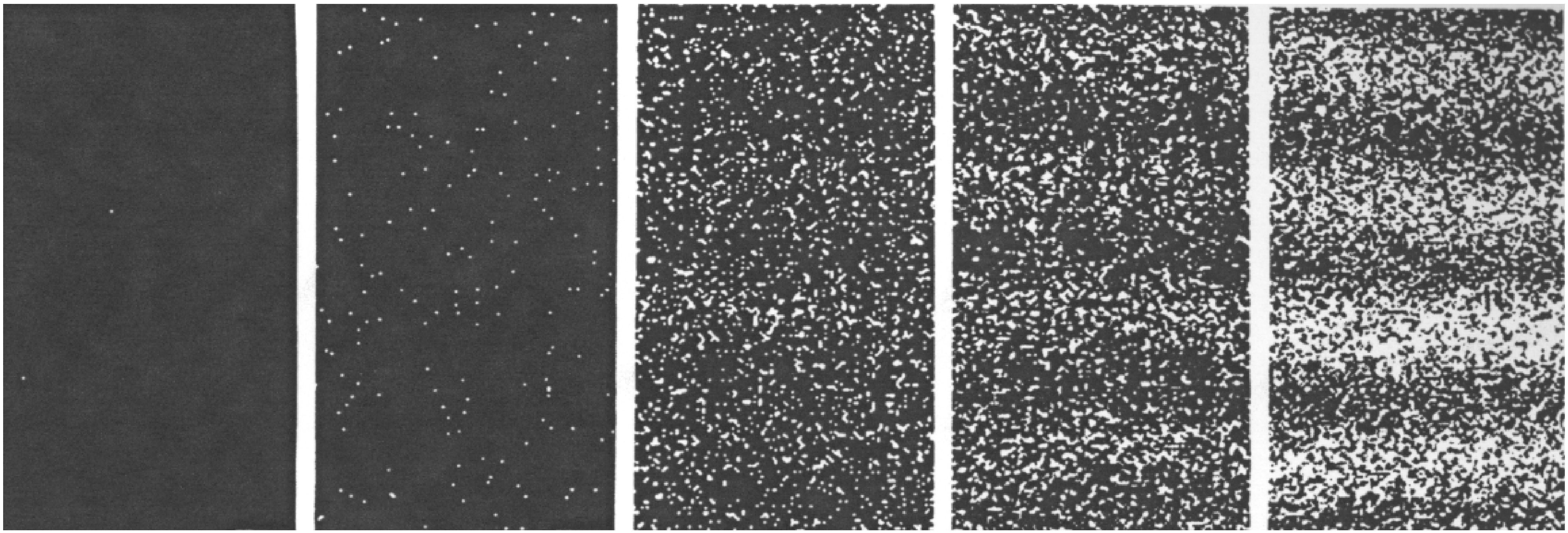
Pilot-wave theory, Bohmian metaphysics, and the foundations of quantum mechanics

Lecture 4

The theory of measurement and the origin of randomness



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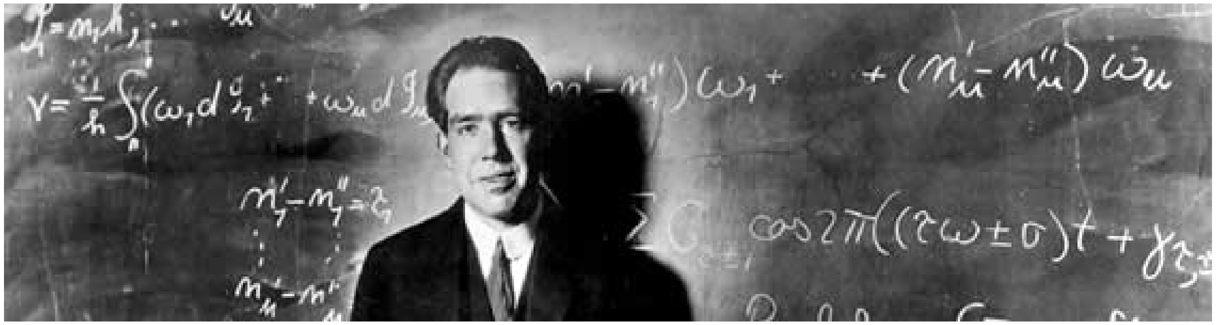
www.tcm.phy.cam.ac.uk/∼mdt26/pilot_waves.html

MDT

An undesired heritage



Physics is about measurement and nothing else.

Pilot-wave theory is obviously not about measurement. It is a theory about reality.

- However, since standard QM is about measurements and apparently plagued with the measurement problem, and since pilot-wave theory is supposed to be a correct quantum-mechanical description of nature, we need to understand how pilot-wave theory claims to resolve the measurement problem.

- Measurements can be described as 'the reading of apparatus states', a problem clearly belonging to the classical world. Obviously related to the pilot-wave classical limit problem (Lecture 2).

The endlessly quotable John Bell..

> "*The problem of measurement and the observer is the problem of where the measurement begins and ends, and where the observer begins and ends. . . . I think that – when you analyse this language that physicists have fallen into, that physics is about the results of observation – you find that on analysis it evaporates, and nothing very clear is being said.* "

J.S. Bell (1986, Interview in Davis and Brown's *The Ghost in the Atom*)

I'm sorry, it's just addictive..

"*And when I look at quantum mechanics I see that it's a dirty theory. The formulations of quantum mechanics that you find in the books involve dividing the world into an observer and an observed, and you are not told where that division comes - on which side of my spectacles it comes, for example - or at which end of my optic nerve.*"

J.S. Bell (1986, Interview in Davis and Brown's *The Ghost in the Atom*)

What pilot-wave theory says about measurement

"*Most of what can be measured is not real and most of what is real cannot be measured, position being the exception.*"

Dürr and Teufel (2009)

First attempt to define measurement

Experiment where a system wave function gets correlated with a pointer wave function.

- Note outcome of any conceivable experiment may be expressed in terms of *positions* of macroscopic objects. Nothing to do with physical observable being measured; relates to how we as humans receive information through our senses.
- **Support** of Ψ is domain in configuration space on which $\Psi \neq 0$. **Pointer wave function** *macroscopic* with support tightly concentrated around region (of $\sim 10^{23}$ particles) that make up pointer in physical space pointing in some direction.
- Different pointer positions belong to 'macroscopically disjoint' wave functions whose supports are macroscopically separated in configuration space [precise requirement: *overlap* extremely small in square norm over any macroscopic region].

Let system coords be $\mathbf{X}$ and apparatus coord $\mathbf{Y}$. Assume system's wave function is $\psi_1(\mathbf{x})$ or $\psi_2(\mathbf{x})$. Possible apparatus wave functions: 'null' $\Phi_0(\mathbf{y})(Y \in \text{supp } \Phi_0)$ or $\Phi_1(\mathbf{y})(Y \in \text{supp } \Phi_1)$ or $\Phi_2(\mathbf{y})(Y \in \text{supp } \Phi_2)$. For ideal measurements expect pointer positions to correlate with system wave functions under Schrödinger evolution:

$$\psi_1\Phi_0 \longrightarrow \psi_1\Phi_1 \quad \text{and} \quad \psi_2\Phi_0 \longrightarrow \psi_2\Phi_2$$

Note however, that if system wave function is a superposition $\Psi(\mathbf{x}) = c_1\psi_1(\mathbf{x}) + c_2\psi_2(\mathbf{x})$ then

$$\Psi\Phi_0 = c_1\psi_1\Phi_0 + c_2\psi_2\Phi_0 \longrightarrow c_1\psi_1\Phi_1 + c_2\psi_2\Phi_2$$

which involves a *macroscopic superposition* of pointer wave functions. Hmmm. Miaow..

The measurement problem in standard QM

- **1**. Assumption of classical background leads to undefinable division between microscopic and macroscopic worlds.

- **2**. Measurement itself apparently not physical process describable in purely quantum-theoretic terms. Observers must be added as extra-physical elements.

- **3**. Schrödinger evolution of Ψ in time gives linear superposition of all possibilities for ever. When correlated with measuring apparatus, get a *macroscopic* superposition of quantum states, which is not what one sees. And which means what, exactly? (Schrödinger's cat problem).

- **4**. Need to postulate non-local 'collapse' in which time-dependent wave function suddenly stops obeying Schrödinger eqn and does something else when 'observed'.

- **5**. Not possible to use standard QM in cosmological problems.

NB: *What exactly is considered a 'problem' depends fundamentally on whether you believe wave function is a real object that is part of structure of individual system, or it represents 'knowledge of the system'(whose?), or it is merely a mathematical tool for calculating and predicting the measured frequencies of outcomes over an ensemble of similar experiments.*

1. A homogeneous account of the world

- Standard QM as described in textbooks practically successful but seemingly fundamentally ill-defined. Clear dividing line between ‘microscopic indefiniteness’ and the definite states of the classical macroscopic realm, but such distinctions defy sharp and precise formulation.

- What happens to the definite states of the everyday macroscopic domain as one goes to smaller scales? Where does macroscopic definiteness give way to microscopic indefiniteness? Does the transition occur somewhere between pollen grains and macromolecules, and if so, where? On which side of the line is a virus?

