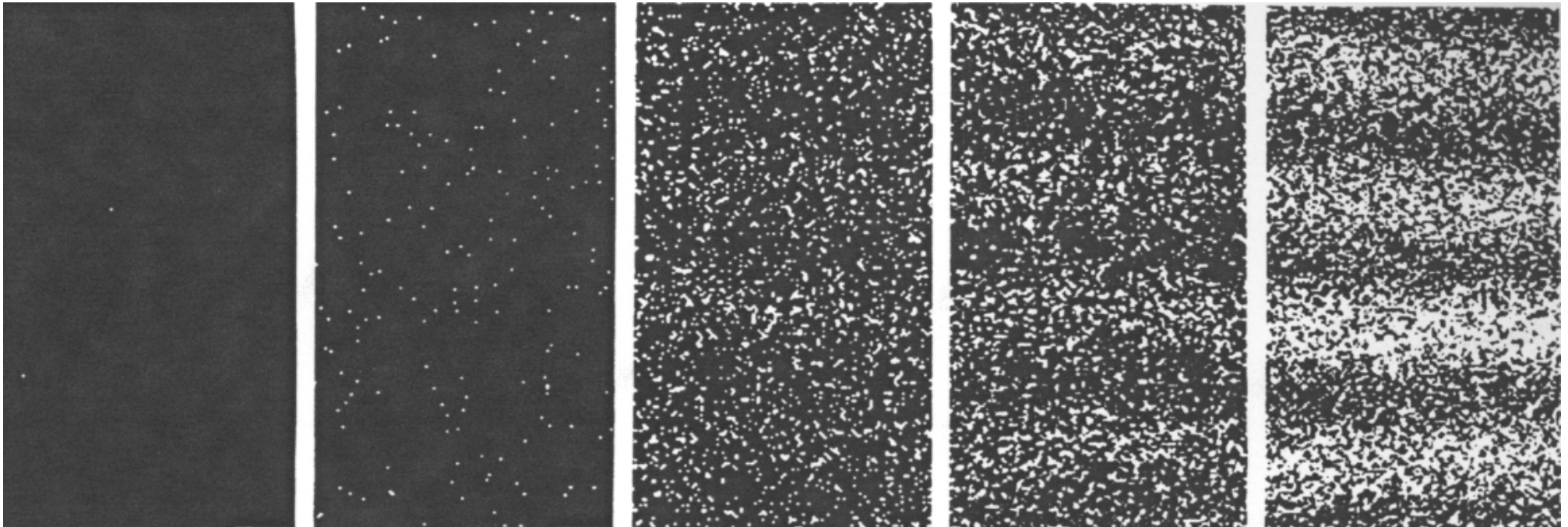


Pilot-wave theory, Bohmian metaphysics, and the foundations of quantum mechanics

Lecture 1

An introduction to pilot-wave theory



Mike Towler

TCM Group, Cavendish Laboratory, University of Cambridge

www.tcm.phy.cam.ac.uk/~mdt26 and www.vallico.net/tti/tti.html

mdt26@cam.ac.uk

Acknowledgement

The material in this lecture is largely a summary of standard works (books and articles) by Peter Holland, David Bohm, Basily Hiley, Detlef Dürr, Stefan Teufel, Antony Valentini, Sheldon Goldstein, and others. The book '*Quantum Dialogue*' by Mara Beller is extensively quoted in the discussion of the Copenhagen interpretation. A list of works consulted in the preparation of these slides is available on the course web site.

MDT

Opinion Poll

In this course we shall claim that all the interpretative problems raised in non-relativistic quantum mechanics are essentially solved by the *pilot-wave approach*.

What is your current opinion on pilot-wave theory? (a.k.a. *de Broglie-Bohm theory*, *Bohmian mechanics*, the *causal* or *ontological interpretation of QM*).

A: I have never heard of it.

B: I have heard of it and..

B1: ..believe it is the best available formulation of non-relativistic QM.

B2: ..believe is interesting and stands equal with one or more other formulations.

B3: ..believe while internally self-consistent it is pointless metaphysical speculation.

B4: ..understand that it has been shown to be incorrect or impossible.

B5: ..don't know enough details to have an opinion.

B6: ..have come to this lecture course just to have fun and slag off Mike.

B7: ..am a paid assassin sent by the orthodox physics inquisition to torture and liquidate everyone who attends this filthy heretical course. Viva Bohr!

