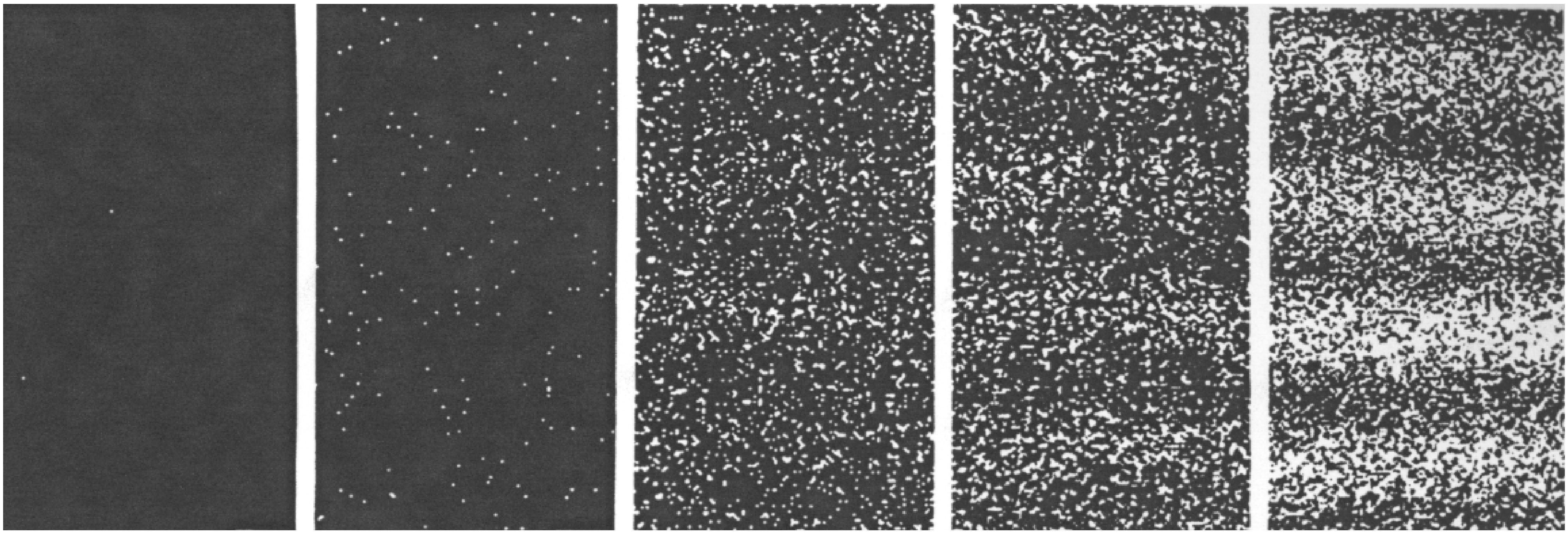
Pilot-wave theory, Bohmian metaphysics, and the foundations of quantum mechanics

Lecture 5

Nonlocality, relativistic spacetime, and quantum equilibrium



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A philosophical note

Metaphysics: *The study of the ultimate nature of reality and our place in it.*

An audience member has observed I have **too many quotations** and not enough equations. Fair enough. But physicists should consider that *philosophical* texts largely consist of repeated quotations followed by discussion. In following this tradition (and adding equations to the mix) I acknowledge that these lectures are about both physics *and* philosophy..

So far we considered consequences of dropping 'logical positivism' (science must be about things that can be observed or measured) in *quantum mechanics*. Today we shall see positivism had an equally essential role in the epistemological foundations of *relativity*; the consequences of dispensing with it are very interesting indeed!

> "*The central event* [in philosophy] *during the second half of the twentieth century has been the downfall of positivism and the reopening of virtually all the traditional problems in philosophy.*" **T. Burge** (1992)

This philosophical event has largely gone unnoticed in theoretical physics. In realizing positivism must be abandoned, it becomes clear people who study metaphysical questions must no longer *automatically* be considered by physicists to be nutters.

But do check with the 'Crackpot Index': `math.ucr.edu/home/baez/crackpot.html`

J.S. Bell - a quote for every occasion

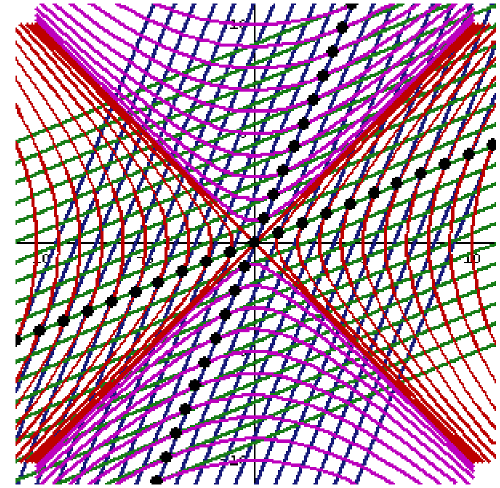
“ [The usual quantum] *paradoxes are simply disposed of by the 1952 theory of Bohm, leaving as the question, the question of Lorentz invariance. So one of my missions in life is to get people to see that if they want to talk about the problems of quantum mechanics - the real problems of quantum mechanics - they must be talking about Lorentz invariance.* ”

(1990, Interview with philosopher Renée Weber)

Lorentz invariance

A symmetry that leads to the laws of physics being the same for all observers.

Key property of spacetime that follows from special relativity.



1. *Physical quantity* is Lorentz covariant if it transforms under representation of group of all *Lorentz transformations* of Minkowski spacetime - the setting for all (non-gravitational) physical phenomena. Such quantities are built from scalars, four-vectors, four-tensors, and spinors. In particular, a scalar (like the space-time interval) remains same under Lorentz transformations - it is a *Lorentz invariant*.
2. An *equation* is Lorentz covariant (or invariant!) if it is expressible in terms of Lorentz covariant quantities. Key property: if equations hold in one inertial frame they hold in any inertial frame i.e. require all non-gravitational laws to make same predictions for identical experiments taking place at same spacetime event in two different inertial frames of reference.

Lorentz transformation converts between two observers' measurements of space and time if one in constant motion with respect to other (different inertial frames). Classically, only conversion believed necessary was $x' = x - vt$ (Galilean relativity) - describes how origin of one observer's coord system slides through space with respect to other at speed v and along x-axis of each frame - in special relativity only good approx if $v \ll c$. Don't just offset x coords; lengths and times distorted too.

Standard configuration (coord systems aligned; same origin; v along x-axis) Lorentz transformations:
$t' = \gamma(t - vx/c^2)$, $x' = \gamma(x - vt)$, $y' = y$, $z' = z$ with the Lorentz factor $\gamma = 1/\sqrt{1 - v^2/c^2}$.

A dissident in the audience..

See MD problems on `www.bss.phy.cam.ac.uk/∼mjd1014`.
I asked Antony Valentini to address them:

MD: On a technical level, the main problem, as far as I can see, with the Bohm interpretation is compatibility with special relativity theory. AV: "*Well, it is compatible with special relativity in the equilibrium domain we experience. Special relativity was developed for and tested in that domain. Why assume it must remain valid outside that domain? Not a scientific argument. Theories often end up failing at some deeper level.*"

MD: Nonlocality is also much more difficult to make sense of in Bohm theory than in a many-minds interpretation. AV: "*Actually, pilot-wave theory is the one theory that gives an explicit and clear account of nonlocality as a fact of nature.*"

MD: Another issue has to do with quantum field theory. In conventional quantum field theory, second quantization gives a well-defined mathematical framework within which both quasi-classical particle-like states and quasi-classical field-like states exist. These states can be used, in the appropriate contexts, to describe the same physical objects. Correspondingly, there are two forms of the Bohm interpretation; one in which the fundamental entities are quasi-classical particles and one in which they are quasi-classical fields. These fundamental entities are piloted by a wave function, which is Everett's [Schrödinger's!!] non-collapsing universal wave function. Unfortunately, in Bohm theory, the two types of fundamental entity would seem to be totally irreconcilable. They cannot be both used for the same objects in different circumstances. AV: "*Perhaps standard QFT needs two different descriptions of the same object, but not so in de Broglie-Bohm. The best current pilot-wave models use fields for bosons and particles for fermions. In all cases studied, the usual predictions are reproduced. One doesn't need both a field and particle ontology for the same object.*"

The scope of this lecture

The issues raised by Matthew and Antony touch on a large number of issues, many of them mathematically sophisticated and beyond the scope of this course. Today we shall only consider a few of them. In particular we shall examine the following topics:

(1) What is quantum nonlocality in a general sense and what does it tell us about space and time? If you don't like what it tells you, is there any way around it?

(2) Is pilot-wave theory compatible with special relativity? How does it give an account of nonlocality?

(3) How does *quantum equilibrium* work and what is its relevance to relativistic questions? Are there circumstances in which we *could* send signals faster than light?

(4) Could pilot-wave theory be telling us anything important which has relevance to development of physical theories with broader scope?

We shall **not** *consider:*

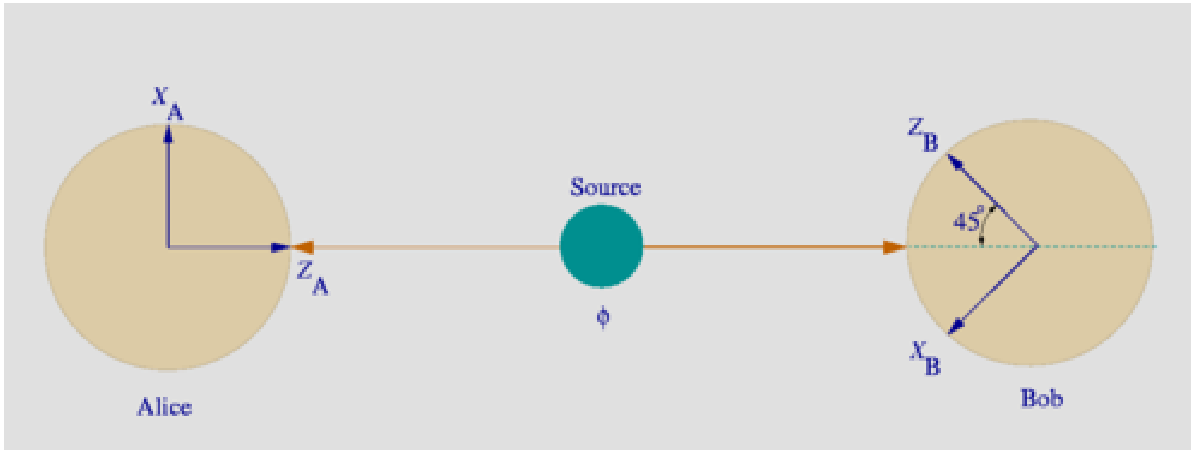
(1) Quantum field theory, string theory, creation and destruction of particles etc..

(2) Pilot-wave field theories, and proper treatments of spin or bosons.

For now we just want to understand non-relativistic Schrödinger theory in its domain of validity, and a few consequences of trying to use relativistic wave equations.

Nonlocality

Definition: *a direct influence of one object on another, distant object, contrary to our expectation that an object is influenced directly only by its immediate surroundings*.



What Einstein-Podolsky-Rosen (EPR) experiment implies:

- Measurement on one side instantly predicts result on other (parallel analyzers).
- If do not believe one side can have causal influence on other, require results on both sides to be determined in advance. But this has implications for non-parallel settings which conflict with quantum mechanics (*Bell*).

Bell's analysis showed that *any account* of quantum phenomena needs to be non-local, not just any 'hidden variables' account i.e. nonlocality is implied by the predictions of standard quantum theory itself. Thus, if nature is governed by these predictions (which it is, according to real experiments) then nature is non-local.

You might expect I should now prove Bell's theorem, but this is well known for 40 years: look it up.

But that's crazy!

Nonlocality - instantaneous action at a distance - sounds strange and yet it is experimentally verifiable. If I want to refuse to believe it, what are my options?

- Loopholes: claim that improving detector efficiencies in the EPR-style experiments will invalidate the results. Usually taken to be desperate clutching-at-straws.
- Deny, in one way or another, that there is a material world - the description of which is the task of physics. Without objective reality, there is nothing to be objectively nonlocal. But can reality be seriously questioned?
- Be a many-worlds person, i.e. make two problems - nonlocality and macroscopic superpositions in measurement - go away, at the cost of believing in something apparently ludicrous (bazillions of ontologically-real extra universes) on the basis of assigning an entire universe to each term in a mathematical expansion.
- Allow time travel into the past. 

The following is apparently an option, but can be demonstrated to be false:

- Say universe is necessarily Lorentz invariant and Lorentz invariance of a physical theory requires locality. (Nonlocal Lorentz-invariant Wheeler-Feynman electrodynamics good counterexample). Should be clear from meaning: 'Lorentz invariance' describes behaviour of a theory under certain transformations of reference frame. 'Locality' implies there is no action at a distance.

On objections to the concept of nonlocality

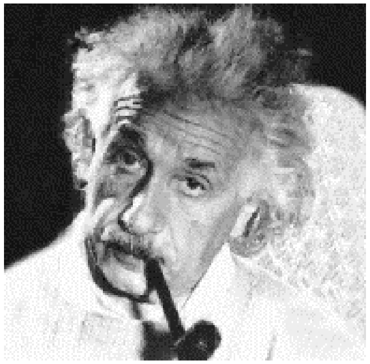
General mode of explanation currently acceptable either action through contact or action propagated by fields. Common belief that nonlocality concept incompatible with possibility of doing science at all!

However..

- Nonlocality perfectly rational in way used in QM. Leads neither to internal logical contradiction nor disagreement with any facts.
- Requires nothing that disagrees with scientific approach. Might think no way to do science if everything strongly nonlocally connected as we can't isolate system enough to study it, but can show nonlocal effects not significant at large scale level.
- Doesn't violate special relativity since (see later) no faster-than-light signal transmission via the statistical measurements basic to standard QM (or in pilot-wave theory). Thus no way to exert instant control over what happens at faraway places, nor to transmit *information* to such places.
- Require systems to have interacted and become entangled in past - so arguments like 'Aliens on the planet Zarb can make my head fall off from the other side of the universe' not strictly relevant.
- Often hear c *limiting* speed for causal processes or signalling but not so - in the first instance it is the *invariant* speed of the theory. Best known counterexample is *tachyons* - hypothetical particles with negative mass that move faster than c and fully respect relativity.
- Faster than light communication implies superluminal signals received before emitted in some frames, so could arrange for return signal that results in original signal not being transmitted. Clearly such loops must be impossible. Though loops can't be removed from strictly Lorentz covariant theories with equivalent frames, this isn't necessarily a problem *in general* (later).

For now we shall presume (with most physicists) that non-locality has been experimentally confirmed.

An implication of the experimental confirmation of nonlocality



"*We have to give up Einstein's interpretation of special relativity and return to Lorentz's interpretation and with it to . . . absolute space and time. . . . The reason for this assertion is that the mere existence of an infinite velocity entails* [the existence] *of an absolute simultaneity and thereby of an absolute space. Whether or not an infinite velocity can be attained in the transmission of signals is irrelevant for this argument: the one inertial system for which Einsteinian simultaneity coincides with absolute simultaneity . . . would be the system at absolute rest - whether or not this system of absolute rest can be experimentally identified.*"

K.R. Popper (1982)

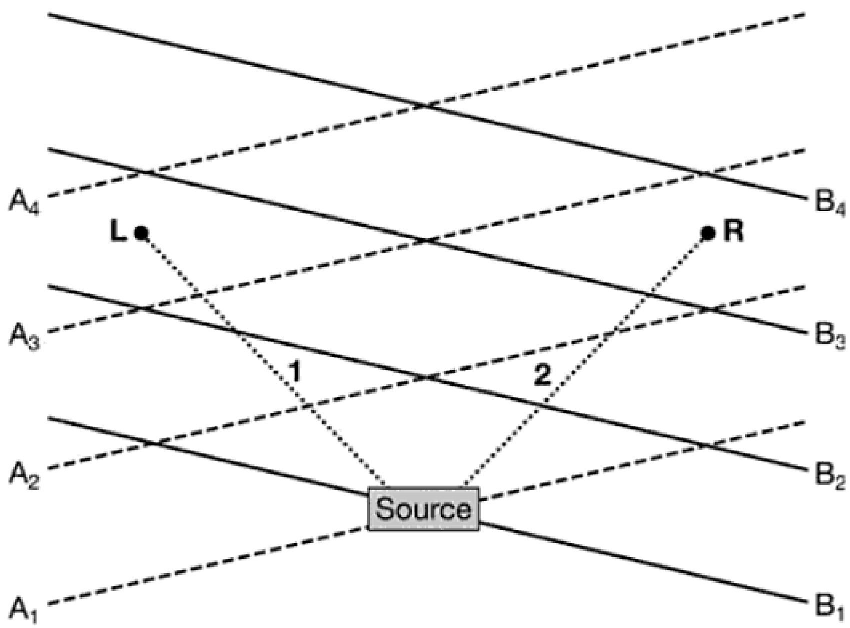
Lorentz ether theory

Popper is essentially suggesting a return to the Lorentz ether theory (LET) instead of special relativity (SR). Before you run screaming from the room at the mention of the word 'ether', let us examine whether this claim makes sense.

- *Ether*, or *aether*, is a supposed space-filling immobile substance or field, thought long ago to be necessary as a transmission medium (something for 'electromagnetic waves to wave in'). Also defines state of absolute rest (preferred reference frame). Experimentally found undetectable.
- From 1892-1906 Lorentz and Poincaré progressively abstracted away all ether mechanical properties except reference frame. Moving relative to this frame introduces length contraction/time dilation via Lorentz transformations. Clocks in this frame show 'real' time. Simultaneity *not* relative.
- Einstein suggested Lorentz transformation represents fundamental relations between space and time coordinates of inertial frames of reference (of which none is 'preferred') $\Rightarrow$ SR.
- Not possible to distinguish LET/SR by experiment - thus they are 'interpretations' of relativity theory. SR usually preferred to LET as existence of undetectable reference frame assumed and validity of relativity principle seems coincidental. SR thus *positivist metaphysical preference*. Similar to QM: Can't detect trajectories $\longrightarrow$ no pilot-waves. Can't detect preferred frame $\longrightarrow$ no Lorentz.

Popper pointing out **nonlocality defines preferred frame**. *Also - away from 'quantum equilibrium'* **can** *detect trajectories* **and** *preferred frame (see later). Premature to discard 'ether'/pilot-waves..?*

Absolute simultaneity: why nonlocality defines preferred frame



Two spin-$\frac{1}{2}$ particles in singlet state emerge from common source. Particle 1 moves to **L** *and particle 2 to* **R**. *At event* **L**, *particle 1 is measured for x-spin, and at* **R** *(spacelike related to* **L**, *i.e. out of speed of light contact) particle 2 is measured for z-spin.*

In Minkowski diagram, relativity of simultaneity implies separate path axis for differently moving observers. Each observer interprets all events on line parallel to his path axis as simultaneous. Such lines A$_i$ and B$_i$ shown for observers A and B at four different times. In frame of A then **R** happens first. In frame of B then **L** happens first.

A1: $\frac{1}{\sqrt{2}}(|\uparrow_x\rangle^1|\downarrow_x\rangle^2 - |\downarrow_x\rangle^1|\uparrow_x\rangle^2)$
A3: $|\downarrow_z\rangle^1|\uparrow_z\rangle^2$
A4: $|\uparrow_x\rangle^1|\uparrow_z\rangle^2$

B1: $\frac{1}{\sqrt{2}}(|\uparrow_x\rangle^1|\downarrow_x\rangle^2 - |\downarrow_x\rangle^1|\uparrow_x\rangle^2)$
B3: $|\uparrow_x\rangle^1|\downarrow_x\rangle^2$
B4: $|\uparrow_x\rangle^1|\uparrow_z\rangle^2$

- Above shows for real wave 'collapse' theory History A completely different from History B as a matter of fact. With hidden variables, even *final outcome* depends on who seen to measure first.
- Thus if 'something' is real (be it particle, wave, both, or something else) then nonlocality both exists and defines preferred frame (unless one denies correlations occur at all as in many-worlds).

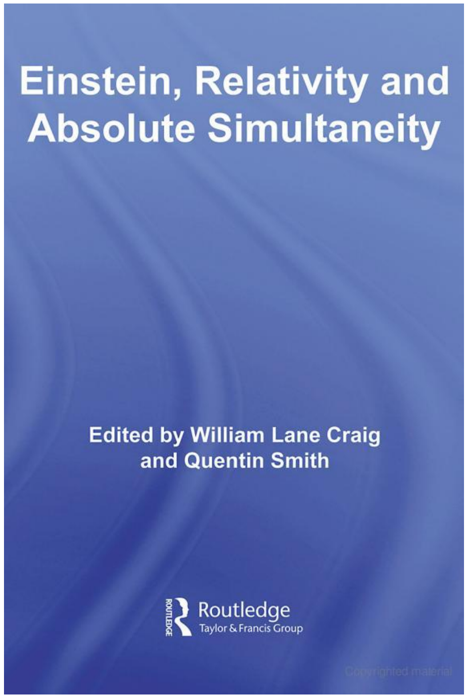
So 'wave function collapse' (e.g. in GRW) doesn't make sense in usual Minkowski spacetime. Collapse can be instantaneous in at most one reference frame, so either (i) some feature of situation picks out preferred frame, or (ii) collapse not instantaneous at all (which has weird implications).

Bell ponders the non-locality experiments..

"I think it's a deep dilemma, and the resolution of it will not be trivial; it will require a substantial change in the way we look at things. But I would say that the cheapest resolution is something like going back to relativity as it was before Einstein, when people like Lorentz and Poincaré thought that there was an aether - a preferred frame of reference - but that our measuring instruments were distorted by motion in such a way that we could not detect motion through the aether. . . . that is certainly the cheapest solution. Behind the apparent Lorentz invariance of the phenomena, there is a deeper level which is not Lorentz invariant. . . . what is not sufficiently emphasized in textbooks, in my opinion, is that the pre-Einstein position of Lorentz and Poincaré, Larmor and Fitzgerald was perfectly coherent, and is not inconsistent with relativity theory. The idea that there is an aether, and these Fitzgerald contractions and Larmor dilations occur, and that as a result the instruments do not detect motion through the aether - that is a perfectly coherent point of view. . . . The reason I want to go back to the idea of an aether here is because in these EPR experiments there is the suggestion that behind the scenes something is going faster than light. Now if all Lorentz frames are equivalent, that also means that things can go backwards in time. . . . [this] *introduces great problems, paradoxes of causality, and so on. And so it is precisely to avoid these that I want to say there is a real causal sequence which is defined in the aether."*

J.S. Bell (1986, Interview in Davis and Brown's *The Ghost in the Atom*)

A fun book to read..



"2005 marked the centenary of one of the most remarkable publications in the history of science, Albert Einstein's 'On the Electrodynamics of Moving Bodies' in which he presented a theory that later came to be known as the Special Theory of Relativity. This 1905 paper is widely regarded as having destroyed the classical conceptions of absolute time and space, along with absolute simultaneity and absolute length, which had reigned in physics from the times of Galileo and Newton to the dawn of the twentieth century. As we embark upon a new century, the Special Theory is now 100 years old, and a great deal has transpired in both philosophy and physics since its first publication. This volume is a timely reappraisal of the theory's central claims, especially concerning the elimination of absolute time and absolute simultaneity."

"This collection draws together essays by both philosophers and physicists and reflects the cutting edge of research and thought on the question of absolute simultaneity. The issues discussed in the book include Aspect's confirmation of Bell's theorem, de Broglie-Bohm's quantum mechanics, the privileged cosmic time series in a Friedman universe, Lorentz's ideas and neo-Lorentzian theory and other relevant issues. Almost all the contributors are convinced that the received view that simultaneity is not an absolute relation is not only unwarranted but false, and it is hoped that this collection will stimulate discussion among both philosophers and physicists concerning the warrant for and problems with assertions of the relativity of simultaneity on the basis of Einstein's theory."

See especially the Introduction and the essays by Craig, Smith, Maudlin, and Valentini.

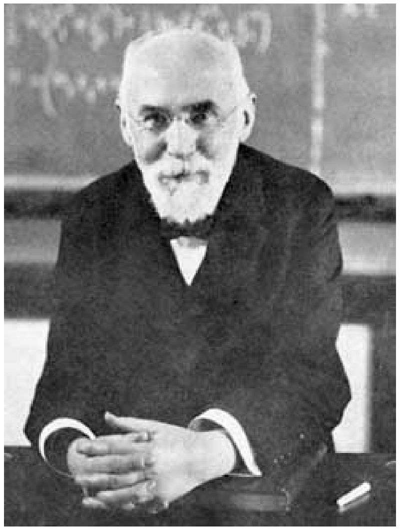
Very interesting!

Three interpretations of relativity theory

Experimentally verifiable facts: $\mathbf{x}$ *and* t *coords of events, measured in any inertial reference system, are related to* $\mathbf{x}$ *and* t *coords of same events, as measured in any other inertial reference system, by the Lorentz transformations.*

1. **Einsteinian interpretation**: Classical 3+1 ontology of space *and* time. Inertial frames all equivalent. Problems: Can be regarded as fantastical as events pop in and out of reality as we switch reference frames. Length contraction/time dilation *real physical effects* (not observation perspective) but unclear why objects enduring through time suffer these (reciprocal!) effects just because in relative motion.
2. **Minkowksian interpretation**: 3d objects don't suffer length contraction/time dilation for simple reason that 3d objects don't exist. Reality is *4-dimensional* i.e. spacetime. 'Ball moving in space' actually 4d *worldtube* with all 3d spacelike slices equally real. Length contraction/time dilation 'perspective effects'. Problems: No objective distinction between past, present and future - can't speak of objective present world in space or of 'temporal becoming' (consciousness?). All 3d observers at different times equally real - nothing singles out one as more real than others.
3. **Lorentzian interpretation**: Preserves classical notions of 3+1 space and time. Single preferred frame ('ether' if you like) and absolute simultaneity. Causal explanation for length contraction/time dilation with respect to absolute space. *Only* interpretation with unified objective reality, temporal becoming, and causal explanations. Problems: Usually stated can't detect preferred frame (not true?). Dangerous - supporters have a tendency to get beaten up (rather like Bohmists).

We shall take the Lorentzian viewpoint in this course



- Quantum nonlocality defines absolute 3+1 slicing of spacetime, leading us back to 'more natural' view that world is single 3d spatially-extended reality, capable of change, parametrized by time t. Surely better than 4d Minkowski view that an uncountable multiplicity of equally-real 3d observers exist with no temporal becoming. Logically possible (like many worlds) but Lorentz view seems more satisfying; spacetime slicing by quantum nonlocality enables us to take that view.
- Superluminal signalling violates causality only *if* one assumes a locally Minkowski structure for spacetime, and not with Lorentz. Historically, Minkowski structure was developed for a local physics. If Nature turns out to be nonlocal, then one should consider revising that structure.
- We love siding with the underdog, clearly.

How do things work in a Lorentzian universe?

- Nonlocality defines universe-wide absolute simultaneity $\Rightarrow$ global absolute separation of past and future $\Rightarrow$ cosmological basis for universal time measure t (same for resting or moving bodies). Unique spacetime foliation $\Rightarrow$ absolute space where $\mathbf{x}$ coords measure real, not apparent, values.
- Galilean transformation (adding velocities) relates moving body coords to rest frame coords. Omitting this is the one mathematical step leading to Einsteinian relativity rather than Lorentzian.
- Anything at rest in absolute space ('ether') obeys Maxwell equations, but Galilean transformations don't allow Maxwell's equations to be expressed similarly in moving frames. More complex expressions using Lorentz transformed variables $\mathbf{x}'$ and t' needed to express them in moving frames.
- For moving observer effect of motion in ether means velocity addition law no longer true. Causal explanation: spatial length of object shortened in direction of motion by factor $(1 - v^2/c^2)^{1/2}$. Can be though of as a sort of 'Doppler effect': bodies held together by electrical forces; electric field of moving charge distribution undergoes distortion with longitudinal components affected by the motion but transverse ones not (Heaviside - hard maths). Thus moving rod contracts/moving clock dilates because of how it is made up and not because of nature of its spatiotemporal environment.
- In moving frames, coords just record contraction effects caused by motion through space. Causal activity explains why lengths contract and physical processes are retarded . Also explains why speed of light appears constant in every frame, enabling Maxwell's equations to appear same in any frame, absolutely at rest or absolutely moving.
- In Einstein STR, constant speed of light in all inertial frames essentially bizarre. Lorentz gives reasonable explanation: light speed constant with respect to ether and measured as isotropic *only* in that frame. Contracted moving experimenters assuming speed of light still c in all directions adjust their clocks at different space points so settings differ by $-vx/c^2$ (to lowest order in v/c).

Relativistic extensions of pilot-wave theory

So far pilot-wave theory based on Schrödinger equation of non-relativistic QM. Therefore suitable only for low energy processes. Can trajectory concept be retained in relativistic quantum domain?

- No problem for *single-particles*. Use wave function $\psi(x^\mu)$ and particle trajectory given by integral curve of a four-vector field j^μ (of current naturally associated with e.g. the wave function of the Dirac equation). Dirac current a timelike future-oriented vector: relevant curves run from $t = -\infty$ to $t = +\infty$, never backwards in time, with velocity everywhere bounded by c. Equivariant density $j^0 = \psi^\dagger\psi$ gives 'quantum equilibrium' in all Lorentz frames at all times.
- For many-particle systems [$\Psi(\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_N, t)$] not obvious how to construct strictly Lorentz invariant realistic quantum theory or even whether possible at all. Problem stems from *unavoidable* nonlocality of any precise quantum theory [See Bell *Speakable and Unspeakable* book p. 171, 194.]

Unresolved issue here is extent to which should require a quantum theory to be 'relativistic'. In pilot-wave theory relativity turns out to be *statistically* valid but individual events do not have an intrinsically relativistic character. Does this matter?

Interesting suggestion which I read somewhere: physical interpretation of QM is best effected in the Schrödinger picture of the underlying theory.

Kinematics and dynamics

Nature described by *kinematics* - which defines structure of space-time - and *dynamics* - accounts for motion within this structure (in so far as it deviates from 'natural' force-free state).

Division not uniquely defined. In general relativity assume curved spacetime (phenomena accounted for by kinematics of spacetime itself) but also may think in terms of flat spacetime on which some 'field' distorts rods and clocks to give *appearance* of curved geometry (phenomena accounted for by dynamical influences on flat spacetime). Impossible to say which picture is 'true'.

- That said, flat spacetime usually regarded as mistake since whole purpose of kinematics is to embrace *universal* features of the dynamics. Any effects found to be independent of the particular material bodies involved are best assumed to be part of the kinematics.
- Leads to 'zeroth law of mechanics': *Behaviour of free bodies is independent of their mass and composition*. Definition of 'free bodies' implicitly assumes (i) forces have their origin in other bodies; (ii) effect of these forces diminishes with distance. This is inappropriate in pilot-wave theory, where force has origin in wave function and doesn't necessarily diminish with distance.
- If *given* a theory, natural definition of kinematics singled out by theory itself; to embrace universal features of the dynamics ask the theory what 'universal features' there are in Nature.
- So issue is not whether fundamental Lorentz invariance is possible, but whether it is *suitable* (as always with kinematics). Does pilot-wave theory define a '**natural kinematics**'?

[See A. Valentini, 'On Galilean and Lorentz invariance in pilot-wave dynamics', *Phys. Lett. A* **228**, 215 (1997).]

Valentini's Aristotelian spacetime

So possible in principle, in virtually any theory, to adopt virtually any spacetime structure, providing add appropriate compensating dynamical factors. What structure suitable for pilot-wave theory?

First find natural definition of 'free system':

- **Newtonian mechanics**: Forces cause of motion. System considered free if RHS of $m\ddot{x} = -\nabla V$ zero. Set of natural motions $x(t) = x(0) + vt$ with v and $x(0)$ arbitrary constants. Uniform motion natural state. Galilean invariance fundamental symmetry.
- **General relativity**: Define 'free body' as one on which no nongravitational forces act. Resulting trajectories independent of body's mass and composition so assume property of spacetime itself. Hence picture of *curved geodesics*.
- **Pilot-wave theory**: System 'free' if 'Aristotelian force' appearing on RHS of $m\dot{x} = \nabla S$ vanishes. Natural motions then $x(t) = x(0)$ with $x(0)$ arbitrary constants. Trivial trajectories independent of mass $\Rightarrow$ 'Aristotelian spacetime' in which rest is only reasonable definition of 'natural' or 'unforced' motion, essentially because the dynamics is first order in time.

Galilean invariance not a fundamental symmetry of the standard low-energy pilot-wave theory. The search for a Lorentz-invariant extension thus seems misguided. In Valentini's view, the difficulties encountered in such a search are no reflection on the plausibility of the pilot-wave theory. Rather, they show that the theory is not being interpreted correctly.

Pilot-wave theory then has a remarkable internal logic - both structure of dynamics, and operational possibility of nonlocal signalling away from equilibrium (see later) independently point to existence of natural preferred state of rest.

Given nonlocality exists, why no superluminal communication?

- Correlations between different EPR subsystems nonlocal, in that they cannot be explained in a local manner in terms of prexisting properties before measurement. But can't exploit to transmit *information* since choice of state to which system 'collapses' is random. Can't choose to transmit message you want.
- In other words, the statistical distributions of properties at either end are just the normal ones. So quantum nonlocality cannot in fact be used for practical signalling at a distance; it is hidden by an all-pervading quantum noise.. This means that if there were a preferred rest frame, it would be undetectable in practice under normal conditions.
- However, turns out this is not fundamental constraint. From pilot-wave perspective, it is a peculiarity of a special 'quantum equilibrium' distribution of the 'hidden variables'. Our inability to detect rest frame not an uncomfortable conspiracy seemingly built into laws of physics: it's just an accident of our living in a state of quantum equilibrium whose statistical noise masks the underlying nonlocality.

Faster than light signals away from quantum equilibrium?

Signal-locality theorem of pilot-wave theory: *in general there are instantaneous signals at the statistical level if and only if ensemble in quantum* **non***equilibrium* $\rho \neq |\psi|^2$.

Have 2-state system with quantum ('spin') observables of form $\hat{\sigma} = \mathbf{m} \cdot \hat{\sigma}$, with $\mathbf{m}$ unit vector specifying point on Bloch sphere (the 'measurement axis') and $\hat{\sigma}$ the Pauli spin operator. Values $\sigma = \pm 1$ obtained on quantum measurement of $\hat{\sigma}$.

- In any (deterministic) hidden-variables theory, for every run of experiment with measurement axis $\mathbf{m}$ there are hidden parameters collectively denoted λ that determine outcome $\sigma = \pm 1$ according to some mapping $\sigma = \sigma(\mathbf{m}, \lambda)$. Over ensemble of experiments, observed distribution of outcomes explained by some distribution $\rho(\lambda)$ of parameters λ. Expectation value given by:

$$\langle \sigma(\mathbf{m}, \lambda) \rangle = \int \rho(\lambda) \sigma(\mathbf{m}, \lambda) \, \mathrm{d}\lambda.$$

- Clear distinction between λ (*initial conditions*) and mapping $\sigma(\mathbf{m}, \lambda)$ to final outcomes (*dynamical law*). Can thus have *arbitrary* $\rho(\lambda) \neq \rho_{QM}(\lambda)$ while *retaining* dynamical law; generically, such 'non-equilibrium' ρ give expectation values/statistics of outcomes that disagree with QM.
- For *pair* of widely-separated 2-state systems at A and B, QM predicts for singlet state $\sigma_A, \sigma_B = \pm 1$ in ratio 1:1 at each wing with a correlation $\langle \Psi | \hat{\sigma}_A \hat{\sigma}_B | \Psi \rangle = -\mathbf{m}_A \cdot \mathbf{m}_B$. Distant settings have no effect on expectation values $\langle \hat{\sigma}_{A,B} \rangle$ or on probabilities $\rho^{\pm}(\mathbf{m}_{A,B}) = \frac{1}{2}$ at each wing. No non-local signalling if $\rho = \rho_{QM}$.
- Bell's theorem tells us that to reproduce this correlation a hidden-variables theory must take the non-local form $\sigma_A = \sigma_A(\mathbf{m}_A, \mathbf{m}_B, \lambda)$, $\sigma_B = \sigma_B(\mathbf{m}_A, \mathbf{m}_B, \lambda)$. For arbitrary ensemble with $\rho \neq \rho_{QM}$, in general $\langle \sigma_A \sigma_B \rangle = \int \rho(\lambda) \sigma_A(\mathbf{m}_A, \mathbf{m}_B, \lambda) \sigma_B(\mathbf{m}_A, \mathbf{m}_B, \lambda) \, \mathrm{d}\lambda \neq -\mathbf{m}_A \cdot \mathbf{m}_B$ and outcomes in each wing not in ratio 1:1. Non-local instantaneous signalling!

Dynamical relaxation to quantum equilibrium

Pauli objection: Taking a particular distribution $P = |\Psi|^2$ as an initial condition is unjustified in a fundamentally deterministic theory, therefore de Broglie's 'theory' is rubbish. [in *Louis de Broglie: physicien et penseur*, 1953. Why can't he just leave poor Louis alone? And in a Festschrift too. Nasty man..]

However, Pauli is right: this should be *derived* from the dynamics, for QM truly to emerge as the statistical mechanics of an underlying deterministic theory.

Easy to show if $P(\mathbf{x},t) = |\Psi(\mathbf{x},t)|^2$ at any t it will always remain so under Schrödinger time evolution ('*equivariance*'). Can also show $|\Psi(\mathbf{x},t)|^2$ is *only* distribution with this property i.e. 'quantum equilibrium' is unique [Goldstein, Struyve 2007]. It is analagous to thermal equilibrium $P = \frac{\exp(-H/kT)}{Z}$.

Now if we want to search for quantum non-equilibrium, need to understand how get $P = |\Psi|^2$ in first place. Can use 'typicality' arguments to show particles expected to be distributed this way in 'typical universe'. For our purposes we must define approach to equilibrium through dynamical (or possibly stochastic) relaxation. We shall use an analogy with classical statistical mechanics to do this.

With all deterministic hidden-variable theories, including pilot-wave theory, the Born distribution should not be regarded as an axiom. Rather, it should be seen as dynamically generated, in the same sense that one usually regards thermal equilibrium as arising from a process of relaxation based on some underlying dynamics.

Approach to equilibrium in classical statistical mechanics

For a classical isolated system, both the probability density ρ and the volume element $\mathrm{d}\Omega$ (on phase space) are preserved along trajectories (Liouville's theorem).

- Despite fact that $\mathrm{d}\rho/\mathrm{d}t = 0$ we find ρ evolves in a highly complex 'filamentary' manner over energy surface so *on a coarse-grained level* ρ becomes uniform as expected (whatever its initial shape). 'Coarse graining' involves dividing phase space into little cells of volume δV and working with *average* of ρ in each cell ($\bar{\rho}$).
- Can quantify difference between ρ and $\rho_{uniform}$ with *classical H-function*, i.e. $H_{class} = \int \rho \ln \rho \, \mathrm{d}\Omega$. This is minus relative entropy of ρ with respect to $\rho_{uniform}$ (standard measure of difference between two distributions). H bounded below by zero, and equals zero if and only if ρ uniform on energy surface (equilibrium).
- Classical $H_{class} = \int \rho \ln \rho \; \mathrm{d}\Omega$ is *constant in time*. If replace fine-grained ρ by coarse-grained $\bar{\rho}$ and assume $\bar{\rho}_0 = \rho_0$ at $t = 0$, then $\bar{H}_{class}(t) \leq \bar{H}_{class}(0)$ for all t - which is the **classical coarse-graining H-theorem**: i.e. $\bar{H}_{class}$ either decreases or remains constant, $\mathrm{d}\bar{H}_{class}/\mathrm{d}t \leq 0$. Decrease of $\bar{H}_{class}$ corresponds to formation of structure in ρ and consequent approach of $\bar{\rho}$ to uniformity.
- Relies on assumption $\bar{\rho}(0) = \rho(0)$ in phase space, i.e. no fine-grained microstructure in initial conditions (which could lead to 'unlikely' entropy-decreasing behaviour). Assumption necessary owing to time-reversibility of the theory.

We can make analogy with subquantum case if we let $\mathrm{d}\Omega \to |\Psi|^2 \mathrm{d}\mathbf{x}$ and $\rho \to P$.

Subquantum H-theorem

A quite general argument (due to Valentini) for the relaxation $P \rightarrow |\Psi|^2$ may be framed in terms of an analogy with the classical coarse-graining H-theorem.

- For 'sufficiently complex' system assume have initial distribution $P(\mathbf{x}, 0)$ of configurations $\mathbf{x}(0)$ each guided by the same Ψ, with $P(\mathbf{x}, 0) \neq |\Psi(\mathbf{x}, 0)|^2$.
- By definition $P(\mathbf{x}, t)$ satisfies continuity equation $\partial P/\partial t + \nabla \cdot (\dot{\mathbf{x}} P) = 0$, and Schrödinger equation implies this is also satisfied by $|\Psi|^2$. Since $\dot{\mathbf{x}} = \nabla S/m$, clear that Ψ actually determines time evolution of P. So ratio $f = P/|\Psi|^2$ is preserved along trajectories: $\mathrm{d}f/\mathrm{d}t = \partial f/\partial t + \dot{\mathbf{x}} \cdot \nabla f = 0$.
- Initial deviations $P \neq |\Psi|^2$ thus forever carried along trajectories and never disappear, appearing to imply equilibrium unreachable (as with ρ in classical stat mech). We now define the *subquantum H-function*: $H = \int |\Psi|^2 f \ln f \ \mathrm{d}\mathbf{x} = \int P \ln(P/|\Psi|^2) \ \mathrm{d}\mathbf{x}$. Continuity equation and $\mathrm{d}f/\mathrm{d}t = 0$ imply $\mathrm{d}H/\mathrm{d}t = 0$ i.e. exact fine-grained H constant as in classical case.
- Divide config space into cells of volume δV and define coarse grained-quantities e.g. $\bar{P} = (1/\delta V) \int_{\delta V} P \ \mathrm{d}\mathbf{x}$ etc.. For coarse-grained H have $\mathrm{d}\bar{H}/\mathrm{d}t \leq 0$; necessary and sufficient condition for $\bar{H}$ to have minimum value is $P = |\Psi|^2 \Rightarrow$ *equilibrium*. Decrease of $\bar{H}$ corresponds to a 'stirring' of the two 'fluids' P and $|\Psi|^2$ by the same velocity field $\dot{\mathbf{x}}$ (since satisfy same continuity equation), making P and $|\Psi|^2$ less distinguishable on a coarse-grained level.

See literature for e.g. defining quantum equilibrium of *subsystems*, and defining 'relaxation times'.

Dynamical origin of quantum probabilities

Proc. Roy. Soc. A **461**, 253 (2005).

In this paper, Valentini and Westman show using explicit numerical simulations that $\rho \to |\Psi|^2$ arises naturally even from a grossly non-equilibrium particle distribution.

- System is a single particle in a 2D box with configuration $q = (x, y)$ and a (pure state) wave function $\psi(x, y, t)$ satisfying Schrödinger equation ($\hbar = 1$)

$$i\frac{\partial \psi}{\partial t} = -\frac{1}{2}\frac{\partial^2 \psi}{\partial x^2} - \frac{1}{2}\frac{\partial^2 \psi}{\partial y^2} + V\psi.$$

- Have ensemble of independent particles each guided by same ψ, so define density $\rho(x, y, t)$ of actual configurations. Guidance law $\mathrm{d}\mathbf{q}/\mathrm{d}t = \mathrm{Im}\,\nabla \ln \psi = \nabla S$ defines velocity field $(\dot{x}, \dot{y})$ which determines evolution of ρ via continuity equation:

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho \dot{x})}{\partial x} + \frac{\partial(\rho \dot{y})}{\partial y} = 0$$

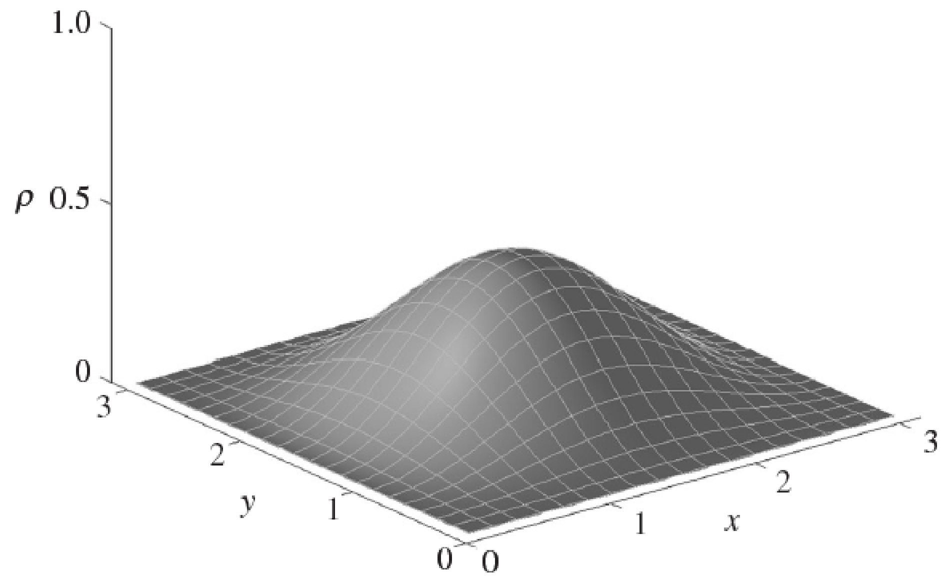
- Box has sides of length π with infinite barriers. The energy eigenfunctions are

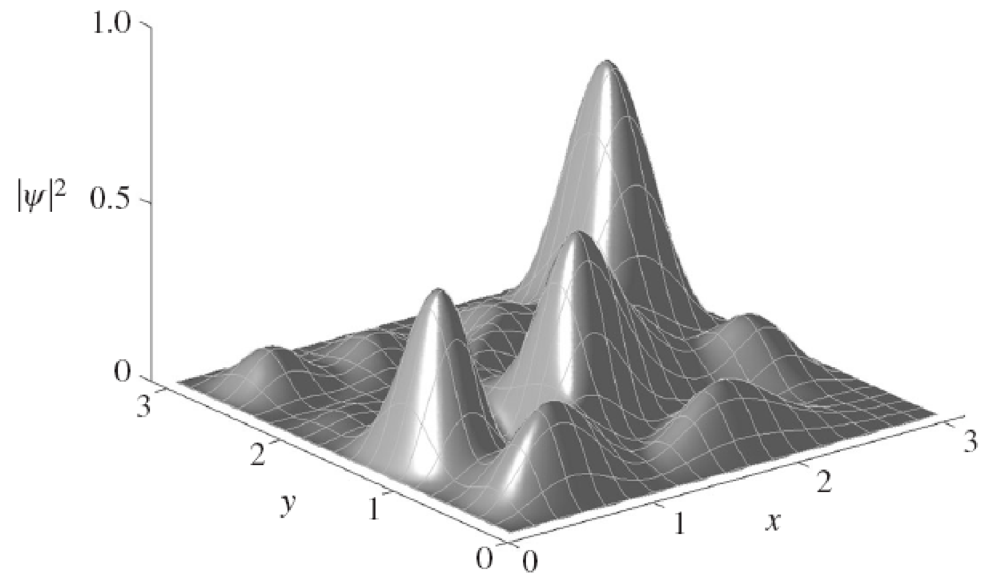
$$\phi_{mn}(x, y) = \frac{2}{\pi}\sin(mx)\sin(ny)$$

with energy eigenvalues $E_{mn} = \frac{1}{2}(m^2 + n^2)$, where $m, n = 1, 2, 3, \ldots$

Dynamical origin of quantum probabilities: numerical simulations

Starting conditions for the simulation: initial $\rho \neq |\Psi|^2$





Initial ψ is superposition of first 16 modes, $m, n = 1, 2, 3, 4, \ldots$ with equal amplitudes but randomly chosen phases θ_{mn}:

$$\psi(x, y, 0) = \sum_{m,n=1}^{4} \frac{1}{4} \phi_{mn}(x, y) \exp(i\theta_{mn})$$

$$\psi(x, y, t) = \sum_{m,n=1}^{4} \frac{1}{4} \phi_{mn}(x, y) \exp i(\theta_{mn} - E_{mn}t)$$

Note ψ periodic in time with period 4π (since $4\pi E_{mn}$ is always an integer multiple of 2π).

Want grossly non-equilibrium starting distribution for particles. Choose distribution equal to square of ground-state wave function:

$$\rho(x, y, 0) = |\phi_{11}(x, y)|^2$$

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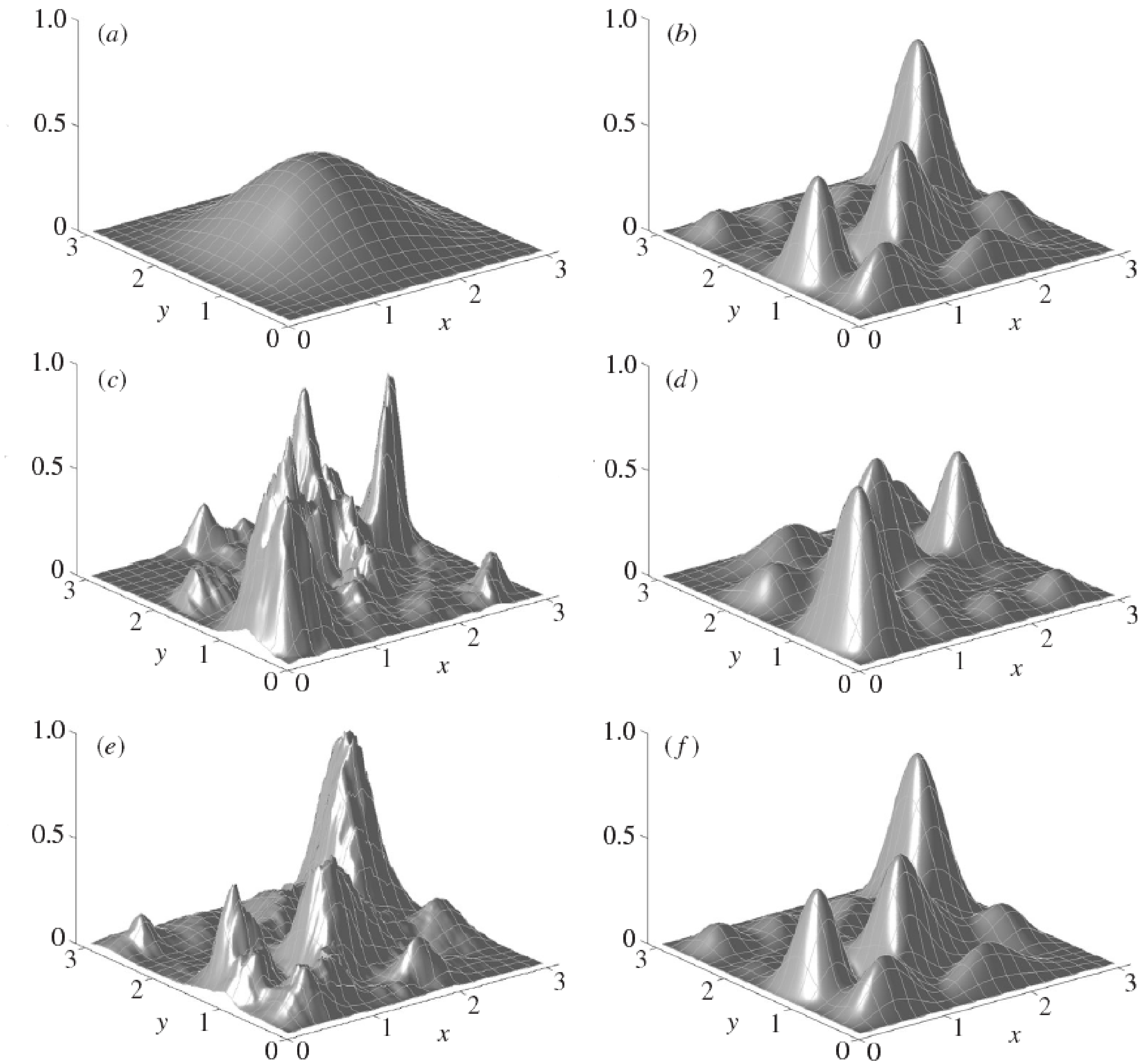
Dynamical origin of quantum probabilities: numerical simulations

Results of evolution

ρ



$|\Psi|^2$

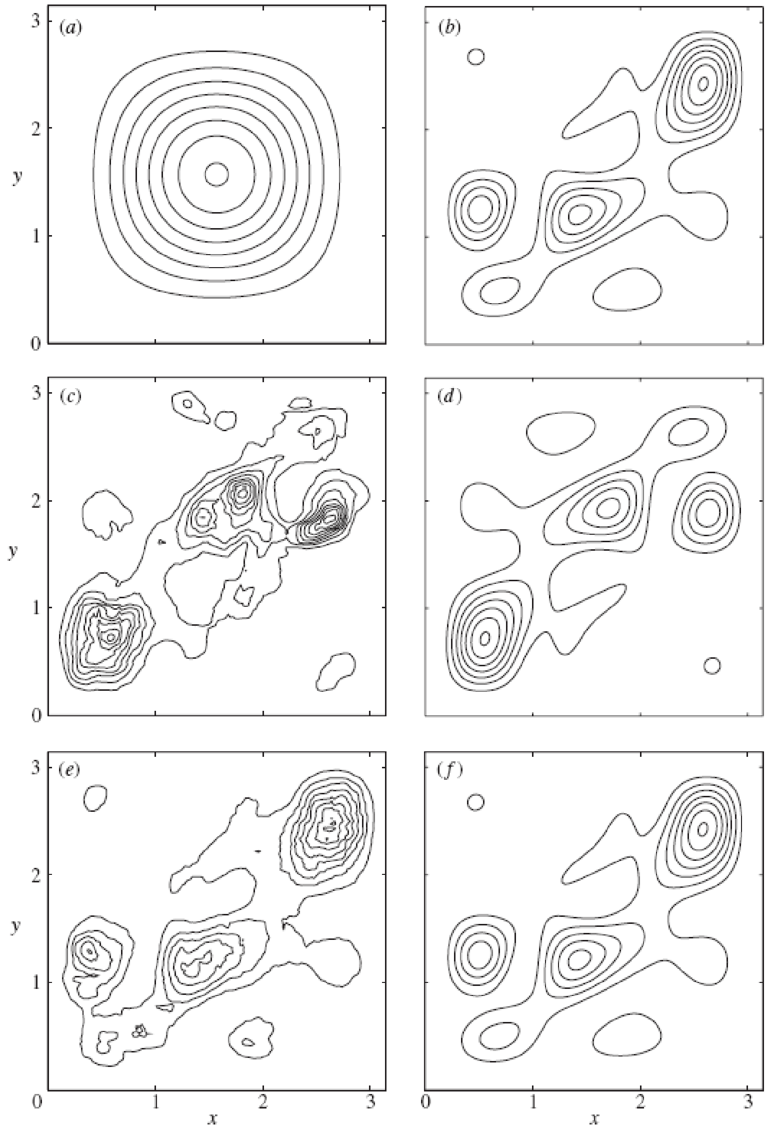
Results for $t = 0$ *(a,b), for* $t = 2\pi$ *(c,d) and for* $t = 4\pi$ *(e,f).*

While $|\Psi|^2$ *recurs to its initial value, the smoothed particle distribution* ρ *shows a remarkable evolution towards quantum equilibrium!*

Dynamical origin of quantum probabilities: numerical simulations

Results of evolution: contour plots

ρ



$|\Psi|^2$

Results for $t = 0$ *(a,b), for* $t = 2\pi$ *(c,d) and for* $t = 4\pi$ *(e,f).*
Same data as previous slide displayed as contour plots.

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Dynamical origin of quantum probabilities: numerical simulations

Character of the trajectories

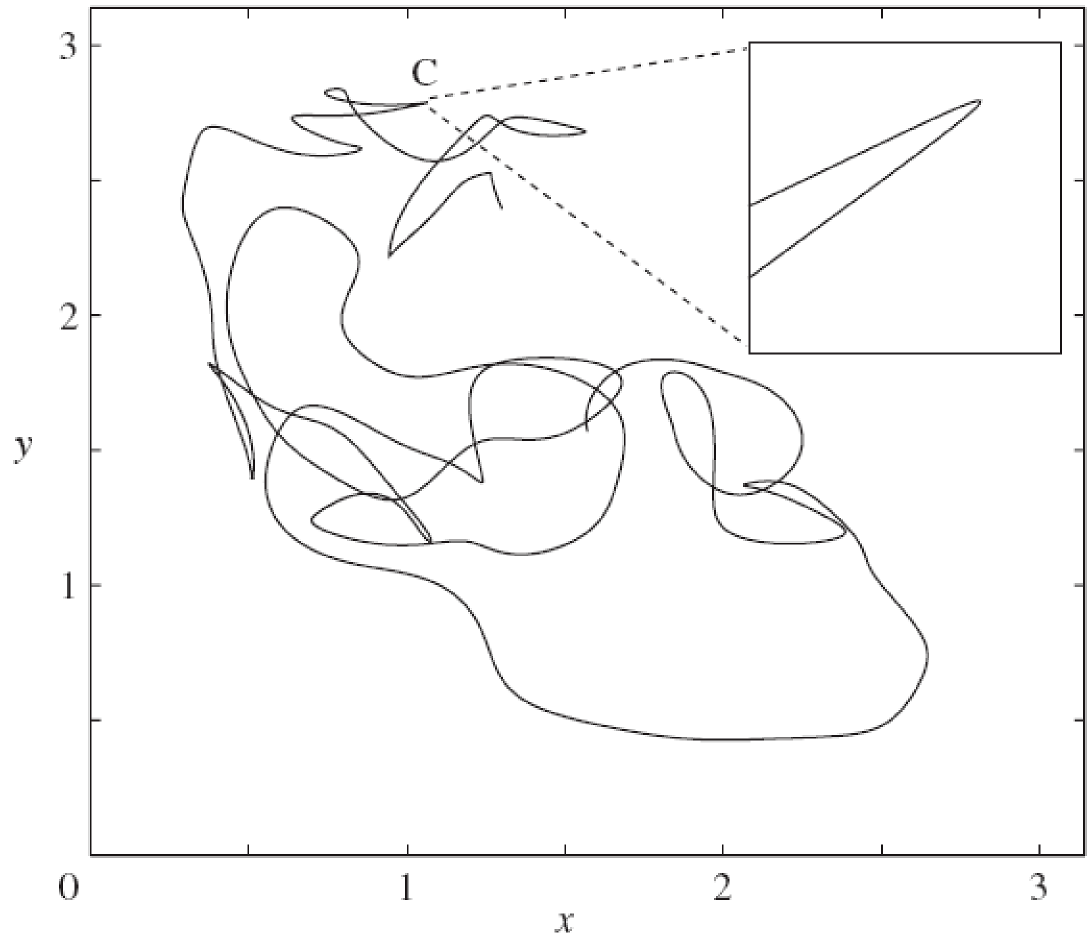
Particle velocity components at t:

$$\frac{\mathrm{d}x}{\mathrm{d}t} = \frac{i}{2|\psi|^2}\left(\psi\frac{\partial\psi^*}{\partial x} - \psi^*\frac{\partial\psi}{\partial x}\right)$$

$$\frac{\mathrm{d}y}{\mathrm{d}t} = \frac{i}{2|\psi|^2}\left(\psi\frac{\partial\psi^*}{\partial y} - \psi^*\frac{\partial\psi}{\partial y}\right)$$

Calculate trajectory $x(t), y(t)$ by numerical integration of above. Typical trajectory shown here - in general they are rather irregular.

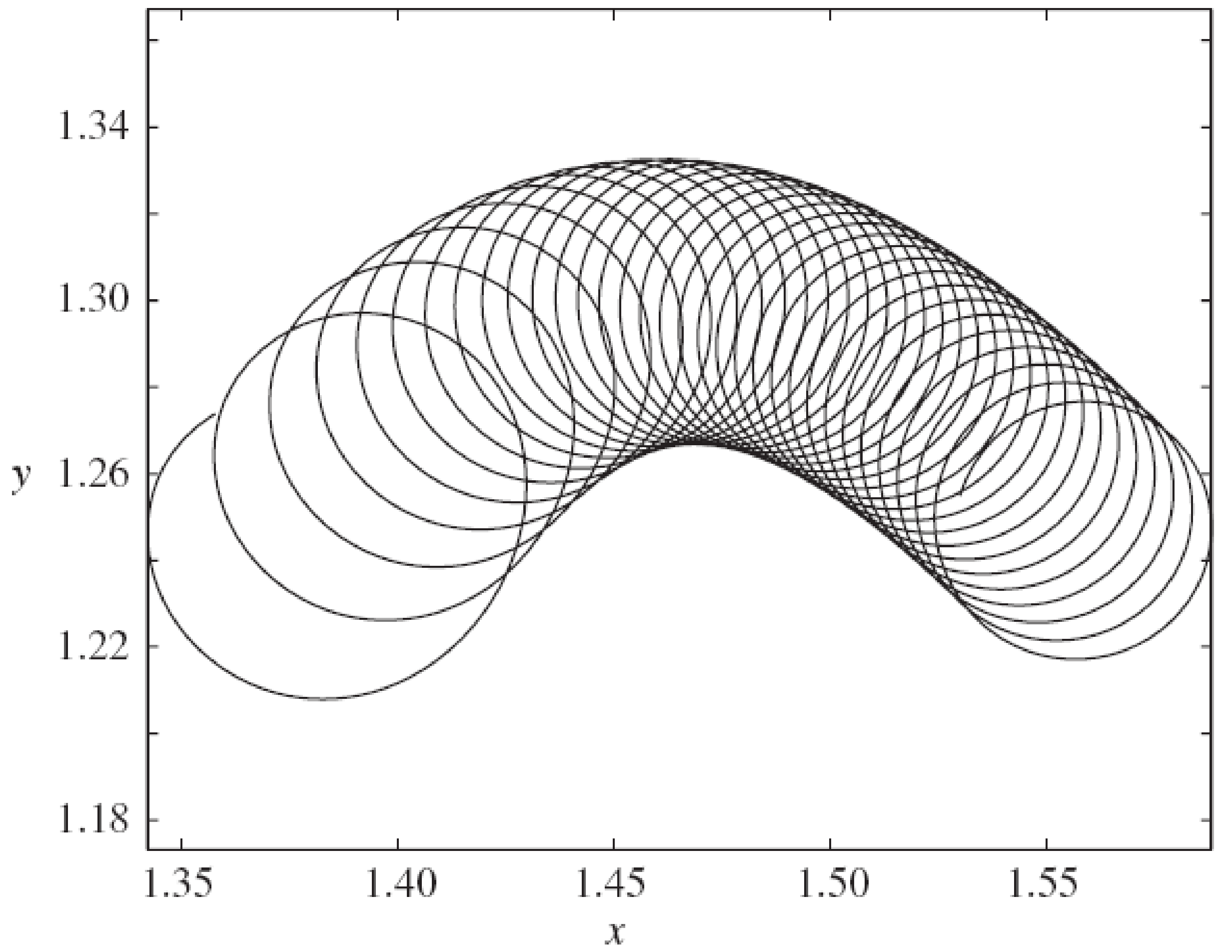
Point C looks like a cusp but tangent not actually discontinuous (particle turning round *slowly*).



Note velocities ill-defined at nodes (where $|\psi| = 0$) and tend to diverge as nodes are approached. This is because - close to a node - small displacements in x and y can generate large changes in phase $S = \mathrm{Im}\ln\psi$ corresponding to large gradient ∇S. Because ψ is smooth, single-valued function, small displacement $(\delta x, \delta y)$ produces small change $\delta\psi$ in complex plane. However, close to a node $\delta\psi$ lies near origin of complex plane and so can correspond to large phase change δS.

Dynamical origin of quantum probabilities: numerical simulations

Close-up of a trajectory near a node



Motion rapid in regions where $|\psi|$ is small. Above close-up of trajectory near nodal or quasi-nodal point where $|\Psi|$ very small (but not known to be strictly zero). Spatial region shown *ca.* 0.3% of whole box. Particle follows rapid circular motion around point moving from right to left - and moving point is a node or quasi-node at which $1/|\psi|$ is highly peaked.

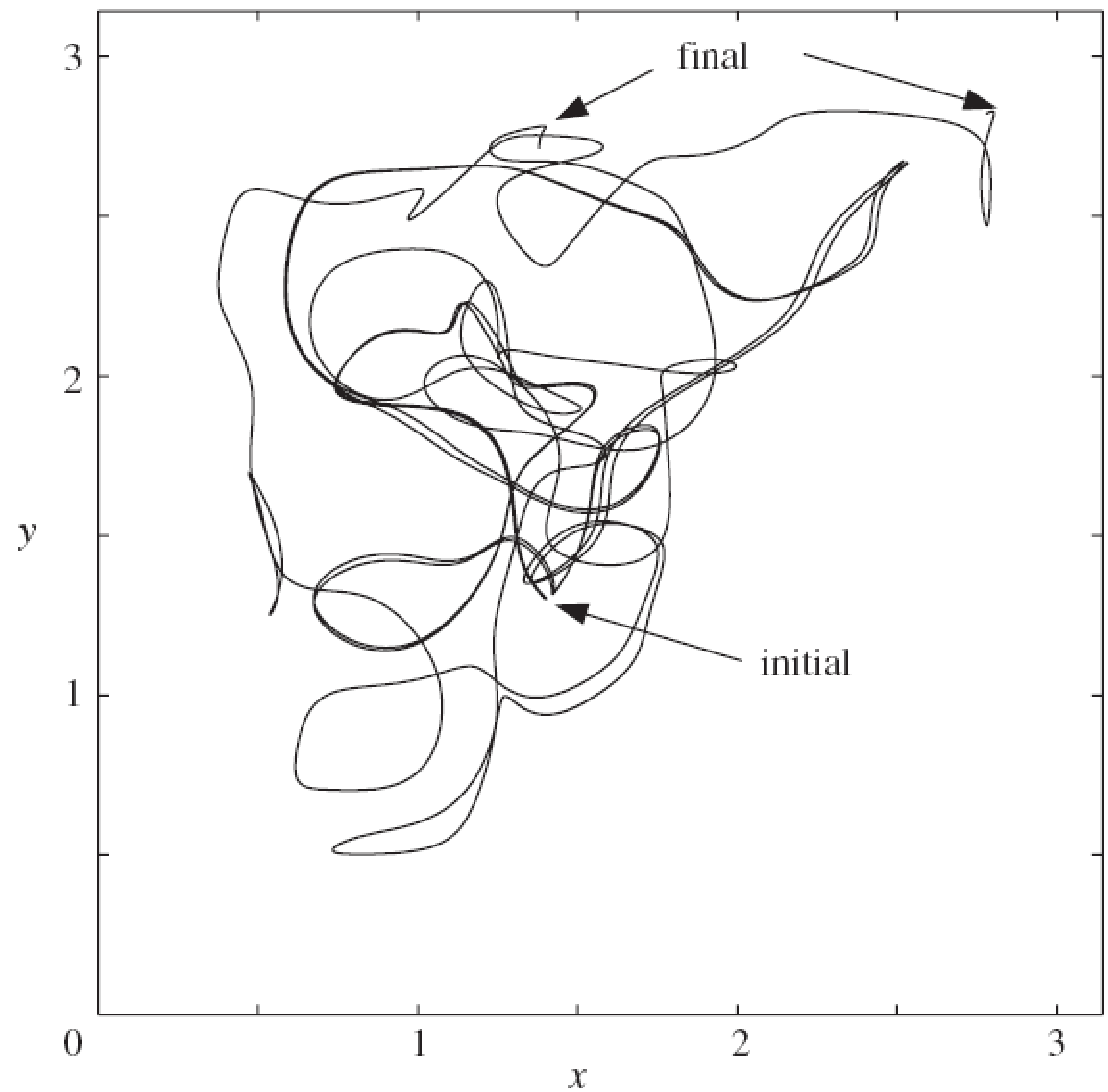
Dynamical origin of quantum probabilities: numerical simulations

Chaotic nature of the trajectories

Two distinct but very close initial positions evolve after $t = 4\pi$ *into widely separated final positions.*

Relevant quantity is *Lyapunov exponent* which characterizes rate of separation of infinitesimally close trajectories. Separation rate depends on orientation of initial separation vector, thus whole spectrum of n Lyapunov exponents - where n is dimensionality of the phase space. Usually use largest one - the *Maximal Lyapunov exponent* (MLE) as it determines predictability of a dynamical system. Positive MLE usually taken as indication that system is *chaotic*.

Difficulty to even *define* 'quantum chaos' in standard QM with no trajectories!



In pilot-wave dynamics, one sees the importance of *nodes* in generating chaotic motion. Numerical simulations suggest a proportionality between Lyapunov exponent and number of nodes.

[See e.g. Frisk *Phys. Lett. A* **227**, 139 (1997)]

Summary of these simulations

Normally say get fast relaxation to equilibrium on coarse-grained level for large numbers of particles. Clear from these simulations that large N in fact not needed for relaxation to occur. Even for one particle, relaxation occurs rapidly if its Ψ is a superposition of even a modest number of energy eigenfunctions.

Relaxation occurs because P and $|\Psi|^2$ evolve like two fluids 'stirred' by same velocity field. Most efficient mixing found to occur in neighbourhood of nodes or quasi-nodes, where Ψ is small. These points move around inside box, rather like 'electric mixers' or stirring devices moving through a fluid, generating an efficient relaxation everywhere.

Typicality and other views

Opposing camps exist in pilot-wave world differing in approach to $P = |\Psi|^2$. Having world's worst PR problem perhaps should be nicer to each other but the opposing camps typically dismiss the others' views with barbed footnotes† in their papers.

- **Bell**: 'It is *assumed* that the particles are so delivered initially by the source'.
- **Holland**: Lists the $|\Psi|^2$ distribution as one of four basic postulates of pilot-wave theory.
- **Dürr, Goldstein, Zanghì** *et al.*: $|\Psi|^2$ regarded as natural measure of probability or 'typicality' for initial configurations of whole universe (taking Ψ as the universal wave function), yielding Born rule for all subsystems at all times. [See Dürr and Teufel book for good discussion].

Valentini finds these 'incorrect and deeply misleading'; postulates about initial conditions should have no *fundamental* status in theory of dynamics. Fair enough.

† *Some people [hawk, spit!] believe that [FILL IN VIEW HERE] is a suitable position but they're obviously madder than a bucket of frogs.*

New experimental tests of quantum theory

Important to test QM in new and extreme conditions: as with any scientific theory, its domain of validity can be determined only by experiment. Helpful to have theories that agree with QM in some limit: hidden-variables theories with nonstandard probability distributions ('quantum nonequilibrium') or models of wave function collapse (such as GRW) are good examples. While it's possible QM might break down in a completely unexpected way or place, the chances of a successful detection of a breakdown would seem higher, the better motivated the theory describing the breakdown.

Once quantum equilibrium reached it is not possible to escape from it (leaving aside remote possibility of rare fluctuations). A universe in quantum equilibrium is then analagous to a universe stuck in a state of global thermal equilibrium or thermodynamic 'heat death'.

Main focus for decades demonstrating violations of Bell's inequality. Valentini suggests some new tests:

Astrophysical and cosmological tests of QM, A. Valentini, J.Phys. A: Math.Theor. **40**, 3285 (2007)

- Find some nonequilibrium matter and look for anomalous blurring of fringes in single-particle interference, breakdown of Malus' law for two state systems, anomalous vacuum fluctuations etc.
- Where to find non-equilibrium matter? Normal matter has 'relaxed' to equilibrium because it has had a long and violent astrophysical history and has interacted with many degrees of freedom in the past. So look for residual nonequilibrium matter in distant past i.e. the early universe (cosmic microwave background thermal anistropy, 'relic particles'). Look for the *generation* of nonequilibrium today (e.g. through gravitation, near black holes etc.).

Other interesting consequences of non-equilibrium in subquantum measurements, quantum computers, cryptography, black hole information problem etc.. See Valentini's 'Subquantum information and computation' and similar papers listed on the course website.

Philosophy important in doing physics?

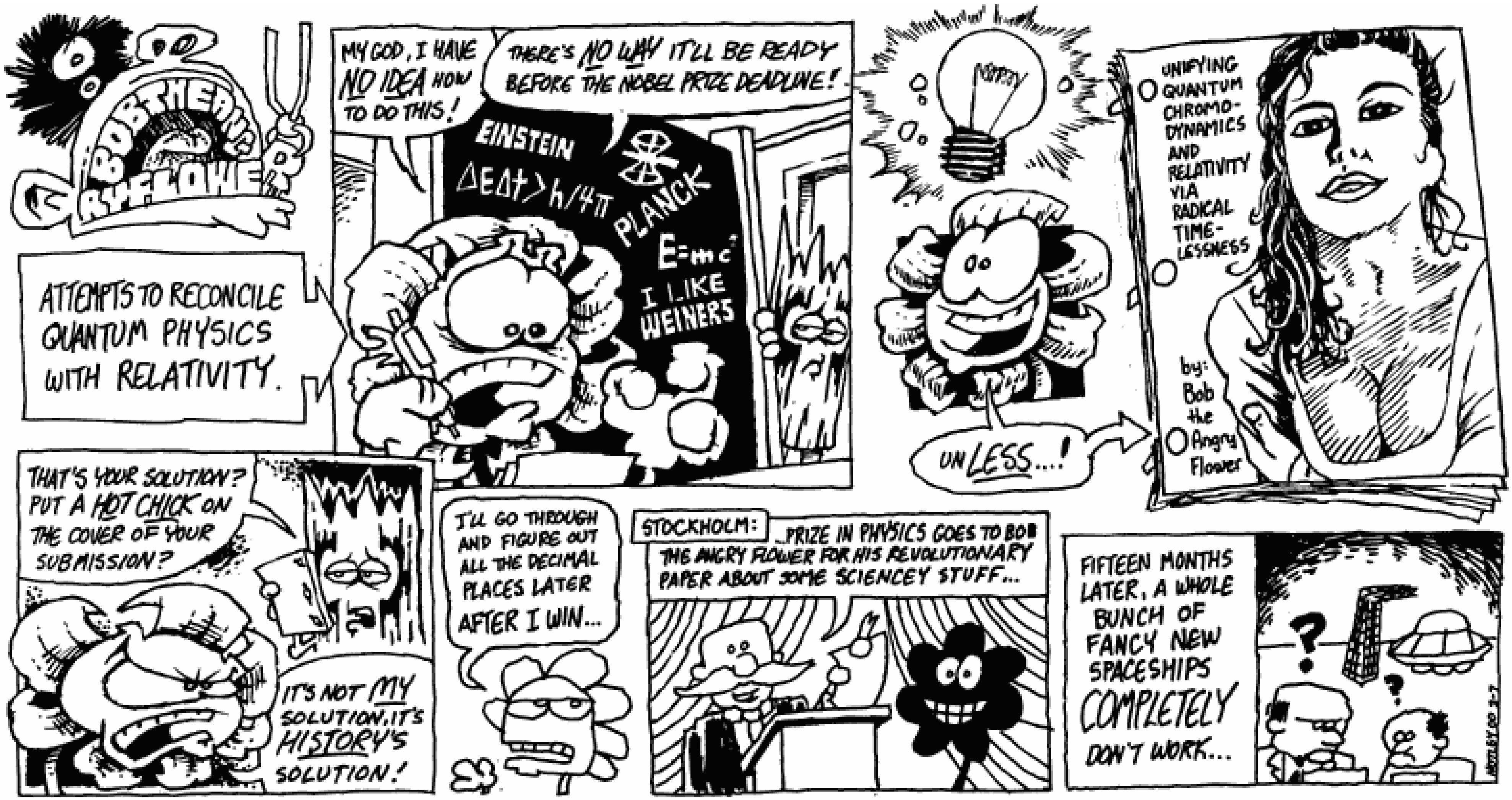
We have seen that a dogmatic insistence on fundamental Lorentz invariance can lead pilot-wave theorists astray. Our built-in beliefs based on Minkowski spacetime encourage us to believe every particle must have its own 'personal time', and that $\Psi(\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_N, t)$ which appears to have a global absolute time must be radically restructured. Not the case if we adopt the (perfectly coherent) viewpoint of Lorentzian relativity; then relativistic Bohmian theories become Lorentz invariant 'on average'. Philosophical musings thus encourage us to be more flexible. Some authors go a little bit further:

'A radical rethinking of quantum gravity: rejecting Einstein's relativity and unifying Bohmian quantum mechanics with a Bell-neo-Lorentzian absolute time, space and gravity'
Quentin Smith, in *Einstein, Relativity, and Absolute Simultaneity* (Routledge, 2008).

"The surprise is that this paper does not present some magnificent mathematical structure that is a new and closer approximation to the 'Master Equation' of quantum gravity, such as Witten's M-theory or the Hartle-Hawking theory, but rather that the results we reach show how easy it is to unify a classical (non quantum) theory of space, time, gravity and cosmology with QM. What's the catch? The key move is to (a) reject Einstein's classical GTR and substitute for it a classical neo-Lorentz GTR; and to (b) select only Bohm's 1952 interpretation of QM.."

So he's saying that the well-known failure to develop a quantum theory of gravity is a result of unfortunate ontological and epistemological choices, and if people had only listened to Bohm everything would have been sorted out by now. If that's not controversial, I don't know what is.. but it's certainly fun.

The Bad Flower continues his physics studies..



Rest of course

Lecture 1: 21st January 2009
An introduction to pilot wave theory

Lecture 2: 28th January 2009
Pilot waves and the classical limit. Derivation and justification of the theory

Lecture 3: 4th February 2009
Elementary wave mechanics and pilot waves, with nice examples

Lecture 4: 11th February 2009
The theory of measurement and the origin of randomness

Lecture 5: 18th February 2009
Nonlocality, relativistic spacetime, and quantum equilibrium

Lecture 6: 25th February 2009
Calculating things with quantum trajectories

Lecture 7: 4th March 2009
Not even wrong. Why does nobody like pilot-wave theory?

Lecture 8: 11th March 2009
Bohmian metaphysics : the implicate order and other arcana
Followed by a GENERAL DISCUSSION.

Slides/references on web site: `www.tcm.phy.cam.ac.uk/~mdt26/pilot_waves.html`