- Why is quantum ‘indefiniteness’ confined to the atomic level? The Moon is made of atoms, so why does the Moon have a definite macroscopic state (especially when there are no observers present)?

- Lack of sharp boundary between ‘classical domain’, in which *real state* is a valid concept, and the ‘quantum domain’ in which it is not. No *ontology* for atoms.

To get round this, can introduce ‘hidden variables’ as in pilot-wave theory, or can use other means (e.g. many worlds, or theories of dynamic wave function collapse such as GRW). All such theories assume wave function is real.

2. Measurement as a physical process

Macroscopic equipment subject to laws of physics like any other system and it ought to be possible to describe operation of such equipment in terms of most fundamental theory. Attempts to do so are notoriously controversial and apt to result in paradox and confusion.

- Particular problem with *biological systems* - led Wigner to conclude that sentient subjects of experiments are in 'a state of suspended animation' until he speaks with them (see Wigner's friend paradox - later).

- Generally implied that human beings, unlike any other physical systems, have special properties by virtue of which they cannot be treated by ordinary physical laws but generate deviation from those laws (but only the laws of quantum theory, not the laws of e.g. gravity or thermodynamics).

- Seems clear that ideally we would prefer a 'quantum theory without observers', in the sense that observers are physical systems obeying the same laws as all other systems, and should *not* have to be added to the theory as extra-physical elements.

3. Macroscopic quantum superpositions

All measurements entail amplifying to the macroscale (e.g. sum of dead and alive cat in a box) linear superpositions at the microlevel (e.g. electron position smeared everywhere in two-slit experiment, linear combination of spin states in EPR, sum of decayed nucleus and undecayed nucleus):

$$c_1\psi_1\Phi_0 + c_2\psi_2\Phi_0 \longrightarrow c_1\psi_1\Phi_1 + c_2\psi_2\Phi_2$$

- If Ψ all there is, this has *no physical meaning* unless Ψ is instrument for computing probabilities of finding some pointer position. But that would mean there *is* a pointer, and that might as well be pilot-wave one. Need say nothing more.
- However, physicists were convinced that innovation of QM is something like '*the macroscopic world is real, but it cannot be described by microscopic constituents*'. On the other hand the pointer moves from 0 to 1 to 2, like the cat either dies or stays alive so some movement is going on. Why should this not be describable?

How is a mathematical superposition of macroscopically-distinct states (a dead-and-alive cat in a box) related to the definite macroscopic states seen in the real world?

No problem if Ψ just tool giving frequencies of outcomes over ensemble of experiments (*'statistical'* or *'ensemble interpretation'*). But then QM *incomplete theory* referring only to ensembles (of particles with unknown dynamics) - no description of individual quantum systems or relation to real individual macroscopic states. No boundary between 'macroscopic' objects with individual (non-ensemble) description and 'microscopic' objects with no such description. How does interference happen again?

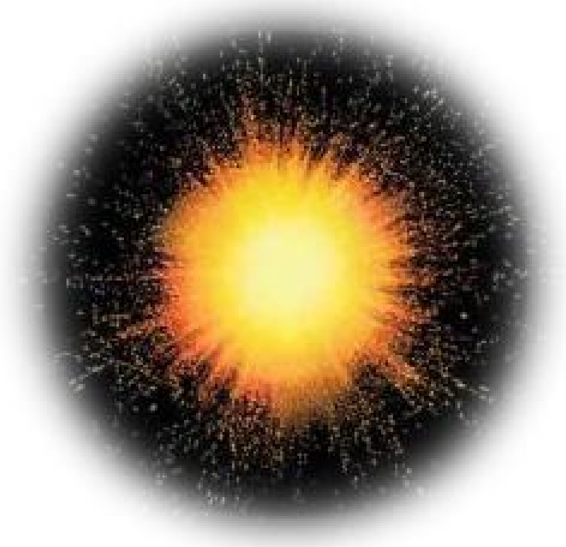
4. Wave function collapse

The pure wave dynamics described by Schrödinger's equation does not yield any account of which result is actually realized in an individual measurement operation, so we introduce an 'observer' who makes everything alright by non-locally 'collapsing' the pointer wave function to a definite outcome.

What's wrong with the collapse hypothesis?

- Not told when collapse is supposed to occur, how long it takes, or what brings it about. Must happen at infinite speed everywhere throughout the universe all at once.
- Processes involved in measurements not different from those expected to occur all the time when physical systems interact with their environments. If cannot account for this with usual Schrödinger theory this implies massive incompleteness in QM treatment of general natural processes as collapse must be constantly occurring.
- Can avoid this by claiming collapse takes place only when human observer becomes aware of pointer reading. But do we really have to invoke consciousness at this level of physics (particularly as no-one understands what it is)?
- What does pointer do when not being looked at? Collapse hypothesis only gains physical content if actual coordinates for collapsed system are posited. Schrödinger once thought that a cat is a big enough pointer to get that point across.

5. Quantum cosmology



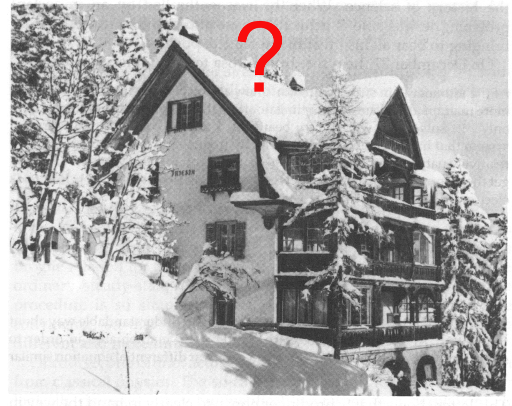
- Problem with description of distant past before life evolved on Earth
- Problem with description of universe as a whole in epochs before life existed.
- No observer possible for whole universe, except maybe God.

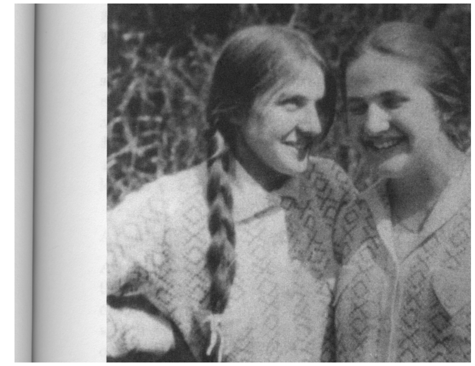
"The Copenhagen view depends on the assumed a priori existence of a classical level to which all questions of observation may ultimately be referred. Here, however, the whole universe is the object of inspection: there is no classical vantage point, and hence the interpretation question must be reargued from the beginning." [DeWitt, 1967]

Main motivation for development of *many-worlds approach* where observation is not an issue. Pilot-wave theory also has this attractive feature and can in principle be applied in cosmology; it has the additional advantage of not being *utter madness*.

Example 1: Schrödinger's woman (aka Wigner's friend)

Choose one from: wife, 'dark lady of Arosa', teenage nymphette twins Ithi and Withi.



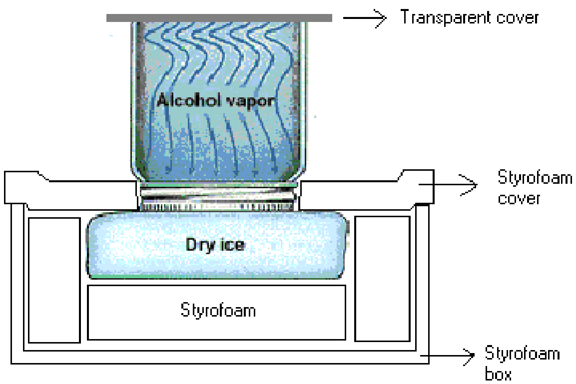


So put - say - Ithi in a box, with an experiment to do involving microscopic system initially in superposition of energy states $\psi = \frac{1}{\sqrt{2}}(|E_1\rangle + |E_2\rangle$. Schrödinger (unlucky!) is outside the box. Ithi does experiment and presumably finds one outcome E_1 or E_2 but from Schrödinger's view she is in a superposition of *girl found* E_1 and *girl found* E_2. Paradox is contradiction between following statements (referring to ensembles) concerning physical state of Ithi just before Schrödinger decides what to do next:

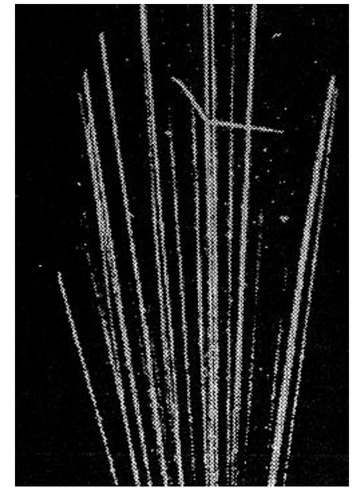
I: There is no definite state of Ithi, because Schrödinger can if he wishes perform measurements showing the presence of interference between different states.
II: There is a definite state of Ithi, because Ithi is human, and instead of testing for interference Schrödinger can simply open the box and ask Ithi what she saw.

Standard QM leads one to suppose that Ithi "was in state of suspended animation before she answered the question" (Wigner) since assume reality of other minds (no solipsism!) and conscious beings ought always to have definite state of consciousness.

Cloud chamber



Sealed chamber with supersaturated vapour kept near condensation point by regulating T. Ionizing radiation leaves trail of charged ions that serve as condensation centers. Vapour condenses around them. Radiation *path* thus indicated by tracks of tiny liquid droplets in supersaturated vapour.



- If α-particle emission undirected - so emitted Ψ spherical - how account for straight particle track revealed by cloud chamber? Intuitively would think it ionizes atoms at random throughout space.
- If only α-particle quantum (only its coords in Ψ) vapour is 'external measuring equipment'. On producing visible ionization α-particle wave packet 'collapses' then spreads until more visible ionization then collapse occurs again etc. Prob for resulting 'trajectory' concentrated along straight lines. Similar result [Mott, 1929] if consider interaction in configuration space with all atoms.
- So in standard QM trajectories emerge only at macroscopic level and are constructed by successive wave packet collapses. Works only because α-particle largely 'classical' (has billions of eV but requires only a few eV to ionize one atom so preserves its identity). In pilot-wave theory macroscopic trajectories simple consequence of microscopic trajectories.

Classical measurements

Measurements in classical physics are the means by which we come to know current state of mechanical system without appreciably disturbing it. Interact with system in such a way that this ideal may in principle be approached arbitrarily closely (ignoring practical problems). Reading meter reveals what has already happened and has no influence on course of phenomena. Conscious observers not required.

Simple model with essential characteristics:

Measure quantity $A(x, p_x)$ associated with particle at x, p_x by letting it interact with apparatus 'particle' at y, p_y. Interaction Hamiltonian $H = gA(x, p_x)p_y$ with coupling constant g. Note $\mathrm{d}A/\mathrm{d}t = \{A, H\} = 0$ with $\{\}$ *Poisson brackets* so evolution generated won't change A. Get y correlated with x so observing y reveals A.

Hamilton's equations:

$$\dot{x} = \partial H/\partial p_x = g(\partial A/\partial p_x)p_y \qquad \dot{y} = \partial H/\partial p_y = gA$$

$$\dot{p}_x = -\partial H/\partial x = -g(\partial A/\partial x)p_y \qquad \dot{p}_y = -\partial H/\partial y = 0$$

Given initial conditions x_0, y_0, p_{x0}, p_{y0} can integrate over period of impulse T to get post-measurement values. In general x and p_x get changed by the measurement process but since $\dot{p}_y = 0$ can make change negligible in limit that $p_y \to 0$:

$$x = x_0, \quad y = y + gAT, \quad p_x = p_{x0} \quad p_y = p_{y0}$$

Can measure x, p_x *simultaneously* with negligible disturbance of either. Can also use statistical distribution $f(x, p_x, y, p_y)$ of initial phase space coords evolving as Liouville's equation $\partial f/\partial t + \{H, f\} = 0$. Rms scatter in results e.g. Δp_x represents ensemble spread of p_x before measurement. Note f 'collapses'! [Holland, Ch. 8.1]

Quantum measurement

Series of causally connected states: two initially independent systems come into contact, mutually transform one another, separate, and one system undergoes irreversible change making subsequent interference practically impossible. In quantum case, probe is as significant as the probed so cannot calculate away its influence to leave pure information about preexisting properties of object as in CM.

- Empirical content of QM relates both to:
  Outcomes of individual measurements, where the dynamical variables are found to be eigenvalues of Hermitian operators.
  An ensemble of similarly prepared individual measurements, where the eigenvalues are predicted to be distributed according to specified probability law.

- Attempting a *physical* treatment of measurement interactions with standard QM involving special but typical many-body processes fail. If Ψ all there is, how do we give objective meaning to notion of a meter reading, and what actually is it that the eigenvalue is a property of?

- The main problem in standard QM turns out to be that the Schrödinger equation cannot map a pure state into a *proper mixture* i.e. it cannot single out a single branch - a definite outcome of the measurement. This is often stated to be solved through technically intimidating arguments about the '*vanishing of the off-diagonal elements of the reduced density matrix*' (a solution from within quantum mechanics!) but this is not the case, as we shall see.

Now who thinks the following would be a really good idea?

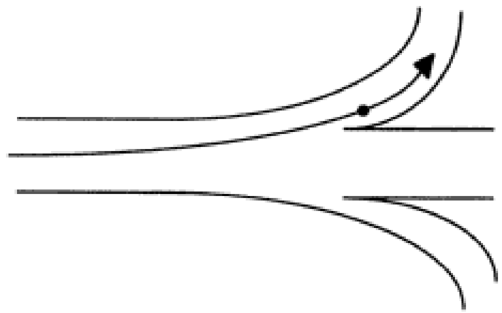
Given classical physics shows experimental operation represented by $H = gAp_y$ realizes correct measurement of A, let's use classical measurements as a guide to tell us how to do quantum measurements. Specifically, to measure an observable A using an apparatus pointer y let's switch on a Hamiltonian operator $\hat{H} = g\hat{A}\hat{p}_y$ i.e. we 'quantize' the classical procedure.

- What does this analagous quantum procedure actually accomplish? It just generates a branching of the total wave function, with branches labelled by eigenvalues A_n of the linear operator $\hat{A}$.
- For example, if system is a particle with position x, the initial wave function $\Psi_0(x, y) = [\sum_n c_n\psi_n(x)]\phi_0(y)$ - where $\hat{A}\psi_n = A_n\psi_n$ and ϕ_0 is the initial (narrow) pointer wave function - evolves into $\Psi(x, y, t) = \sum_n c_n\psi_n(x)\phi_0(y - gA_nt)$. The effect of the experiment is simply to create this branching.
- As we know, Newton's laws give great results for analyzing the dynamics of particles in a two-slit experiment, so they sound like just the ticket for our quantum measurement process!

'Quantum measurement' procedures are - when we have real objects with measurable properties - generally not correct measurements; they are merely experiments of a certain kind designed to respect a formal analogy with **classical** *measurements.*

Measurements in pilot-wave theory: general points

In pilot-wave theory, total configuration $q(t)$ ends up in the support of one of the (nonoverlapping) branches of the wave function:



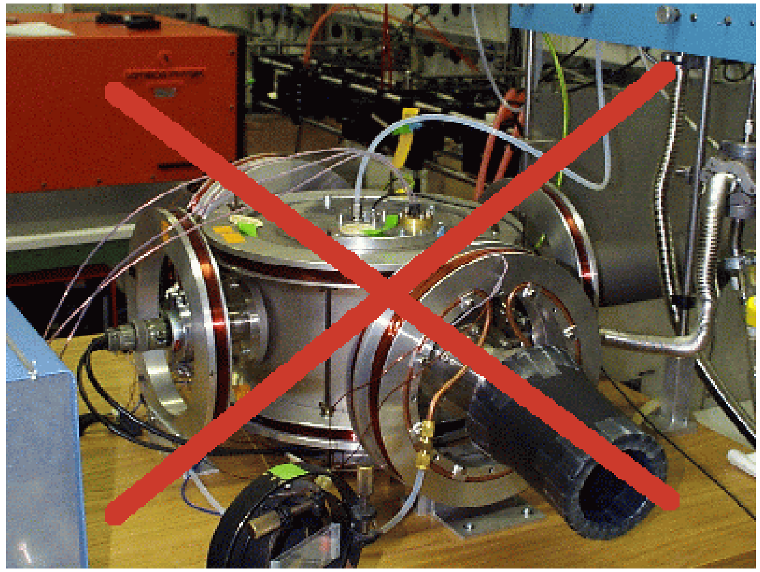
- If over ensemble x and y have initial $P_0(x,y) = |\Psi_0(x,y)|^2$, then a fraction $|c_n|^2$ of trajectories $q(t) = x(t), y(t)$ end in (support of) nth branch $\psi_n(x)\phi_0(y - gA_nt)$.

- From this perspective eigenvalues A_n have no particular ontological status. Just have complex-valued field on configuration space obeying linear wave equation. The time evolution may often be conveniently analyzed using linear functional analysis methods (just like you can with a classically vibrating string - see later).

- Can't stress enough that generally speaking one has *not measured anything* here. Normally if pointer found to occupy nth branch say '*observable A therefore has the value A_n*'. But at end of experiment only result is that system trajectory $x(t)$ is guided by the (effectively) reduced wave function $\psi_n(x)$.

- Doesn't usually imply system has/had property with value A_n (at end or beginning of experiment) since with pilot waves no general relation between eigenvalues and ontology. Incorrect to identity eigenvalues with values of real physical quantities.