Feynman on the two-slit experiment

- *“A phenomenon which is impossible, absolutely impossible, to explain in any classical way, and which has in it the heart of quantum mechanics. In reality it contains the only mystery.”*
- *“Do not keep saying to yourself, if you can possibly avoid it, ‘But how can it be like that?’ because you will get ‘down the drain,’ into a blind alley from which nobody has yet escaped. Nobody knows how it can be like that.”*
- *“Many ideas have been concocted to try to explain the curve for P_{12} [that is, the interference pattern] in terms of individual electrons going around in complicated ways through the holes. None of them has succeeded.”*
- *This experiment “has been designed to contain all of the mystery of quantum mechanics, to put you up against the paradoxes and mysteries and peculiarities of nature one hundred per cent.”*
- *“How does it really work? What machinery is actually producing this thing? Nobody knows any machinery. Nobody can give you a deeper explanation of this phenomenon than I have given; that is, a description of it.”*

What problems are we trying to address?

- Though QM treats statistical ensembles in a satisfactory way, we are unable to describe individual quantum processes without bringing in unsatisfactory assumptions, such as the collapse of the wave function.
- We do not really understand the well-known nonlocality that has been brought out by Bell in connection with the EPR experiment.
- What is the mysterious ‘wave-particle duality’ in the properties of matter that is demonstrated in a quantum interference experiment?
- Also where is the boundary between a quantum system and the rest of the world (including apparatus and experimenter)? Where does macroscopic definiteness give way to microscopic indefiniteness? Somewhere between pollen grains, viruses and macromolecules? No defined boundary between ‘classical domain’ where ‘real state’ is valid concept and ‘quantum domain’ where it is not.
- Above all, there is the inability to give a clear notion of what the reality of a quantum system should be.

Why Pilot-wave theory?

“Hardly any other interpretation accepts existence of an outside reality, agrees with predictions of quantum formalism, and is both consistent and not utter madness..”

'Standard quantum mechanics': the Copenhagen interpretation

"A philosophical extravaganza dictated by despair" (Schrödinger)

1. System *completely described* by wave function ψ representing observer's knowledge of the system, or 'potentiality'. The wave function is all there is. 'Hidden variables' *impossible*. (Heisenberg)
2. Description of nature essentially probabilistic. Probability of event related to square of amplitude of wave function related to it (Born rule). 'Measurement' randomly picks out exactly one of the many possibilities allowed for by the state's wave function through nonlocal 'collapse process'.
3. Heisenberg's uncertainty principle: observed fact that it is not possible to know values of all properties of system at same time; those properties not known with precision must be described by probabilities. Properties in fact supposed to be *indeterminate* not uncertain.
4. Complementarity principle: there is no logical picture (obeying classical propositional logic) that can simultaneously describe and be used to reason about all properties of a quantum system. Example: matter exhibits a wave-particle duality. An experiment can show the particle-like properties of matter, or wave-like properties, but not both at the same time. (Niels Bohr)
5. Measuring devices are essentially classical devices, and measure classical properties such as position and momentum.
6. The 'correspondence principle' of Bohr and Heisenberg: the quantum mechanical description of large systems should closely approximate to the classical description.

Now well-known that Copenhagen cannot be reconstructed as a coherent philosophical framework - it is a collection of local, often contradictory, arguments embedded in changing theoretical and sociopolitical circumstances.. ..riddled with vacillations, about-faces and inconsistencies. [See Mara Beller book 'Quantum Dialogue']

How did Founding Fathers get people to believe this?

- Positivist/antirealist arguments (i.e. confusing 'must not' with 'need not') nevertheless valuable for protecting science from undesirable extensions of scientific metaphysics. Greatest chance of achieving consensus in scientific community. Bohr's antirealist philosophy uniquely designed for this purpose.
 - Bohrian rhetoric of finality and inevitability: *'We see that it cannot be otherwise', 'This is something there is no way round', 'The situation is an unavoidable one', [complementarity] is 'most direct expression of a fact..as the only rational interpretation of quantum mechanics', 'obvious', 'evident', 'clear from the outset', 'a simple logical demand', 'we must recognize', 'it is imperative to realize'. **(Circular) demonstrations of consistency disguised as compelling arguments of inevitability.** [Those who do not agree are] *'unable to face the facts'* and disagreeing with the masters of the universe thus becomes bad for your career..*
 - Bohr's numerous illustrations of experimental arrangements with bolts, springs, diaphragms and rods made experimentalists feel at ease in unfamiliar quantum territory. No mathematical training and only most elementary physics needed to follow his arguments.
 - Paint oneself as an intellectual revolutionary. Intentionally ambitious individuals like Heisenberg particularly prone to using revolutionary rhetoric. Present opponents (Einstein! Schrödinger!) as dogmatic conservatives unable to digest revolutionary novelties - even though just disagree with him.
- "One would expect proponents of Copenhagen were in possession of some very strong arguments, if not for inevitability, at least for high plausibility. Yet a critical reading reveals that all the far-reaching, or one might say far-fetched, claims of inevitability are built on shaky circular arguments, on intuitively appealing but incorrect statements, on metaphorical allusions to quantum 'inseparability' and 'indivisibility' that have nothing to do with quantum entanglement and nonlocality."* [Beller]

What to take home from this course

- Today it is simply untenable to regard the views of Bohr and Heisenberg as in any sense standard or canonical. They are more 'smoke and mirrors' than a unique compelling world view forced on us by experiment. The meaning of quantum theory is today an *open question*.
- Rejection of Bohmian hidden variables theories by Bohr, Heisenberg, Pauli *et al.* - not merely as hopeless but as downright meaningless - was ultimately irrational. Acknowledging this would have been embarrassing. Pilot-wave theory is intuitive, internally consistent, in agreement with experiment, and very plausible.
- Interpretations pointless unprovable metaphysics, says you, but ask yourself what general public knows about QM: the '*bizarre mind-boggling spooky quantum world where cats are alive and dead at the same time in many universes at once!!!!*'. But all weird stuff (except nonlocality) can be seen as interpretational artefact. Go on - you enjoy your mum thinking you're profound don't you..?
- Pilot-wave theory is of course **wrong** like any other non-relativistic/no gravity theory. But understanding QM in different ways can provide guidance on new theoretical and experimental directions. Bell's theorem and Aspect's experiments confirming nonlocality came *directly from trying to understand why Bohm was wrong*. 'All impossibility theorems proved is lack of imagination'. New perspectives lead to new directions - e.g. Valentini and nonlocality (lecture 5).

Alternatives to Copenhagen

1. Ensemble interpretation, or statistical interpretation
2. Participatory Anthropic Principle (PAP)
3. Consistent histories
4. Objective collapse theories (GRW etc.)
5. Many worlds
6. Stochastic mechanics
7. The decoherence approach
8. Many minds
9. Quantum logic
10. Transactional interpretation
11. Relational quantum mechanics
12. Modal interpretations of quantum theory
13. Incomplete measurements
14. Pilot-wave interpretation (a.k.a. Bohmian mechanics, de Broglie-Bohm interpretation, causal interpretation, ontological interpretation, hidden variables)

Only the blue ones will be mentioned again in this course. Look the others up if you must!

Interpretation	Deterministic?	Waveform real?	Unique history?	Hidden variables?	Collapsing wavefunctions?	Observer role?
Copenhagen interpretation (Waveform not real)	No	No	Yes	No	NA	NA
Ensemble interpretation (Waveform not real)	No	No	Yes	Agnostic	No	None
Copenhagen interpretation (Waveform real) Objective collapse theories	No	Yes	Yes	No	Yes	None
Consistent histories (Decoherent approach)	Agnostic ¹	Agnostic ¹	No	No	No	Interpretational ²
Quantum logic	Agnostic	Agnostic	Yes ³	No	No	Interpretational ²
Many-worlds interpretation (Decoherent approach)	Yes	Yes	No	No	No	None
Stochastic mechanics	No	No	Yes	No	No	None
Many-minds interpretation	Yes	Yes	No	No	No	Interpretational ⁴
Bohm-de Broglie interpretation ("Pilot-wave" approach)	Yes	Yes ⁵	Yes ⁶	Yes	No	None
Transactional interpretation	No	Yes	Yes	No	Yes ⁷	None
Copenhagen interpretation (Waveform real) PAP	No	Yes	Yes	No	Yes	Causal
Relational Quantum Mechanics	No	Yes	Agnostic ⁸	No	Yes ⁹	None
Incomplete measurements	No	No ¹⁰	Yes	No	Yes ¹⁰	Interpretational ²

What is Ψ ?

The most puzzling object in quantum mechanics

- Is it subjective or objective?
- Does it merely represent information or does it describe an observer-independent reality?
- If it is objective, does it represent a concrete material sort of reality, or does it somehow have an entirely different and perhaps novel nature?
- What's going on with the collapse?

There is very little agreement about the answers to these questions, but we can all agree that the following crudely-expressed possibilities must be correct: [Goldstein]

- The wave function is everything.
- The wave function is something (but not everything).
- The wave function is nothing.

The Ψ -function as information

The idea that Ψ represents information, and does not describe an objective state of affairs, raises many questions and problems:

- Information about what?
- What about quantum interference? How can terms of a quantum superposition interfere with each other, producing observable interference pattern, if such a superposition is just an expression of our ignorance?
- Problem of vagueness: QM supposed to be fundamental physical theory. As such it should be precise. But if fundamentally about information, then presumably concerned directly either with mental events or, more likely, with behaviour of macroscopic variables. But notion of 'macroscopic' is intrinsically vague.
- Simple physical laws are to be expected, if at all, at the most fundamental level - of the basic microscopic entities - and that messy complications should arise at level of larger complex systems. It is only at this level that talk of information, as opposed to microscopic reality, can become appropriate.
- The very form of the Hamiltonian and wave function strongly points to a microscopic level of description. Why else $\Psi(\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_N)$?
- There is a widespread belief that large things are built out of small ones, and that to understand the large we need to understand the small.

In pilot-wave theory, Ψ is a real physical field which influences particle trajectories. At the same time it represents the greatest possible information about the system.

Basic idea of pilot-wave theory

De Broglie: “A freely moving body follows a trajectory that is orthogonal to the surfaces of an associated guiding wave.”

Used a proto-pilot-wave theory to predict ‘diffraction’ of electrons by small holes.

Ph.D. thesis in 1924 \implies Nobel prize in 1929!

- System has wave function $\Psi(x, t)$ (de Broglie’s idea!) and definite particle configuration $x(t)$ at all times. To define trajectory need velocity vector, which can be deduced from the *probability current density vector* of the standard theory:
 $\dot{x}(t) = \frac{j(x, t)}{|\Psi(x, t)|^2} =$ ratio of quantum probability current j to probability density ρ .

- Recall probability ‘fluid’ has current $\mathbf{j} = \frac{i\hbar}{2m}(\Psi\nabla\Psi^* - \Psi^*\nabla\Psi) = \frac{\hbar}{m}\text{Im}(\Psi^*\nabla\Psi)$ which satisfies the quantum-mechanical continuity equation,

$$\frac{\partial\rho}{\partial t} + \nabla \cdot \mathbf{j} = 0 \quad \text{or} \quad \frac{\partial}{\partial t} \int_V |\Psi|^2 dV + \int_S \mathbf{j} \cdot d\mathbf{A} = 0,$$

i.e. the conservation law for probability density in QM (time derivative of probability of particle being measured in V equals rate at which probability flows into V).

Theory based on this “completely avoids all the quantum paradoxes, all the mysticism of Bohr and Heisenberg, and replaces it with sharp mathematics.”

Basic equations - de Broglie form (1927)

Focussing for simplicity on nonrelativistic quantum system of N (spinless) particles with positions $\mathbf{x}_i (i = 1, 2, \dots, N)$ it is now generally agreed that, with appropriate initial conditions, quantum physics may be accounted for by the deterministic dynamics defined by two differential equations, i.e. the *Schrödinger equation*

$$i\hbar \frac{\partial \Psi}{\partial t} = \sum_{i=1}^N -\frac{\hbar^2}{2m_i} \nabla_i^2 \Psi + V \Psi$$

for a pilot wave $\Psi(\mathbf{x}_1, \dots, \mathbf{x}_N, t)$ in configuration space and the *guidance equation*

$$m_i \frac{d\mathbf{x}_i}{dt} = \nabla_i S$$

for particle trajectories $\mathbf{x}_i(t)$. Phase $S(\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_N, t)$ defined by $S = \hbar \text{Im} \ln \Psi$ of complex polar form $\Psi = |\Psi| \exp[iS/\hbar]$. Compare $\mathbf{j}/|\Psi|^2$. Particles ‘pushed along’ by wave along trajectories perpendicular to surfaces of constant phase.

- Can also guess guidance equation from de Broglie relation $\mathbf{p} = \hbar \mathbf{k}$ (connects particle and wave properties). Wave vector \mathbf{k} defined only for plane wave. For general wave, obvious generalization of \mathbf{k} is local wave vector $\nabla S(\mathbf{x})/\hbar$. Hence $\mathbf{v} = \nabla S/m$. Detailed justifications next week.
- Operators on Hilbert space and all that play no fundamental role, but are exactly right mathematical objects to provide compact representation of the statistics in a de Broglie universe.

Basic equations - Bohm form (1952)

Bohm's 1952 presentation: take first time derivative of guidance equation $m\dot{\mathbf{x}} = \nabla S$ then use TDSE to get second-order theory analagous to Newton's second law:

$$\begin{aligned}\frac{\partial}{\partial t} \nabla_i S(x, t) &= \nabla_i \frac{\partial}{\partial t} \hbar \text{Im} \ln \Psi = \nabla_i \text{Im} \left[\frac{\hbar}{\Psi} \frac{\partial}{\partial t} \Psi \right] = \\ \nabla_i \text{Im} \left[\frac{i}{\Psi} \left(\frac{\hbar^2}{2m_i} \nabla_i^2 \Psi - V \Psi \right) \right] &= -\nabla_i \left[\frac{1}{\Psi} \left(\frac{-\hbar^2}{2m_i} \nabla_i^2 \Psi + V \Psi \right) \right] = -\nabla_i [V + Q]\end{aligned}$$

$$m_i \ddot{\mathbf{x}}_i = -\nabla_i (V + Q) ,$$

where

$$Q \equiv - \sum_i \frac{\hbar^2}{2m_i} \frac{\nabla_i^2 |\Psi|}{|\Psi|}$$

is the so-called 'quantum potential'. Bohm regarded this as the law of motion, with the de Broglie guidance equation added as a constraint on the initial momenta. For de Broglie, in contrast, law of motion for velocity had a fundamental status, and for him represented a unification of the principles of Maupertuis and Fermat.

Usually stated de Broglie did one-particle theory only, and that Bohm generalized it to N particles (as claimed in Bohm's book). However, de Broglie gave the full N -particle theory at the 1927 Solvay conference and also indicated that he knew about above second-order theory (see Valentini book). De Broglie's priority clearly established - thus 'Bohmian mechanics' is probably the wrong name.

Quantum Hamilton-Jacobi theory and the quantum potential

Bohm theory often presented as follows. Substitute amplitude-phase decomposition (polar form) of time-dependent wave function $\Psi(x, t) = R(x, t) \exp(iS(x, t)/\hbar)$ into time-dependent Schrödinger equation. Separate into real and imaginary parts to get pair of coupled evolution equations, namely the continuity equation for $\rho = R^2$:

$$\frac{\partial R^2}{\partial t} + \nabla \cdot \left(\frac{R^2 \nabla S}{m} \right) = 0$$

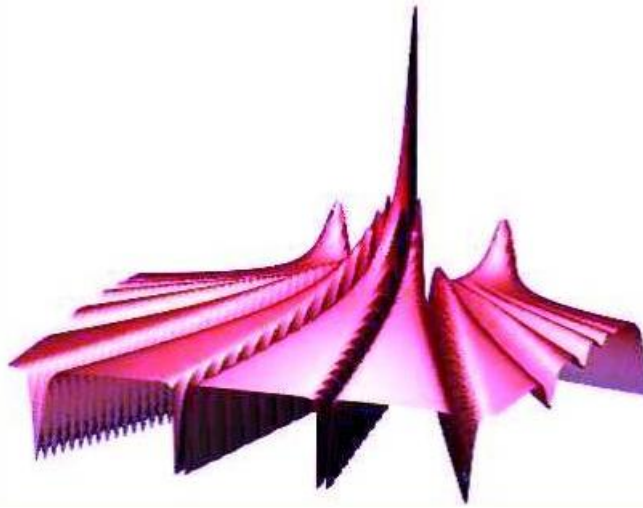
and a modified *Hamilton-Jacobi equation* for S :

$$-\frac{\partial S}{\partial t} = \frac{(\nabla S)^2}{2m} + V + Q \quad \text{where} \quad Q = -\frac{\hbar^2}{2m} \frac{\nabla^2 R}{R}$$

Suggests connection between phase of the Schrödinger wave and the classical 'action' S ? (and thus the Lagrangian function, which is the excess kinetic energy flow over the total potential energy). See next week.

Only difference between Hamilton-Jacobi theory (a standard reformulation of classical mechanics) and this 'quantum version' is the *quantum potential* Q . Thus Newtonian mechanics can be said to emerge from 'Bohmian mechanics' in classical limit. Size of quantum potential provides a measure of deviation of pilot-wave theory from its classical approximation.

The quantum potential Q

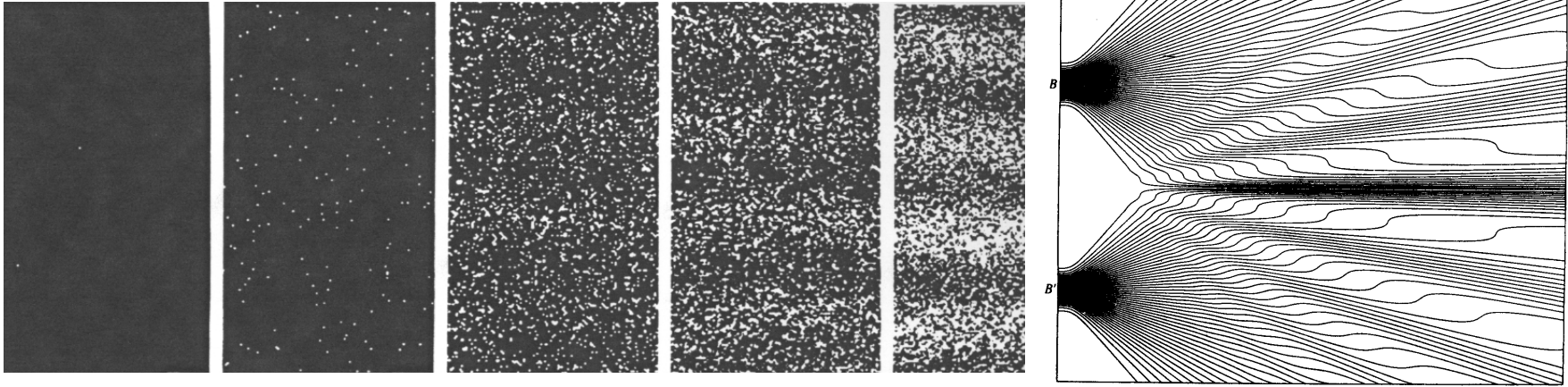


Good points: Q gives measure of pilot-wave deviation from classical approximation. Can be used to develop approximation schemes for solutions to Schrödinger equation.

Bad points: Increased complexity (Schrödinger equation simple and linear. Modified Hamilton-Jacobi equation complicated, highly nonlinear, and requires continuity equation for closure). Gives false impression of lengths to which we must go to convert orthodox QM into something more rational. Suggests in order to make Schrödinger equation into theory accounting for quantum phenomena in 'realistic terms' we must add complicated quantum potential of grossly nonlocal character.

In fact the quantum potential need not be mentioned in the formulation of pilot-wave theory - it is merely a reflection of the wave function, which it shares with orthodox quantum theory. If you don't like it can use de Broglie formulation instead..

The two-slit experiment explained



While each particle track passes through just one of the slits, the wave passes through both; the interference profile that consequently develops in the wave generates a similar pattern in the trajectories guided by the wave.

Compare Feynman's commentary (see earlier) with Bell's:

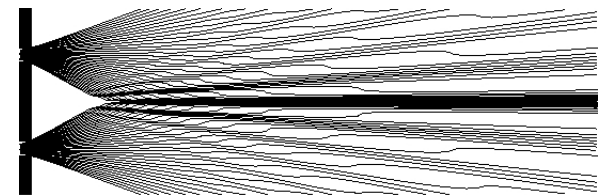
"Is it not clear from the smallness of the scintillation on the screen that we have to do with a particle? And is it not clear, from the diffraction and interference patterns, that the motion of the particle is directed by a wave? De Broglie showed in detail how the motion of a particle, passing through just one of two holes in the screen, could be influenced by waves propagating through both holes. And so influenced that the particle does not go where the waves cancel out, but is attracted to where they cooperate. This idea seems to me so natural and simple, to resolve the wave-particle dilemma in such a clear and ordinary way, that it is a great mystery to me that it was so generally ignored."

But trajectories are forbidden..

..because you can't observe them, aren't they?

- True can't directly observe them - but obvious why not - doesn't mean don't exist. Interaction involved with detection would influence particle's future motion, so won't see trajectory particle would have followed if position hadn't been detected (though result is also a Bohmian trajectory). Remember can't observe wave function either!
- Usual deduction of forbidden trajectories from Heisenberg relations is flawed (Lecture 3). Also can *retrodict* trajectories from final position if you really want to.
- Pilot-wave theory agrees with experiment, thus fact that pilot-wave theory exists proves assumed nonexistence of trajectories cannot be concluded from experiment.

"But to admit things not visible to the gross creatures that we are is, in my opinion, to show a decent humility, and not just a lamentable addiction to metaphysics." [Bell again]



Occam's razor? Best theory has fewest premises! But pilot-wave theory adds no math not already there - everything follows from one semantic change in meaning of probability. Also eliminates the - to most people unconvincing - postulates about measurement (fewer premises..). Gives completely new interpretation of quantum phenomena in which e.g. probability plays no fundamental role. Descriptive content identical but theories probably not equivalent at all. No basis for Occam.

Trajectories have great explanatory power!

The quantum equilibrium distribution: $P(\mathbf{x}, t) = |\Psi(\mathbf{x}, t)|^2$

Pilot-wave theory requires particles to be distributed with probability density $P(\mathbf{x}, t) = |\Psi(\mathbf{x}, t)|^2$. Once so distributed at any t they will always remain so under the Schrödinger time evolution ('equivariance'). Can show $|\Psi(\mathbf{x}, t)|^2$ is the *only* distribution with this property i.e. 'quantum equilibrium' is unique [Goldstein, Struyve 2007] and is analagous to thermal equilibrium $P = \frac{\exp(-H/kT)}{Z}$.

Can use 'typicality' arguments to show particles expected to be distributed this way in 'typical universe', but what happens if they are not (in the early universe, say)?

If guidance equation is exact $\mathbf{v}_i = \nabla_i S(x_i)/m$:

Particles tend towards $|\Psi|^2$ distribution due to essentially random perturbations of wave function coming from environment in which particles are found.

If guidance equation is only true 'on average' $\mathbf{v}_i = \nabla_i S(x_i)/m + \xi_i(t) + ??$:

Particle velocity from guidance equation subject to additional random fluctuations $\xi_i(t)$ (with zero average) from some sub-quantum-mechanical level. This is treatable as simple diffusion. Then, in order to satisfy equivariance, must assume suitable 'osmotic velocity' like in Brownian motion so that in equilibrium the diffusion current is balanced by the osmotic velocity and the *mean velocity* is given by guidance equation. Analagous to Nelson's *stochastic mechanics* which gives entirely classical derivation and interpretation of Schrödinger equation by analogy with Brownian motion.

Note similarity to drift velocity in diffusion quantum Monte Carlo.

The measurement problem - definition

What is the problem?

If you believe that the wave function is all there is, and you include both the system and the measuring apparatus, then the wave function after the measurement is a superposition of all possible results and continues to evolve according to the time-dependent Schrödinger equation. QM thus implies that measurements typically fail to have outcomes of the sort the theory was created to explain.

OR, EQUIVALENTLY

QM has two rules for evolution of wave function:

- A deterministic dynamics given by Schrödinger's equation for when the system is not being 'measured' or observed.
- A random nonlocal collapse of the wave function to an eigenstate of the 'measured observable' for when it is.

Objection because standard QM does not provide a coherent account of how these two apparently incompatible rules can be reconciled. Why should interactions between system and apparatus that we happen to call measurements be governed by laws different from those governing all other interactions?

The measurement problem in pilot-wave theory

If you admit the existence of configurations then:

- The description of the after-measurement situation includes, in addition to the wave function, the values of the variables that register the result, thus there is no measurement problem.
- The wave function of the universe does not collapse, but the wave function of the subsystem being measured *effectively* does ('*decoherence*' arguments).
- You can derive 'effective collapse' of subsystem wave function mathematically. In contrast to orthodox collapse this takes place objectively, takes a finite amount of time, and doesn't depend on anything like the observer's knowledge.
- All measurement paradoxes (Schrödinger's cat, Wigner's friend etc.) disappear.
- Observers not required, thus theory works before life evolved and wave function of universe doesn't depend on observation by God. Nor is it necessary to postulate each term in the superposition forms a whole new universe (cf. *utter madness*).

Better, no? See Lecture 4 for details.

Stationary states

$$\Psi(\mathbf{x}, t) = \Psi_0(\mathbf{x})e^{-iEt/\hbar}$$

Eigenfunctions of the Hamiltonian $\hat{H} = \frac{-\hbar^2}{2m}\nabla^2 + V$ with V independent of time. In $\Psi = R \exp(iS/\hbar)$ form we have $R(\mathbf{x}, t) = R_0(\mathbf{x})$ and $S(\mathbf{x}, t) = S_0(\mathbf{x}) - Et$.

Consequences

- Probability density independent of time: $|\Psi|^2 = R_0^2(\mathbf{x})$ i.e. no time-dependence to where particle likely to be found - there is effectively *no motion*.
- Quantum potential $Q = (-\hbar^2/2m)\nabla^2 R_0/R_0$ is time-independent. Therefore so is the total effective potential $\partial(V + Q)/\partial t = 0$.
- The velocity field is independent of time. If $\Psi_0(\mathbf{x})$ is a real function then the velocity is zero (Recall probability current $\mathbf{j} = \frac{\hbar}{2mi}(\Psi^*\nabla\Psi - \Psi\nabla\Psi^*)$). The particle is at rest where one would classically expect it to move since the quantum force $(-\nabla Q)$ cancels the classical force $(-\nabla V)$.
- The energy of all particles in the ensemble, $-\partial S/\partial t$, is a constant of the motion and equal to the energy eigenvalue E .

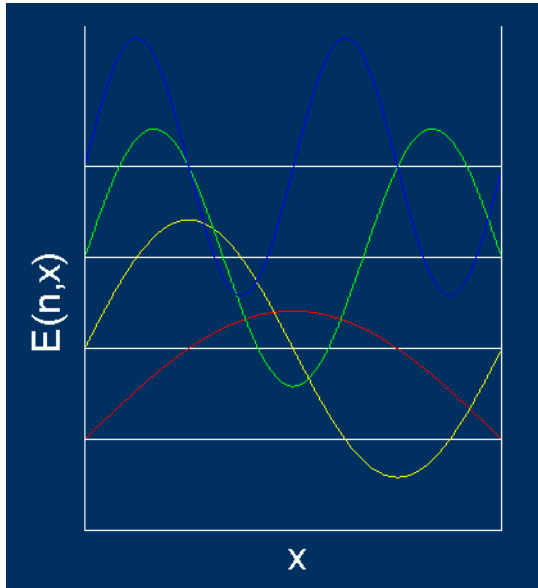
Usually said that stationary states are only theoretical constructs not realizable in nature..

The Uncertainty Principle

$$\Delta x_i \Delta p_j \geq \frac{\hbar}{2} \delta_{ij}$$

- Uncertainty emerges in pilot-wave theory simply as a constraint on our knowledge of quantum systems (as opposed to a constraint on their objective definiteness).
- Put another way, uncertainty principle describes statistical scatter in results of specific pairs of (non-commuting) measurements carried out on ensemble of identically prepared systems. Does *not* describe inherent impossibility for a particle to *have* e.g. a simultaneously well-defined position and momentum.
- Closely related to QEH, which says particle positions are Born-rule distributed. Trivial consequence is that position uncertainty (square root of expectation value of x^2 minus squared expectation value of x) matches that of standard QM.
- Statements about momentum uncertainty more subtle. In principle, momentum not indefinite (if know exact x and Ψ can infer precise velocity from guidance formula). But in general, QEH prevents you from knowing x and hence velocity.
- Thus momentum measurements do *not* simply reveal the value of the pre-measurement velocity (times mass). See next slide.

Momentum



Particle in a 1-D box

$$\Psi \propto e^{ikx} + e^{-ikx}$$

- k chosen so wave function vanishes at walls.
- *Momentum measurement* will give $\hbar k$ or $-\hbar k$.
- These are stationary states so pilot-wave theory says particle *at rest* (oh yes!). Assuming QEH, position uncertainty is correct.

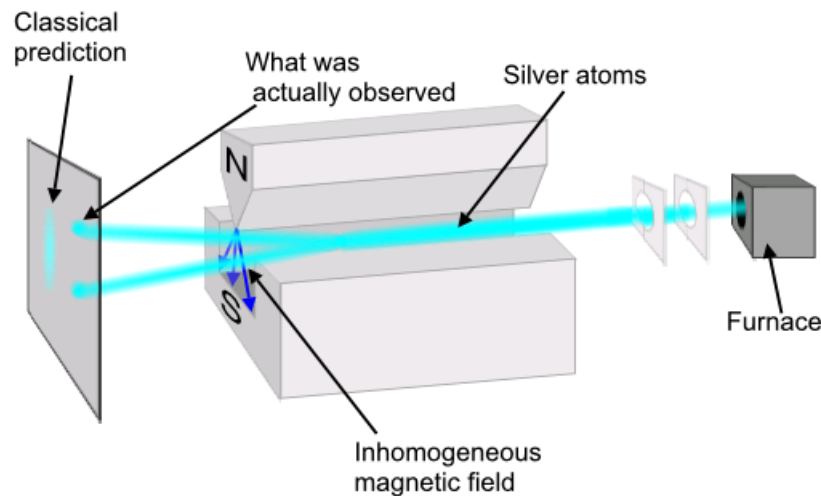
- ‘Momentum’ doesn’t have usual meaning in pilot-wave theory. But I can measure it! How? Time of flight measurement. Take away walls, let particle move freely, then detect position x . Divide by time, multiply by mass \Rightarrow ‘momentum’.
- In pilot-wave theory, wave function forms two packets which move off in the two directions. Particle carried away by one of them (50/50) and *accelerates* up to velocity $\pm \hbar k / m$ at point x . Thus measurement agrees with QM prediction but *not* revealing pre-experiment value of zero! \Rightarrow ‘Naive realism about operators’.
- Pilot-wave momentum uncertainty correct. Says how initial spread of positions relates to spread of momentum values that could be measured if momentum measurement performed (related to Fourier transform of $\Psi(x)$, roughly speaking).

'Naive realism about operators'

- Humans oblivious to trajectories; classical limit intuition arising from appropriately narrow wave packets (and equilibrium ensuring config near bulk of packet). In quantum case use (supposedly fundamental) classical language for measurement. But *energy* and *momentum* only relevant for Newtonian mechanics where they are conserved. Otherwise who's interested in $m \times v$?
- In 'momentum measurement' perform operations that *classically* would measure p . *Quite wrongly* assume same operations would yield 'value of momentum' even for nonclassical system. Instead 'result' (for ideal case) equals eigenvalue p of a linear operator \hat{P} (acting on final Ψ guiding system) which has nothing to do with any real property of system prior to 'measurement'.
- How do we get away with this? Formally possible since (from linearity of Schrödinger equation) there is general mathematical correspondence between linear operators and classical variables. Classical variable 'corresponding' to \hat{Q} arises from guidance by narrow packet with \hat{Q} -spectrum peaked around q : such a system may, phenomenologically, be assigned classical variable with value q . Ensures usual scheme works as phenomenological book-keeping device. Solutions of linear equation conveniently classified in terms of linear vector space.
- **Literal identification of eigenvalues with real physical quantities is fundamental error of quantum 'measurement' theory.** 'Observable has value q ' ultimate source of schizophrenia: Ψ with eigenvalue q generally not eigenfunction of 'incompatible' observable (which then apparently has '*no definite value*'). