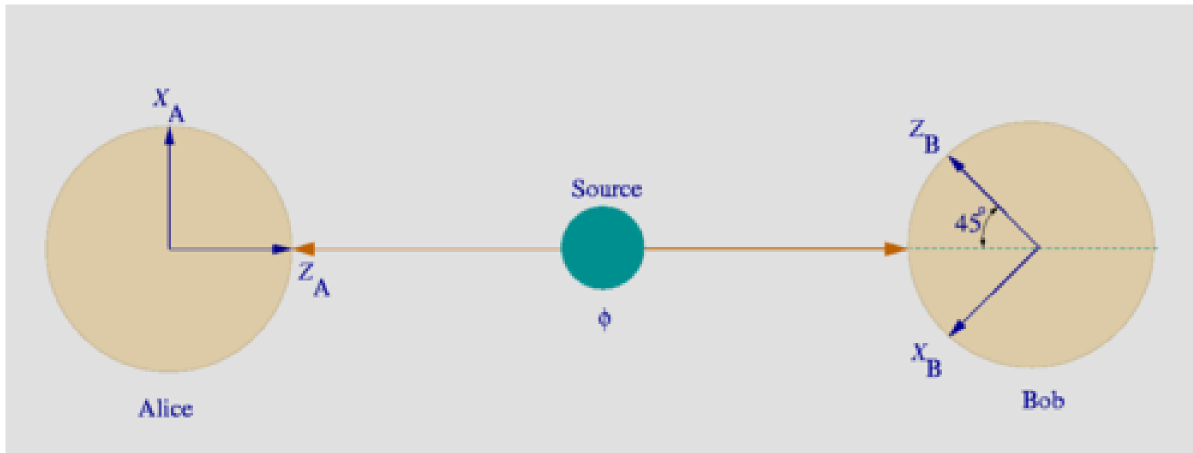


In pilot-wave dynamics, one sees the importance of *nodes* in generating chaotic motion. Numerical simulations suggest a proportionality between Lyapunov exponent and number of nodes.

[See e.g. Frisk *Phys. Lett. A* **227**, 139 (1997)]

Nonlocality

Definition: *a direct influence of one object on another, distant object, contrary to our expectation that an object is influenced directly only by its immediate surroundings*.



What Einstein-Podolsky-Rosen (EPR) experiment implies:

- Measurement on one side instantly predicts result on other (parallel analyzers).
- If do not believe one side can have causal influence on other, require results on both sides to be determined in advance. But this has implications for non-parallel settings which conflict with quantum mechanics (*Bell*).

Bell's analysis showed that *any account* of quantum phenomena needs to be non-local, not just any 'hidden variables' account i.e. nonlocality is implied by the predictions of standard quantum theory itself. Thus, if nature is governed by these predictions (which it is, according to real experiments) then nature is non-local.

If you wish to pursue the study of these matters, you should look up a proof of Bell's theorem. I highly recommend Tim Maudlin's book *Quantum non-locality and relativity* for this and many other things (this is a big and complex subject). For anyone who thinks the study of quantum foundations is pointless, Bell's theorem was a **direct result** of Bell trying to understand why the pilot-wave interpretation is wrong.

Nonlocality and configuration space and relativity

QM and experiment show violation of Bell's inequality (VBE) for events at space-like separations (implying non-locality). **How does this square with relativity?** No problem - remember, light speed is not a speed *limit*; it is the speed which remains *invariant* under certain (Lorentz) transformations of the reference frame. What constraints do VBE+QM imply? According to Maudlin, results unequivocal:

- VBE does *not* require superluminal matter or energy transport.
- VBE does *not* entail the possibility of superluminal signalling.
- VBE does require superluminal *causal connections*.
- VBE can be accomplished only if there is superluminal *information transmission*.

If you don't want to believe non-locality, what options do you have?

(1) Deny reality (*then nothing can be non-local*); (2) Believe many worlds interpretation (*then* **everything** *happens, so you can't say there are non-local correlations*); (3) Allow things to move backwards in time (*Hopeless mix-up if present events depend on future and shape of future in part determined by present. Cramer's transactional interpretation not a solution.*). Of all the apparent bizarrerie, believing that influences (in the above sense) travel very fast seems more appealing.

Both pilot-wave theory and experiment seem to imply existence of preferred reference frame (the one in which non-local correlations are absolutely simultaneous). Suggests *neo-Lorentzian interpretation* of relativity more appropriate than standard Einstein-Minkowski one. (See my course, Lecture 5). Note also that pilot-wave theory can be made relativistic; predictions agree with experiment but disagree with dogma of relativistic metaphysics (preferred frame, Lorentz invariant on average). Highly interesting!

NB: Valentini showed superluminal signalling becomes possible under conditions of *quantum non-equilibrium*, and that pilot-wave theory makes *testable predictions* (potentially measurable in CMB.)

Why configuration space?

Use of wave function $\Psi(\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_N)$ defined on a multi-dimensional configuration space does not imply this space exists in same sense that physical 3d space may be said to exist. (Remember even in classical mechanics we can use a configuration space description).

In CM, config space representation is just convenient summary of positions of all particles; in QM situation different since the physics is different - possibility of entanglement due to non-local interactions. So a simply-connected 3d space alone cannot describe the holistic quantum connectiveness and nonlocality features of multi-particle quantum systems. Instead this is done *formally* by employment of the N-dimensional config space. Problems with such a space *actually existing* are considerable:

1. Need at least three separate dimensions for every particle in the universe.
2. Continual variation in total number of dimensions in universe as particles are created and destroyed.
3. Extra dimensions always completely unnoticeable at macroscopic scales.
4. *Complete lack of any experimental evidence* for the existence of multi-dimensional physical spaces.

- Currently don't know the 'means' by which quantum non-local connections are actualized. Not because of non-relativistic context since non-locality is also present in relativistic versions of QM.
- Given strong reasons against taking multi-dimensional space as real, the strong evidence in favour of physically real wave fields, and absence of info about the 'means' of non-local connections, it is a coherent position to take Ψ to be a mathematical representation of a real field in physical space.
- Notion of an N-particle system described in pilot-wave theory by its trajectory which is traced out in $3N$-dimensional config space. Even though this description is given using a multi-dimensional space, the motion of individual particles can be calculated since there is a natural mapping from the system's trajectory in $3N$-dimensional space to trajectories in 3d space.
- Perhaps when we discover (or develop model of) the 'means' by which quantum non-local connections are actualized then can describe wave field in physical 3d space?

An interesting observation about Feynman's path integral QM

Want to calculate *propagator* K (carries wave function Ψ from past into future).

- In *Feynman's path integral theory* the propagator is

$$K^F(\mathbf{x}_1, t_1; \mathbf{x}_0, t_0) = N \sum_{\text{all paths}} \exp\left[\frac{i}{\hbar}\int_{t_0}^{t_1} L_c(t)\,\mathrm{d}t\right].$$

Here propagator linking two spacetime points calculated by linearly superposing amplitudes $e^{iS/\hbar}$ (obtained by integrating **classical** Lagrangian $L_c(t) = \frac{1}{2}mv^2 - V$) associated with infinite number of **all possible paths** connecting the points. Get future wave function at $\mathbf{x}_1$ from $\Psi(\mathbf{x}_1, t_1) = \int K^F(\mathbf{x}_1, t_1; \mathbf{x}_0, t_0)\Psi(\mathbf{x}_0, t_0)\,\mathrm{d}x_0$.

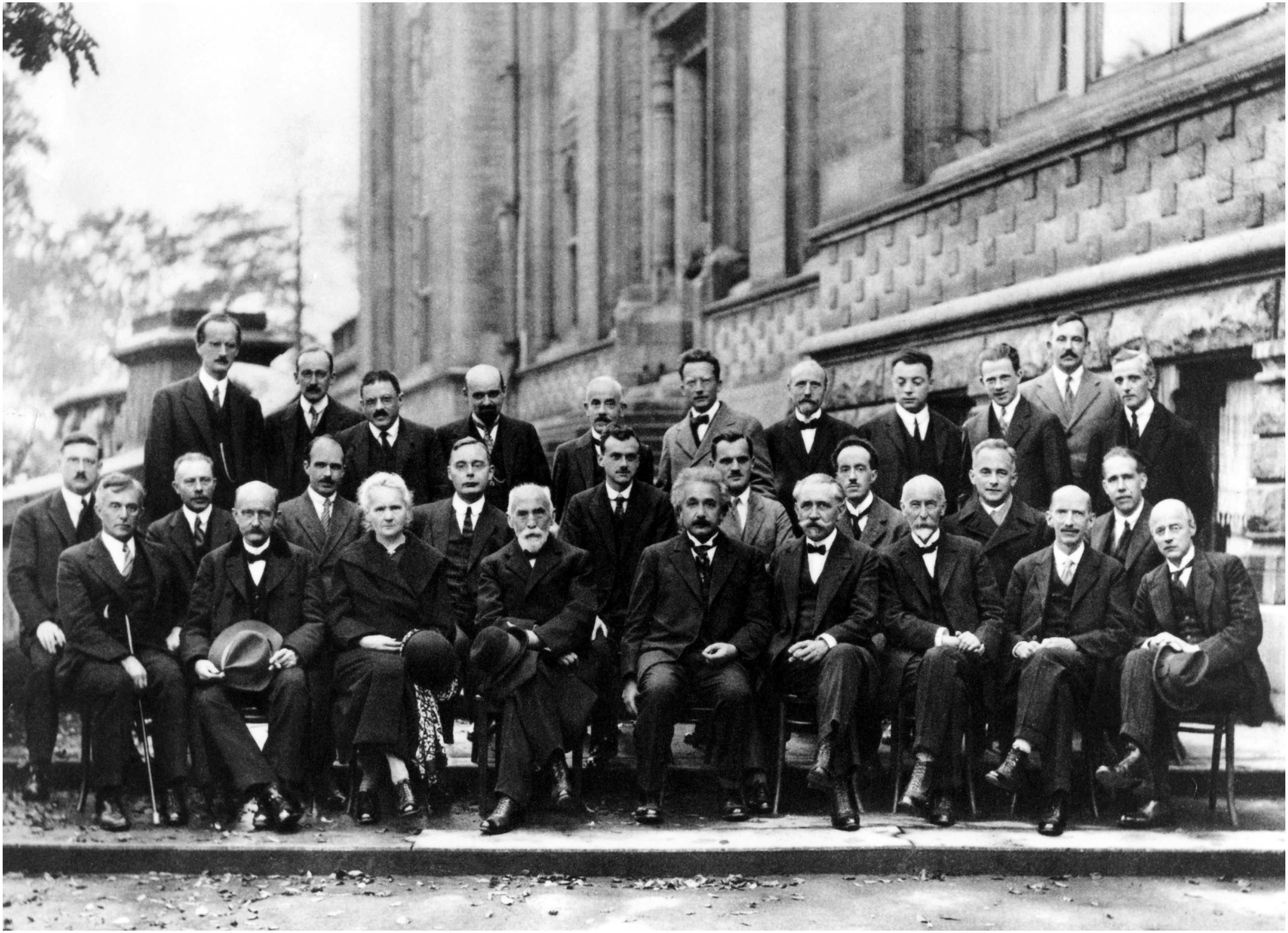
- In the equivalent pilot-wave theory expression the propagator is

$$K^{PW}(\mathbf{x}_1, t_1; \mathbf{x}_0, t_0) = \frac{1}{J(t)^{\frac{1}{2}}} \exp\left[\frac{i}{\hbar}\int_{t_0}^{t_1} L_q(t)\,\mathrm{d}t\right].$$

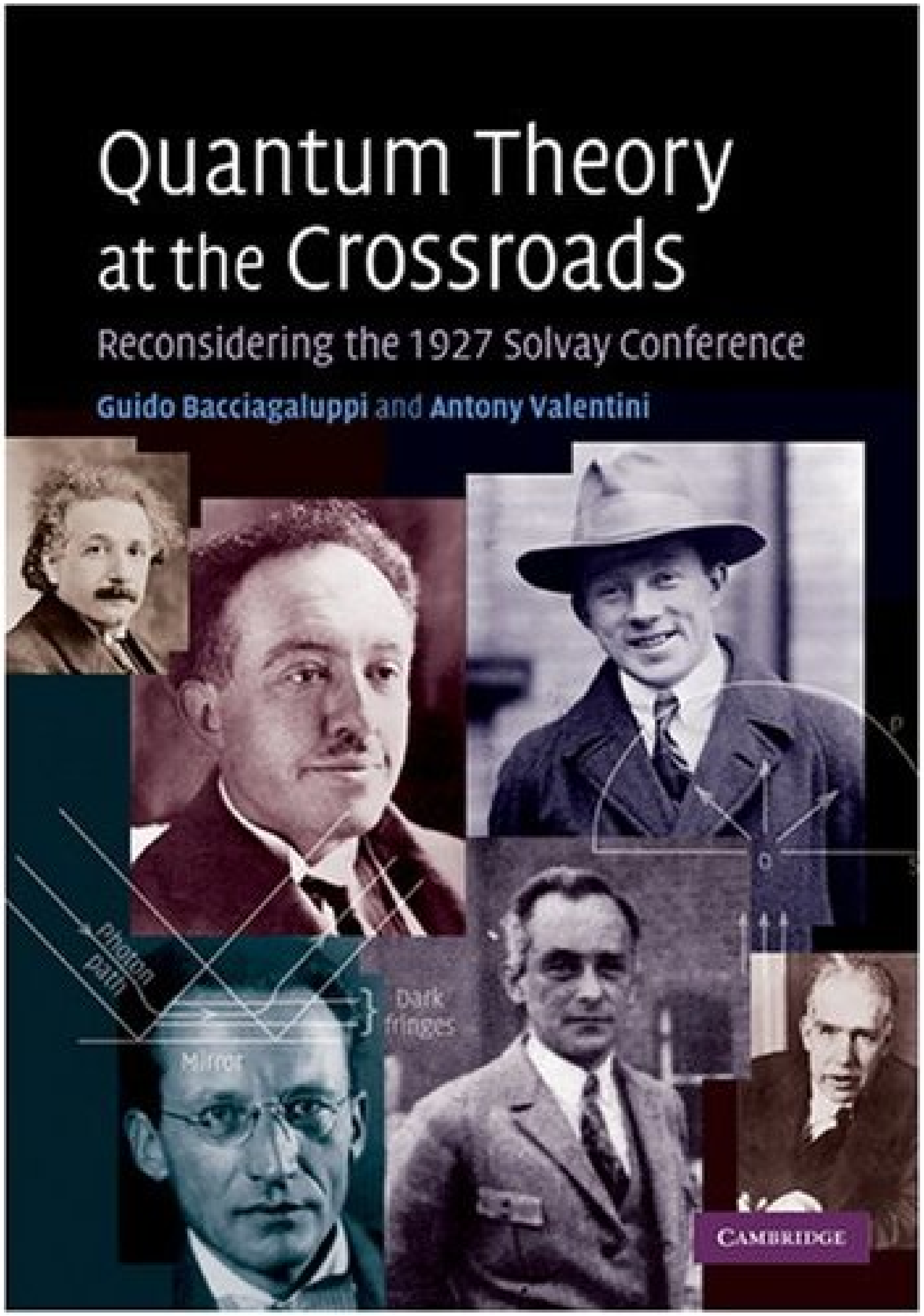
i.e. get same result as Feynman by integrating **quantum** Lagrangian $L_q(t) = \frac{1}{2}mv^2 - (V + Q)$ along precisely **one** path - the one the particle actually follows. Integral over K with different starting points not required since trajectories don't cross, i.e. $\Psi(\mathbf{x}_1, t_1) = K^{PW}(\mathbf{x}_1, t_1; \mathbf{x}_0, t_0)\Psi(\mathbf{x}_0, t_0)$

Not many people know this..

Let's finish with some history: the 5th Solvay conference, 1927



The book to read



Fascinating!

Pilot wave theory in 1927

- **May 1927**: De Broglie publishes 'Wave mechanics and the atomic structure of matter and of radiation', *Le Journal de Physique et le Radium*, **8**, 225 (1927) - the final development of a remarkable progression in thought that had begun in 1923. Paper contains full modern pilot-wave dynamics, though only for single particles.

  [See p. 61 Bacciagaluppi and Valentini (BV) for analysis of this paper.]

  "*One will assume the existence, as distinct realities, of the material point and of the continuous wave represented by the function Ψ, and one will take it as a postulate that the motion of the point is determined as a function of the phase of the wave by the equation $\mathbf{v} = -\frac{1}{m_0}\left(\nabla S + \frac{e}{c}\mathbf{A}\right)$. One then conceives the continuous wave as guiding the motion of the particle. It is a pilot wave.*"

- **August 1927**: "*...it is very rich in ideas and very sharp, and on a much higher level than the childish papers by Schrödinger...*" [Pauli, letter to Bohr of August 1927, referring to this paper of de Broglie].

- **October 1927**: 5th Solvay conference takes place in Brussels. De Broglie presents the main results of his *Journal de Physique et le Radium* paper, but now for a nonrelativistic system of N particles guided by a wave function Ψ in configuration space that determines the particle velocities according to de Broglie's law of motion. The theory published in the Proceedings is absolutely pilot-wave dynamics as we know it today, which is why it is usually called, er.., 'Bohmian mechanics'.

A short play set in 1927

Volunteers please to represent the following characters:

MR. DE BROGLIE: French, highly intelligent, masterful speaker, good in bed.

MR. PAULI: Humourless Austrian Ernst Stavro Blofeld: ‘Today you vere lucky, Mr. Bond’.

MR. EINSTEIN: Everybody loves him. German, warm, avuncular, sounds continually amused.

MR. SCHRÖDINGER: Massively upper class Austrian Anglican priest. Secret pervert.

MR. BOHR: Prissy, pernickety old Danish person. Hopelessly sad.

MR. HEISENBERG: Cockney, sounds like Ray Winstone threatening to cut someone’s legs off.

MR. BRAGG: Posh English. The person sitting to the left of Mr. Heisenberg.

MR. DIRAC: Doesn’t say anything of course. The person sitting to the right of Mr. Heisenberg.

TWO MEN IN WHITE COATS: One behind Heisenberg, one behind Bohr. No violence please.

THE SOLVAY AUDIENCE: Everyone else - please follow stage directions.

For amusement’s sake, please exaggerate foreign accents, even if you happen to be foreign yourself.

Universe I: Pauli has a revelation..

The 1927 Solvay conference

MR DE BROGLIE: *. . . and so we conclude that the dualist representation by corpuscles and associated waves allows us now to see the non-relativistic quantum theory as just statistical mechanics with a different (quantum) dynamics. And with that, gentlemen, Madame Curie, I end my presentation. I thank you for your attention.*

Enthusiastic applause. Intermittent whooping from near the back of the hall.

Mr. Bohr is sitting in the centre of the front row. He applauds and flashes a beaming smile at Mr. de Broglie. However there appears to be something wrong with his eyes.

MR PAULI: *My dear de Broglie, I should - I think - like to congratulate you.. Since I read your very sharp article in the* Journal de Physique *I have been intrigued by this approach, as I have expressed several times to Mr. Bohr. With your apparently successful extension to the many-body case I begin now to see that much of what we have thought up to now is* ganz falsch *- not even wrong. I even see how some doubts I had about inelastic scattering could be resolved. Working alone in Paris away from our little circle has been good for you it seems - I had hitherto suspected the new mechanics would be a German creation..* [He nods in the direction of Mr. de Broglie and sits down.]

Mr. Heisenberg leans against one of the walls of the lecture theatre, smoking a cheroot. He does not smile or clap. His eyes are fixed on the back of Pauli's neck. A thin hiss of smoke escapes from his delicately pursed mouth.

Universe I: One year later, Bohr speaks

MR. BOHR: . . . *so I shall try to describe to you a certain general point of view, which I hope will be helpful in order to harmonize the apparently conflicting views taken by different scientists. I call it* [offstage, a trumpet sounds] *the 'Complementarity Principle'. It says, you see, that with there being this dual wave-particle nature of reality there is - we must all now agree - no logical picture that can simultaneously describe and be used to reason about all properties of a quantum system.* [General hilarity, then silence broken by occasional embarrassed coughing. Someone shouts 'Keep up, Bohr!'.]

MR. EINSTEIN: *But my dear Bohr, is that not precisely what M. de Broglie has provided? It is, if I may say, now generally accepted that the de Broglian mechanics has lifted a corner of the Great Veil.*

MR. HEISENBERG: *Seen one of these so-called 'electrons' when you're not looking at it, have you? Eh?* [Bragg foolishly calls out 'Yes, indeed!'. Heisenberg lunges at him, but is restrained by Mr. Dirac.]

MR. SCHRÖDINGER: *What madness is this? These conflicts Bohr speaks of are in your head and his. You would replace de Broglie's beautiful, logical, comprehensible quantum theory - which so elegantly extends the theory of poor Boltzmann and Mr. Einstein - with such pettifogging mumbo-jumbo? This would lead us down the road to rats being at the same time both dead and alive..*

MR. BOHR: *But.. I am the Father of Quantum Mechanics. I have an Institute. You must listen to me..*

ALL: *Father of My Arse, mate. Hoo, hoo. Get back to Copenhagen.. You belong in an Institute. Etc..* [Enter men in white coats. Bohr and Heisenberg are put into straitjackets and dragged away.]

MR. HEISENBERG: [offstage] *But to hope for so-called hidden variables is like saying 2+2=5. OOoofff!*

Soon afterwards, inspired by the physicists and instead of waiting until the 1960s as expected, the philosophers have all the logical positivists taken outside and shot. All the old problems in philosophy are opened again, and everyone has much more fun. Henceforth, quantum theorists are seen by the public - if seen at all - as rather dull on account of the lack of barking paradoxes. At least until Marie Curie discovers nonlocality in 1935..

Universe II: The one we live in..

The 1927 Solvay conference

MR. DE BROGLIE: *. . . and so we conclude that the dualist representation by corpuscles and associated waves allows us now to see the non-relativistic quantum theory as just statistical mechanics with a different (quantum) dynamics. And with that, gentlemen, Madame Curie, I end my presentation. I thank you for your attention.*

Polite applause. Some photon somewhere goes the other way.

MR. PAULI: *It seems to me that, concerning the statistical results of scattering experiments, the conception of Mr. de Broglie is in full agreement with Born's theory in the case of elastic collisions, but that it is no longer so when one also considers inelastic collisions. I should like to illustrate this by the example of the rotator, which was already mentioned by Mr. de Broglie himself. As Fermi has shown. . .* [there follows a technical argument† . . .] *. . . Mr. de Broglie's point of view does not then seem to me compatible with the requirement of the postulate of the quantum theory, that the rotator is in a stationary state both before and after the collision. · · · In Born's theory, agreement with the quantum postulate is realized thus, that the different partial waves in configuration space, of which the general solution of the wave equation after the collision is composed, are applicable* separately *in a statistical way. But this is no longer possible in a theory that, in principle, considers it possible to avoid the application of notions of probability to individual collision processes.*

† See BV p.511 for the full translation.

The death of pilot-wave theory

The little exchange on the previous slide (and de Broglie's supposedly weak reply) is supposed to be why pilot-wave theory was rejected, and why de Broglie gave it up - which he did by 1930.. Here is how historians usually characterize what happened:

"*It was immediately clear that nobody accepted his ideas. . . In fact, with the exception of some remarks by Pauli. . . de Broglie's causal interpretation was not even further discussed at the meeting. Only Einstein once referred to it* en passant." [Jammer, *The Interpretations of QM in Historical Perspective*].

Like almost all commentary in the literature, this is **factually incorrect on just about every level**. As has now been made clear by Valentini and Bacciagaluppi's recent book, which includes the first full English translation of the proceedings from the original French, the theory was *extensively* discussed by most of the participants, both after de Broglie's presentation, and in the General Discussion on the final day. The only critical remark (apart from a minor one by Kramers) was Pauli's, and as we shall see, Pauli's objection is actually not correct (sadly, it is *not even wrong*). Contrary to popular opinion, de Broglie's reply to Pauli *did* contain the essential points required for a proper treatment of inelastic scattering.

Note the irony: Pauli was basically claiming that de Broglie's theory failed to produce a unique result in an inelastic scattering measurement. Can you think of another well-known theory which fails to produce a unique result after a quantum measurement of anything at all (without simply stating that looking at it makes it have a unique result)? Ah.

Pauli's objection: what was wrong with it?

- Bohm discusses this point in one of his 1952 papers reintroducing pilot-wave theory. He concludes: *"Thus, Pauli's objection is seen to be based on the use of the excessively abstract model of an infinite plane wave."*. However this cannot be true. Not only is it highly unlikely that a physicist of Pauli's abilities would make such an elementary mistake, but Pauli states quite explicitly in his first sentence that Mr. de Broglie's conception is fine for *elastic* collisions (which one would expect to suffer from same problem).
- Real problem with Pauli's objection stems from his "As Fermi has shown.." remark. This refers to misleading optical analogy introduced by Fermi in a more restricted context: *(time-dependent) scattering of an electron in two spatial dimensions by a rotator - a model scattering centre with one rotational degree of freedom - is mathematically equivalent to (time-independent) scattering of a scalar light wave in three spatial dimensions by an infinite diffraction grating*. Unfortunately to be applied in this context one requires a *frequency-dependent* speed of light, and it cannot be applied to a real situation with a finite incident wave (see BV discussion).
- Clear from his answer that de Broglie understood general separation mechanism required to yield definite outcome, but was misled by false optical analogy and phrased his answer in terms of it.

Bohm continued downplaying de Broglie's contribution until his death, see e.g. the following rather naughty extract (from Bohm and Hiley's **1993** textbook). Given the existence of a clear question of priority (which Bohm would lose under any serious analysis) one would expect him to have paid more attention to finding out exactly what it was that de Broglie had done. However, this passage does express the common viewpoint:

"*The idea of a 'pilot wave' that guides the movement of the electron was first suggested by de Broglie in 1927, but only in connection with the one-body system. De Broglie presented this idea at the 1927 Solvay Congress where it was strongly criticised by Pauli. His most important criticism was that, in a two-body scattering process, the model could not be applied coherently. In consequence de Broglie abandoned his suggestion. The idea of a pilot wave was proposed again in 1952 by Bohm in which an interpretation for the many-body system was given. This latter made it possible to answer Pauli's criticism.*"

The green remarks are incorrect or misleading. Bohm's character was such that he was simply not interested in historical questions of priority.

Last week I was at the '*New Perspectives on the Quantum State*' conference at the Perimeter Institute, Canada. Everyone in quantum foundations was there, so I report to you from the cutting edge:

- Only two interpretations were presented in more than one lecture: de Broglie-Bohm pilot-wave theory and 'quantum Bayesianism'. These two approaches dominated the discussions.
- Stating (as is usual) that 99 per cent of physicists prefer the Copenhagen interpretation or the many worlds interpretation is thus certainly *not* true amongst guys who do this for a living.

The Quantum Bayesianists - led with great energy by Christopher Fuchs - are basically quantum information theorists. For them the wave function refers only to *what degrees of belief one has about what the outcomes of measurement will be*. They seem to aggressively promote the idea that anyone who believes otherwise is basically insane. Maybe, but it is interesting that one can take any historical discussion of Ernst Mach and his followers (who refused to contemplate the necessity of believing in atoms c.1900), make some simple substitutions [Mach $\longrightarrow$ Fuchs, Einstein $\longrightarrow$ Valentini (say), Boltzmann $\longrightarrow$ de Broglie or Bohm], and still end up with a paragraph that makes sense. Try it:

"Einstein contrasted the fruitfulness of the realist approach with the sterility of the Machian system. Mach was able to relate the various data of experience to form what Einstein called a 'catalogue'. However for Mach there was no hypothesized centre of realism, which could be used as an intellectual resource even by scientists of moderate ability to produce new ideas and discoveries. Mach himself did produce important results, which suggests that, in the hands of a scientist of the greatest ability, his approach did not prevent the production of useful progress. However.. his devoted followers themselves produced little if any science that should be remembered."

Home and Whitaker, *Einstein's Struggles with Quantum Theory* book

Conclusions: why bother with pilot-wave theory?

If you subtract all the now discredited positivistic philosophy from ordinary QM, what you are left with is pilot-wave theory. It is in no sense a 'weird addition to QM'. Other than it being in full agreement with experiment and the fact that it 'makes sense', I make no claims about whether pilot-wave theory is a correct description of Nature. However, in my opinion, it is is useful to study it because:

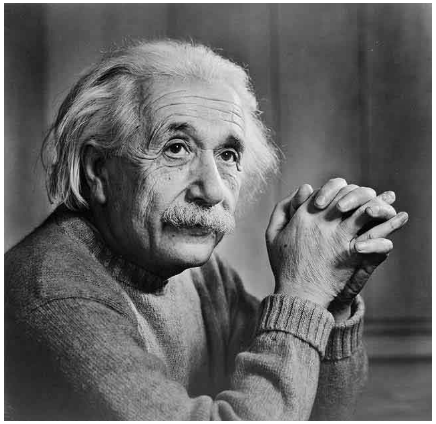
- Everyone needs an ontology - this is the most straightforward one.
- We can explain the probabilities that appear in the instrumental theory.
- We can explain the existence of things that are *assumed* in the instrumental theory.
- It may suggest research towards a theory that might supersede quantum theory.
- It makes testable predictions (e.g. Valentini's non-equilibrium stuff).

At the Perimeter Institute, Travis Norsen suggested current debate about whether Ψ is 'ontic' or 'epistemic' (i.e. reality or 'knowledge') is basically sterile. Should instead first classify theories based on what they say exists - what their *beable*s are - then (later) decide which theory we think is *true*. "*Thus, we have theories according to which* ψ *is a beable.. and theories according to which only* other *things exist, i.e. theories according to which* ψ *doesn't exist.. it is nothing. That it may still serve some practical (epistemic) purpose for human theorists to think about* ψ *- even if it doesn't exist - seems to be of (at best) secondary importance.*" I agree with this (highly unfashionable!) statement.

Listen to Bertrand Russell

"Philosophy is to be studied, not for the sake of any definite answers to its questions since no definite answers can, as a rule, be known to be true, but rather for the sake of the questions themselves: because these questions enlarge our conception of what is possible, enrich our intellectual imagination and diminish the dogmatic assurance which closes the mind against speculation."

On being wrong



Also - you've seen why everyone in my title was wrong about something - principally because they said that what de Broglie and Bohm had done was impossible (despite the fact that it clearly was not). But what about *Einstein*? He supported the idea of hidden variables. He encouraged de Broglie. He encouraged Bohm. In the 1920s he even came up with a pilot-wave type theory himself. However, he withdrew it from publication at the last minute. Years later he in fact said in a private letter to Born: "*Have you heard that Bohm believes (as de Broglie did, by the way, 25 years ago) that he is able to interpret the quantum theory in deterministic terms? That way seems too cheap to me*".

Whatever Einstein thought about the de Broglie-Bohm theory, he was wrong in the following sense. His attitude, I think, was a major tactical mistake in his war with the Bohrians. It would have been quite possible for him to have stressed that it was a clear counter example to Copenhagen, and it would have acted very much as the kind of debating point that Einstein could have used to great effect. At the same time Einstein, if he wished, could have pointed out that it was not a complete and satisfactory solution to the riddles of quantum theory. For that, he might have said, one would have to wait for the coming to fruition of Einstein's own great work.

Perhaps, as always, he knew something we don't. But it still seems wrong to me.

> *With hindsight we can now see how impractical, inhibiting ideas came to dominate and distort the entire development of quantum theory. The early quantum physicists attributed to nature a limitation we can now see was simply a deficiency of contemporary thought.* [Holland, 1993]

Acknowledgements

The material in this lecture is largely a summary of standard works and may contain quotes from and references to the work of P. Holland, D. Bohm, B. Hiley, D. Dürr, S. Teufel, A. Valentini, S. Goldstein, P. Riggs, T. Norsen, H. Wiseman, H. Westman, T. Schick, T. Maudlin, G. Bacciagaluppi, C. Fuchs, L. Vaughan, M. Beller, S. Notley, A. Bennett, and possibly others. A list of works consulted in the preparation of these and other slides is available on the web site of my graduate lecture course (click 'Further Reading').

www.tcm.phy.cam.ac.uk/∼mdt26/pilot_waves.html

MDT