Failing to measure stuff : example



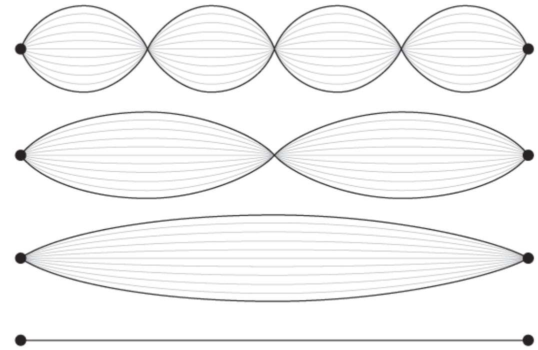
The eigenfunction $\psi_E(x) \propto (\mathrm{e}^{ipx} + \mathrm{e}^{-ipx})$ of the kinetic-energy operator $\hat{p}^2/2m$ has eigenvalue $E = p^2/2m \neq 0$ and yet since $\partial S/\partial x = 0$ the actual de Broglie-Bohm kinetic energy vanishes: $\frac{1}{2}m\dot{x}^2 = 0$.

If the system had this initial wave function, and we performed a so-called 'quantum measurement of kinetic energy' using a pointer y, then the initial joint wave function $\psi_E(x)\phi_0(y)$ would evolve into $\psi_E(x)\phi_0(y - gEt)$ and the pointer would indicate the value E - even though the particle kinetic energy was and would remain equal to zero.

The experiment has not really measured anything.

Linear functional analysis: a classical vibrating string

Consider string held fixed at endpoints, $x = 0, L$*. Assuming wave speed* $c = 1$*, small vertical displacement* $\psi(x,t)$ *obeys wave equation, i.e. the PDE* $\frac{\partial^2\psi}{\partial t^2} = \frac{\partial^2\psi}{\partial x^2}$.



Can be solved using standard methods of linear functional analysis. Can define Hilbert space of functions ψ with a Hermitian operator $\hat{\Omega} = -\partial^2/\partial x^2$ acting on it. Solutions of wave equation expandable in complete set of eigenfunctions $\phi_m(x) = \sqrt{2/L}\,\sin(m\pi x/L)$, where $\hat{\Omega}\phi_m = \omega_m^2\phi_m$ with $\omega_m^2 = (m\pi/L)^2 (m = 1, 2, 3, \ldots)$. Assuming for simplicity $\dot{\psi}(x,0) = 0$, general solution is

$$\psi(x,t) = \sum_{m=1}^{\infty} c_m\phi_m(x)\cos\omega_m t \qquad \left(c_m \equiv \int_0^L \phi_m(x)\psi(x,0)\,\mathrm{d}x\right)$$

or (in bra-ket vector notation)

$$|\psi(t)\rangle = \sum_{m=1}^{\infty} |m\rangle\langle m|\psi(0)\rangle\cos\omega_m t$$

(where $\hat{\Omega}|m\rangle = \omega_m^2|m\rangle$). Can write any solution as superposition of oscillating ‘modes’. Even so, true ontology consists of total displacement $\psi(x,t)$ of string (maybe also velocity and energy). Would not normally regard individual modes in sum as physically real and would certainly not assert ψ composed of ontological multiplicity of strings, with each string vibrating in a single mode. Would say that in general eigenfunctions and eigenvalues have mathematical significance only.

Factorizable and nonfactorizable wave funtions

A factorizable wave function is one which may be written as a product, e.g.:

$$\Psi(x, y, z, t) = \psi_1(x, t)\psi_2(y, t)\psi_3(z, t)$$

- Starting measurement experiment with a product wave function $\psi(x)\phi(y)$ expresses that initially system $\psi(x)$ and apparatus $\phi(y)$ are independent physical entities. The Schrödinger equation for a product wave function separates into two independent Schrödinger equations, one for each factor, if interaction potential $V(x, y) \approx 0$.
- $V(x, y) \approx 0$ on its own does not imply that x and y parts develop independently, since wave function need not be a product.
- Physical independence with pilot waves needs velocity field of system to be function of x alone and that of apparatus a function of y alone. Given $\mathbf{v}^{\Psi} = \frac{\hbar}{m}\,\mathrm{Im}\,\nabla \ln \Psi$, we see that

$$\nabla \ln \Psi(x, y) = \nabla \ln[\psi(x)\phi(y)] = \nabla \ln \psi(x) + \nabla \ln \phi(y) = \begin{pmatrix} \nabla_x \ln \psi(x) \\ \nabla_y \ln \phi(y) \end{pmatrix}.$$

So particle coordinates x indeed guided by system wave function ψ if combined system guided by product wave function.

This is how we split the 'wave function of the universe' into effective wave functions of subsystems.

Pilot-wave theory of measurement

True observables of the theory - the things that immediately present themselves in experiments - are the positions of particles, particularly that of the apparatus pointer. The idea that the 'hidden variables' (i.e. the positions of particles) are 'metaphysical' and/or 'unobservable' is shown by the pilot-wave theory to be a misapprehension. The hidden variables **are** *the observables and the system and apparatus always have definite positions, whatever the quantum state may be.*

- Measurement is typical many-body interaction process - special only since interaction leaves system in particular state (eigenfunction of a Hermitian operator). Apparatus left in state whose subsequent behaviour in no way influences system.
- Consider first *ideal measurements*: don't destroy system and are reproducible in sense that immediate repetition yields same result. Divide naturally into two stages:

  (1) *State preparation* of certain kind where system wave function gets correlated with apparatus wave function and evolves into eigenfunction of Hermitian operator.

  (2) Irreversible act of *amplification* which allows one indelibly to *register* outcome and infer value of physical property of particle corresponding to an operator.

The class of physical variables associated with Hermitian operators does not exhaust the quantities we may wish to determine empirically. Mass (inferred from position in a mass spectrograph), wavelength (inferred from fringe spacing in an interference experiment) and time are examples of quantities that do not correspond to the eigenvalues of such operators.

Stage 1: state preparation

Wave function $\psi(\mathbf{x}, t)$ associated with one-body system. Want info about particle variable $A(\mathbf{x}, t)$ associated with operator $\hat{A}(\hat{\mathbf{x}}, \hat{\mathbf{p}})$ via *local expectation value*:

$$A(\mathbf{x}, t) = \mathrm{Re}\, \psi^*(\mathbf{x}, t)(\hat{A}\psi)(\mathbf{x}, t)/|\psi(\mathbf{x}, t)|^2 \qquad (\text{Lecture } 3)$$

Evaluate along path $\mathbf{x} = \mathbf{x}(t, \mathbf{x}_0)$ for true values of physical quantity. System interacts with apparatus with initial packet $\phi_0(y)$ where y continuously variable location of meter needle (1d). Interaction Hamiltonian $H = g\hat{A}\hat{p}_y$. Initial independence $\Rightarrow$ factorizable wave function $\Psi_0(\mathbf{x}, y) = \psi_0(\mathbf{x})\phi_0(y)$ which becomes entangled in the 4d space during interaction. Solve Schrödinger equation

$$i\hbar \frac{\partial \Psi(\mathbf{x}, y, t)}{\partial t} = -i\hbar g \hat{A} \frac{\partial \Psi(\mathbf{x}, y, t)}{\partial y}$$

by expanding Ψ in complete set of eigenfunctions of $\hat{A}$ (eigenvalue a) - including possible contribution from continuous part of spectrum: $\Psi(\mathbf{x}, y, t) = \sum_a f_a(y, t)\psi_a(\mathbf{x})$. Substitute into Schrödinger equation and (with eigenfunctions orthonormal) find coefficients given by:

$$\frac{\partial f_a(y, t)}{\partial t} = -ga \frac{\partial f_a(y, t)}{\partial y} \implies f_a(y, T) = f_{a0}(y - gaT).$$

Here $f_{a0}(y)$ are initial values and T is period of impulse. Expanding the initial wave function as $\psi_0(\mathbf{x}) = \sum_a c_a \psi_a(\mathbf{x})$ where c_a are constants. Substituting (with a bit of algebra) we find wave function at termination of interaction:

$$\Psi(\mathbf{x}, y, T) = \sum_a c_a \psi_a(\mathbf{x}) \phi_0(y - gaT)$$

Subsequent evolution proceeds according to the free Hamiltonian.

Stage 1: state preparation - continued..

So at the termination of the interaction, the wave function is

$$\Psi(\mathbf{x}, y, T) = \sum_a c_a \psi_a(\mathbf{x}) \phi_0(y - gaT).$$

Wave function no longer factorizable. System point $\mathbf{x}(t), y(t)$ performs a complicated motion during the interaction. Coordinate y now correlated with eigenvalues a.

- Packet centres ϕ_0 move along $y_a = gat$ until $t = T$. When interaction ends, separation of two packets with neighbouring eigenvalues $a, a + \delta a$ is $\delta y_a = gT\delta a$.
- Want impulse strength/duration and subsequent free evolution such that packets ϕ_0 with different eigenvalues have no appreciable overlap (are orthogonal). Condition: width of packet $\Delta y \ll \delta y_a$. If not then process is 'incomplete measurement'.
- Assuming complete measurement, wave function splits into set of non-overlapping config space functions (even though $\psi_a(\mathbf{x})$s may overlap). Particle enters region where one such function finite so - assuming summands don't subsequently overlap (next slide) and since particle cannot cross node - wave function may be effectively replaced for particle dynamics by just one summand (factorizable again!): $\Psi \rightarrow c_a \psi_a(\mathbf{x}) \phi_0(y - gaT)$.
- In region where ath summand finite $A \rightarrow \mathrm{Re}\, \psi_a^*(\mathbf{x})(\hat{A}\psi_a)(\mathbf{x})/|\psi_a(\mathbf{x})|^2$. Thus different value of A (constant throughout region) associated with each outgoing packet. Value of A obtained in ensemble trial, an eigenvalue a, depends on which outgoing packet particle enters. Whatever initial value of A, it has been deterministically and continuously transformed into an eigenvalue (though actual result unpredictable depending on initial values $\mathbf{x}_0, y_0, \psi_0, \phi_0$).

Why Hermitian operators?

$$\int \psi_1^*(\mathbf{x})\hat{A}\psi(\mathbf{x})\,\mathrm{d}\mathbf{x} = \int (\hat{A}\psi(\mathbf{x}))^*\psi(\mathbf{x})\,\mathrm{d}\mathbf{x}$$

- Relevant properties of Hermitian operators are: they have real eigenvalues and their eigenfunctions can be chosen to be orthogonal.

- Many textbooks say we use *Hermitian* operators to represent quantum observables since they have real eigenvalues, and observables are real. But this seems to confuse the metaphysical sense of 'real' (it exists..) with the mathematical sense (it's not complex or imaginary..) of the word. One could imagine - for example - representing an observable and its associated uncertainty by a complex number.

- *In pilot-wave theory, we see that Hermitian operators are important not just for the usual reasons but because their eigenvalues are* **spacetime constants***. When conditions for a complete measurement are met, can infer* A *without needing to know* $\mathbf{x}$*. Eigenfunctions thus play special role of uniquely specifying relevant particle property independently of particle location.*

Stage 2: amplification/registration

Stage 1 **reversible**: outgoing packets can in principle reoverlap significantly - apparatus and object coords then still well-defined but correlated so can no longer infer from y anything on state of $\mathbf{x}$. Get *permanent* results of real measurements if apparatus coord y represents - or is coupled to - a very large number of degrees of freedom.

To represent these apparatus coords introduce variables $z_1, \ldots, z_N$ where $N \sim 10^{23}$. Initial wave function for these is $\xi_0(z_1, \ldots, z_N)$. Total initial Ψ therefore

$$\Psi_0(\mathbf{x}, y, z_1, \ldots, z_N) = \Psi_0(\mathbf{x})\phi_0(y)\xi_0(z_1, \ldots, z_N)$$

After the interaction the further coordinates will be correlated with the y-coordinate:

$$\Psi(\mathbf{x}, y, z_1, \ldots, z_N, T) = \sum_a c_a \Psi_a(\mathbf{x})\phi_0(y - gaT)\xi_a(z_1, \ldots, z_N)$$

This is a linear superposition of nonoverlapping functions in total configuration space. Once particle enters domain where one of summands finite it will stay there, since probability that functions subsequently overlap overwhelmingly low, even if ψ_as or ϕ_0s overlap. Reading of y will yield permanent record of state of system $\mathbf{x}$ at termination of impulse. This is for all practical purposes **irreversible**.

How much to explicitly include? Bell: '*Put sufficiently much into the quantum system that the inclusion of more would not significantly alter practical predictions.* Cloud chamber very good example.

Decoherence

Decoherence implies loss of coherence, i.e. diminuition of interference terms between different branches of the wave function. In context of measurement theory, it is implied that this happens due to establishment of correlations between the quantum system and its **environment** *(as in previous slide).*

- Concept invented by *Bohm* in 1952 - strictly speaking was the only thing added by him to de Broglie's 1927 pilot-wave theory. It was general lack of understanding of its role in measurement theory in 1920s that led to de Broglie being discouraged by people like Pauli. Decoherence arguments began to be discussed again in the 1980s and today they are widely used.

- Not mysterious - just ordinary dynamical Schrödinger evolution. Most important thing to remember:

  Decoherence alone does not solve the measurement problem.

  Merely provides mechanism for the different branches of Ψ to stop interfering. All branches *continue to exist*. Require some appropriate interpretation of the wave function or an addition of 'hidden variables' to say why one branch is *what one sees*.

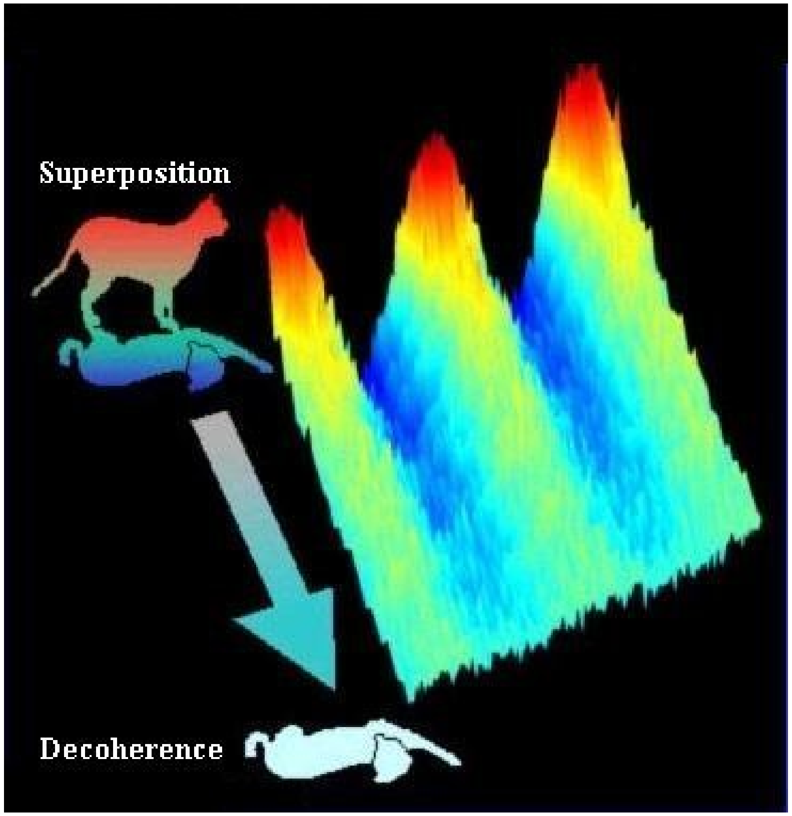
- Bigger the object, faster this happens, so macroscopic wave packets stay narrow. Decoherence-like interactions also affect microscopic systems e.g. α-particle interaction with gas in cloud chamber.

- Useful also in e.g. defining classical limit. Pilot-wave trajectories differing in initial conditions cannot cross (Lecture 2), since wave guides particles by way of 1st-order equation, while Newton's equations are 2nd-order and possible trajectories do cross. However, non-interfering components produced by decoherence can indeed cross, and so will trajectories of particles trapped inside them.

See http://plato.stanford.edu/entries/qm-decoherence/ for a good discussion.

Misunderstanding decoherence..

..is all too common in diagrams like this:



Where has the second cat gone?

Decoherence doesn't make it go away!

[*Perhaps it created its own parallel universe and disappeared off into it. Ha, ha. But what kind of nutter would believe something like that?*]

The density matrix

Decoherence arguments normally formulated in terms of density matrix.

Density matrix : Describes *statistical state* of quantum system. - required with (a) an *ensemble of systems*, or (b) *single system with uncertain preparation history* (don't know with certainty what pure state system is in).

- Normal *pure state* ψ completely determines statistical behavior of measurement for operator A. For any real function F, expectation value of $F(A)$ is $\langle\psi|F(A)|\psi\rangle$.
- Consider *mixed state* prepared by statistically combining two pure states ψ, ϕ each with prob $\frac{1}{2}$, e.g. toss coin and use state preparation for ψ (heads) or ϕ (tails). Statistical properties of observable completely determined but no vector ξ such that $\langle\xi|F(A)|\xi\rangle$. There is a unique *density operator* ρ such that expectation value is $\mathrm{Tr}[F(A)\rho]$, and here ρ is clearly $\frac{1}{2}|\phi\rangle\langle\phi| + \frac{1}{2}|\psi\rangle\langle\psi|$.
- The most general finite dimensional density operator is of form $\rho = \sum_j p_j|\psi_j\rangle\langle\psi_j|$ with non-negative coeffs p_j that sum to one. Measurement expectation value: $\langle A\rangle = \sum_j p_j\langle\psi_j|A|\psi_j\rangle = \mathrm{Tr}[\rho A]$.
- Just as Schrödinger equation describes pure states evolving in time, the *von Neumann Equation* describes how a density operator evolves in time: $i\hbar\frac{\partial\rho}{\partial t} = [H,\rho]$. Analogous to Liouville equation in classical statistical mechanics.

Density matrix and measurement

- Real diagonal elements give particle distribution $\rho(\mathbf{x},\mathbf{x}) = |\psi(\mathbf{x})|^2$. Complex off-diagonal elements encode 'entanglement' or interference - if $\rho(\mathbf{x},\mathbf{x}') = 0$ then $\mathbf{x}$ and $\mathbf{x}'$ lie in non-overlapping supports (branches) of the wave function.

- Density matrix evolution by phenomenological equations that in course of time lead to vanishing off-diagonal elements became celebrated solution of the measurement problem. However, nothing but rephrasing of *for-all-practical-purposes* impossibility of bringing wave packets belonging to different pointer positions to interference.

- Error just came from confusion about proper and improper density matrices and whether or not they submit to the ignorance interpration. Nevertheless many physicists accepted this as a solution - as if Schrödinger had not been aware of the computation. But of course Schrödinger *based* his cat story on the fapp impossibility of interference of macroscopically disjoint wave packets.

  "*There is a difference between a shaky or out-of-focus photograph and a snapshot of clouds and fog banks.*"

- Definiteness of final outcome is property of definiteness of pointer location under all circumstances (whatever the quantum state). This is the crucial point, and not that Ψ is composed of a set of disjoint packets. Latter just a necessary condition that allows us to ascertain from the always well-defined pointer reading unambiguous information on an object property. Not itself the condition for definiteness.

Derivation of Born's statistical postulate

Born's postulate (1926): *If system described by wave function* $\psi_0(\mathbf{x}) = \sum_a c_a \psi_a(\mathbf{x})$ *then probability of finding result* a *when measurement is performed is given by* $|c_a|^2$*.*

In pilot-wave theory, statistical element enters since unable to control initial positions $\mathbf{x}_0$ and y_0. Only know distributed as $|\psi_0|^2$ and $|\phi_0|^2$. How come Born's prescription?

During interaction $H = g\hat{A}\hat{p}_y$ total probability conserved: $(\mathrm{d}/\mathrm{d}t)\int |\Psi|^2 \, \mathrm{d}^3x\mathrm{d}y = 0$. When packets cease to overlap, configuration space probability density given by

$$|\Psi(\mathbf{x}, t, T)|^2 \approx \sum_a |c_a|^2 \psi_a(\mathbf{x})|^2 |\phi_0(y - gaT)|^2$$

since interference terms negligible. Probability that system point lies in volume element $\mathrm{d}^3x \, \mathrm{d}y$ about point $(\mathbf{x}, y)$ in domain where ath summand appreciable therefore

$$P_a \mathrm{d}^3x\mathrm{d}y = |c_a|^2 |\psi_a(\mathbf{x})|^2 |\phi_0(y - gaT)|^2 \, \mathrm{d}^3x\mathrm{d}y$$

Hence total prob that $\mathbf{x}$ lies within $\psi_a(\mathbf{x})$ and y within $\phi_0(y - gaT)$ given by $\int P_a \, \mathrm{d}^3x\mathrm{d}y = |c_a|^2$ (integral over all configuration space). Deduce prob of outcome $\Psi \to c_a\psi_a(\mathbf{x})\phi_0(y - gaT)$, in which true value $A = a$, given by Born formula.

In standard QM Born's postulate is normally just stated rather than being derived.

More about Born's postulate

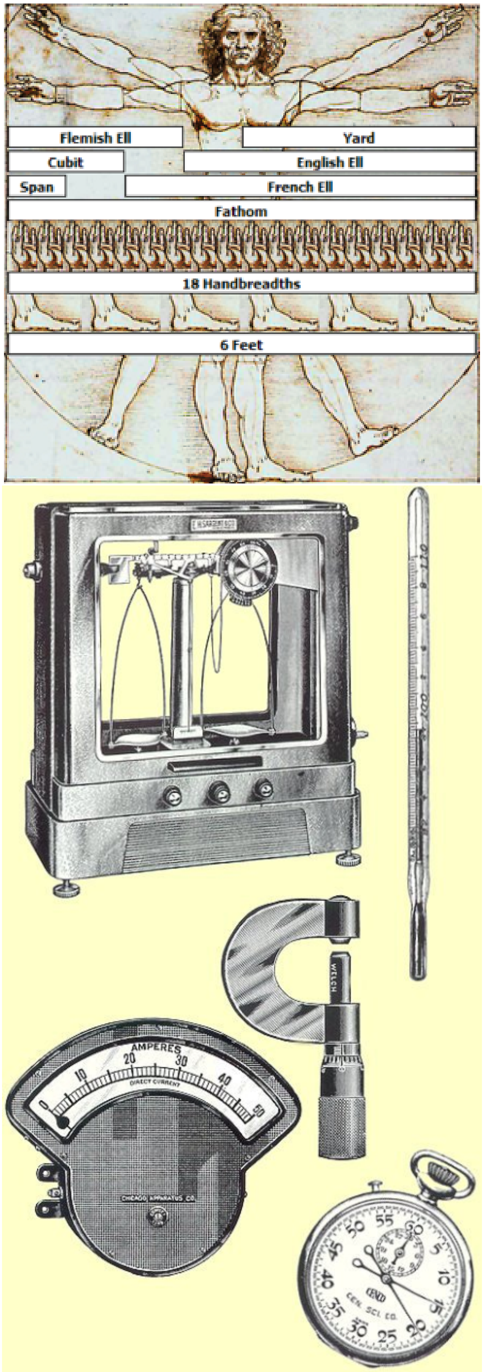
- Notice that the numbers $|c_a|^2$ only have statistical meaning just given when wave $\psi_0(\mathbf{x})$ physically disrupted in way that occurs in a measurement. These numbers do not refer to the probability that the particle *has* the value $A = a$ when the state function is $\psi_0(\mathbf{x})$. The true value of A is in general quite different - given by $A_0(\mathbf{x}_0) = \mathrm{Re}\,\Psi_0^*(\mathbf{x})(\hat{A}\psi_0)(\mathbf{x})/|\psi_0(\mathbf{x})|^2|_{\mathbf{x}=\mathbf{x}_0}$.

- Although measurement changes in fundamental way the prior actual value of the observed quantity in each individual trial, the mean value of this quantity over the ensemble is preserved and is therefore given by the weighted sum of the eigenvalues:

$$\langle \hat{A} \rangle = \int \psi_o^*(\mathbf{x})(\hat{A}\psi_0)(\mathbf{x})\,\mathrm{d}^3x = \sum_a a|c_a|^2$$

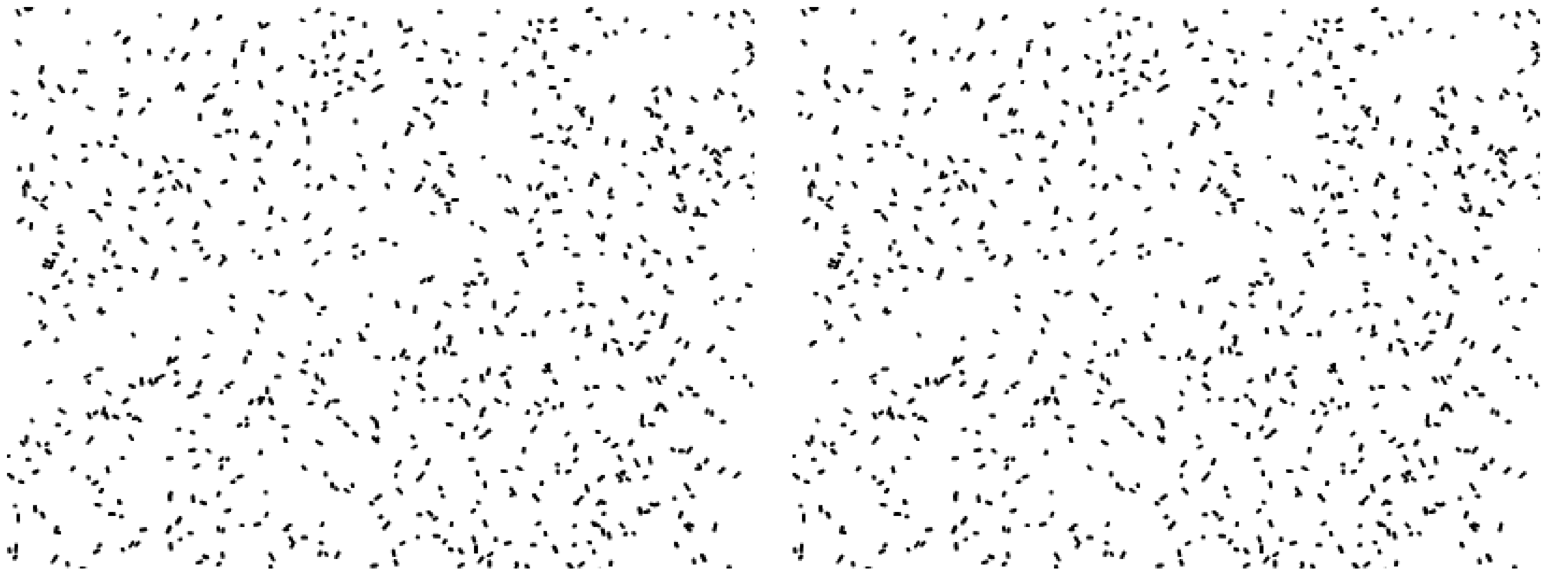
  Thus some information concerning the prior state of the system is revealed by the measurement, albeit statistically. The same is true for the mean of any function of $\hat{A}$, since this commutes with the interaction Hamiltonian.

John Bell (oh no, not again!) on measurement

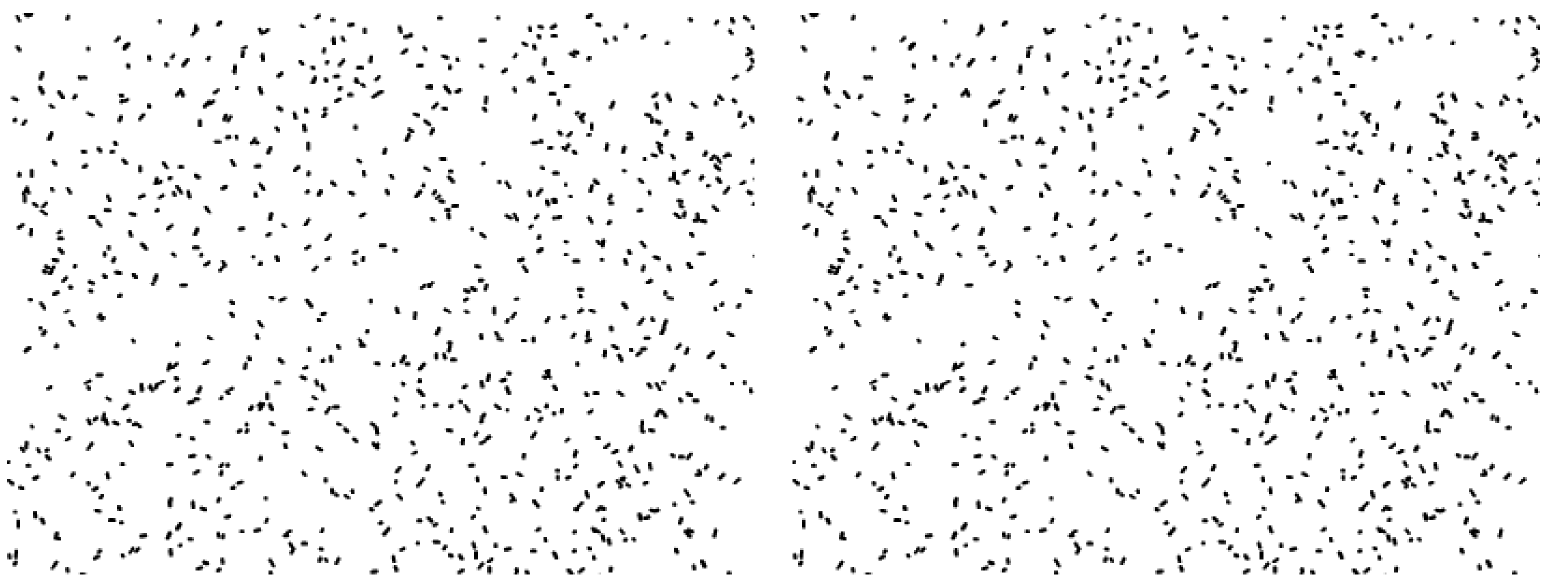


"A final moral concerns terminology. Why did such serious people take so seriously axioms which now seem so arbitrary? I suspect that they were misled by the pernicious misuse of the word 'measurement' in contemporary theory. This word very strongly suggests the ascertaining of some preexisting property of some thing, any instrument involved playing a purely passive role. Quantum experiments are just not like that, as we learned especially from Bohr. The results have to be regarded as the joint product of 'system' and 'apparatus', the complete experimental set-up. But the misuse of the word 'measurement' makes it easy to forget this and then to expect that the 'results of measurements' should obey some simple logic in which the apparatus is not mentioned. The resulting difficulties soon show that any such logic is not ordinary logic. It is my impression that the whole vast subject of 'Quantum Logic' has arisen in this way from the misuse of a word. I am convinced that the word 'measurement' has now been so abused that the field would be significantly advanced by banning its use altogether, in favour for example of the word 'experiment'."

The origin of randomness



It's just like in classical stat mech. With a modified force law and nonlocality.



[From the talk title, I bet you were expecting it to take up half the lecture, weren't you?]

And finally.. the quantum Zeno effect

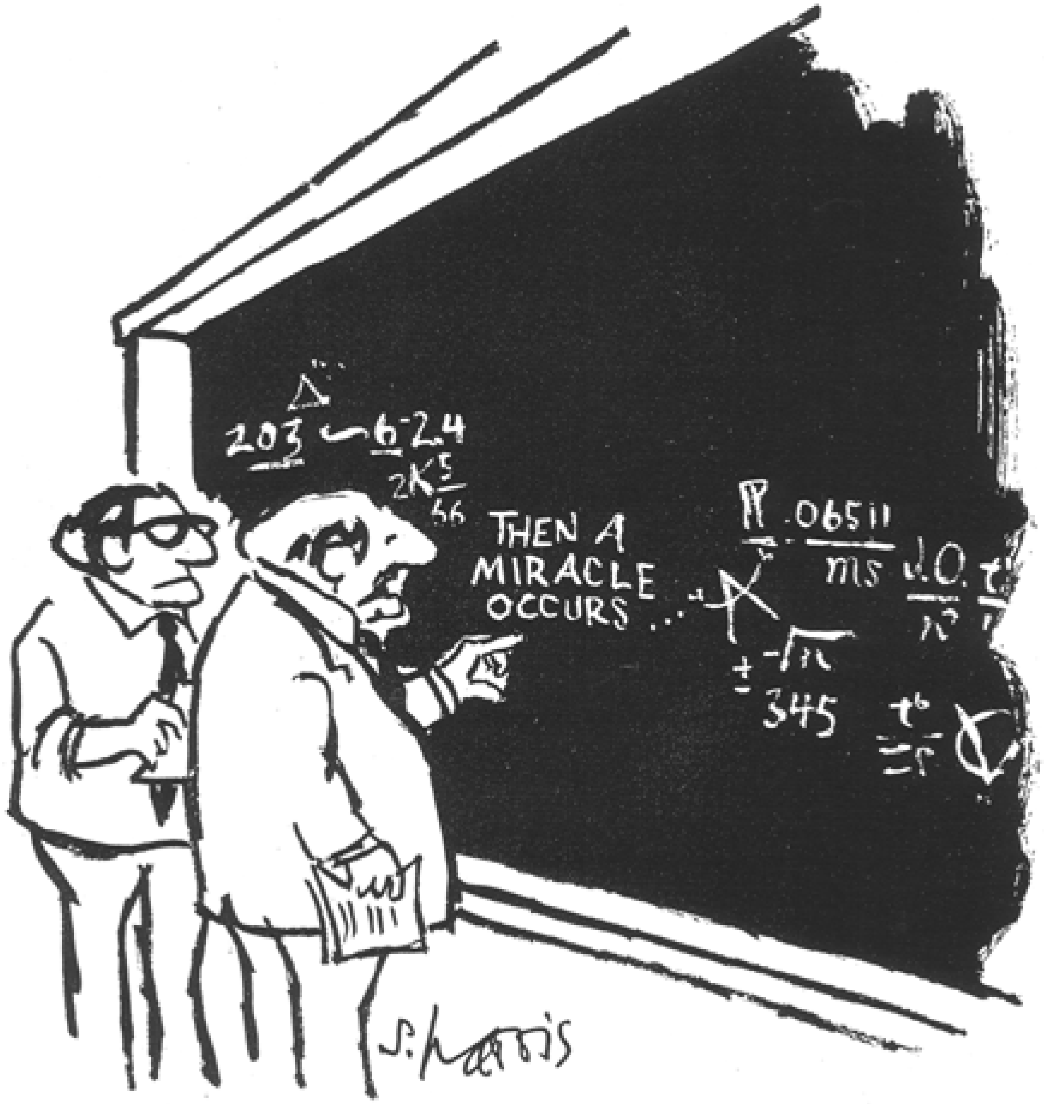
Requested by a member of the audience last week..

The *watched-pot-never-boils effect* - excellent example of false paradox created by usual interpretation. Nowadays its significance is as an impressive *illustration* of the participatory nature of quantum measurements. As Bohm himself wrote:

"*If one supposes that an electron is continually 'watched' by a piece of apparatus, the probability of transition has been shown to be zero. It seems that the electron can undergo transition only if it is not 'watched'. This appears to be paradoxical in the usual interpretation which can only discuss the results of 'watching' and has no room for any notion of the electron existing while it is not being 'watched'. But in* [pilot-wave theory] *with its objective ontology, this puzzle does not arise because the system is evolving whether it is watched or not. Indeed, as the theory of measurement that we have outlined shows, the 'watched' system is profoundly affected by its interaction with the measuring apparatus and, so we can understand why, if it is 'watched' too closely, it will be unable to evolve at all.*"

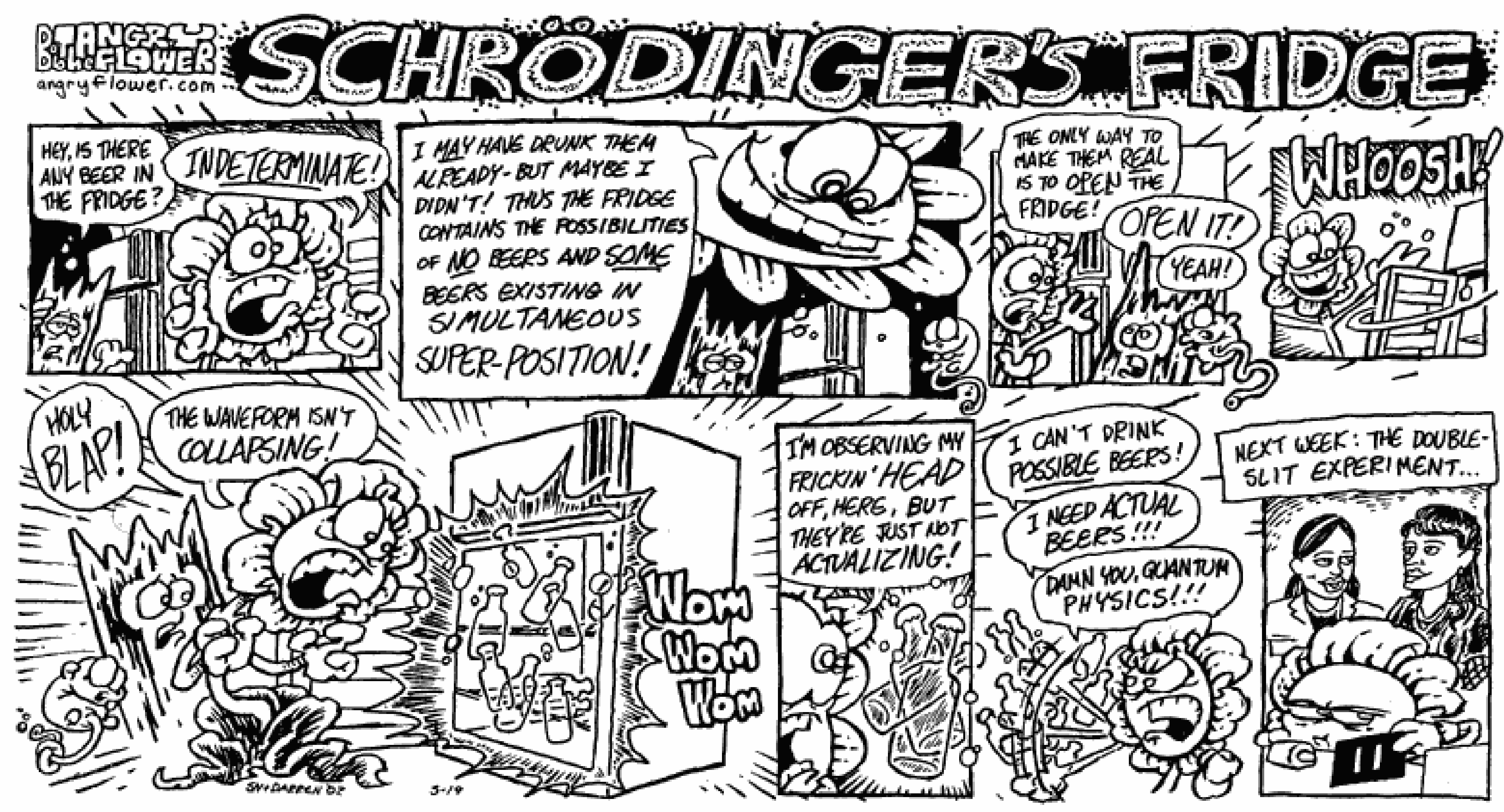
Note also the *reverse Zeno effect*, where Wile E. Coyote, busily chasing the Roadrunner, runs off a cliff but doesn't fall down until he *observes* that he's running in mid-air.

The theory of measurement



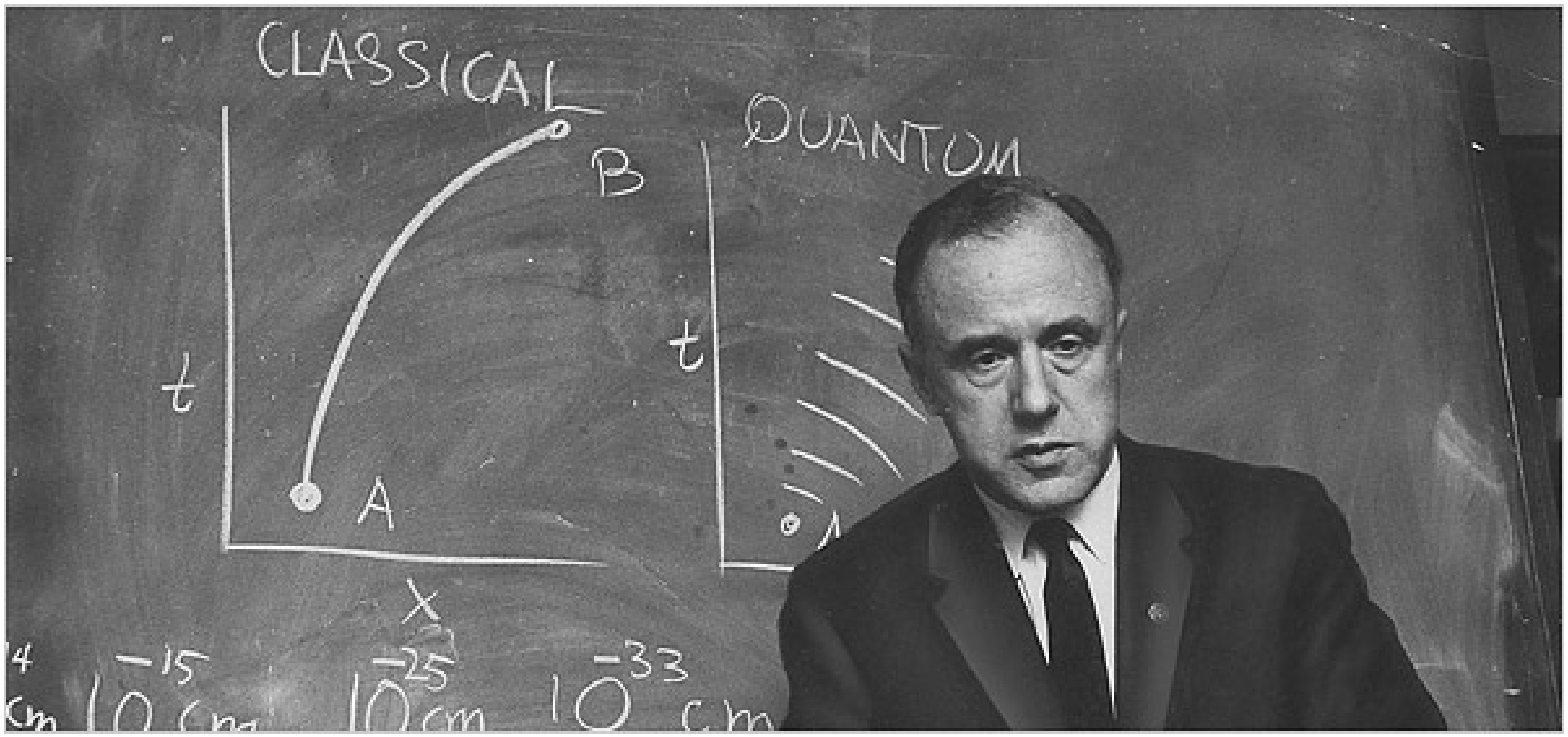
"I THINK YOU SHOULD BE MORE EXPLICIT HERE IN STEP TWO."

Macroscopic superpositions



Note to self

Pilot-wave theory *is* standard quantum mechanics with a *single* semantic change in the meaning of a word: $|\Psi(x)|^2$ is the probability that a particle *is* at point x rather than the probability of being *found there in a suitable measurement*. No extra maths. Nothing.



"*Every attempt, theoretical or observational, to defend such a hypothesis (the notion of hidden variables supplementing the wave function description) has been struck down.*" [J.A. Wheeler (1983)]

Uh?

Rest of course

Lecture 1: 21st January 2009
An introduction to pilot-wave theory

Lecture 2: 28th January 2009
Pilot waves and the classical limit. Derivation and justification of the theory

Lecture 3: 4th February 2009
Elementary wave mechanics and pilot waves, with nice examples

Lecture 4: 11th February 2009
The theory of measurement and the origin of randomness

Lecture 5: 18th February 2009
Nonlocality, relativistic spacetime, and quantum equilibrium

Lecture 6: 25th February 2009
Calculating things with quantum trajectories

Lecture 7: 4th March 2009
Not even wrong. Why does nobody like pilot-wave theory?

Lecture 8: 11th March 2009
Bohmian metaphysics : the implicate order and other arcana
Followed by a GENERAL DISCUSSION.

Slides/references on web site: `www.tcm.phy.cam.ac.uk/~mdt26/pilot_waves.html`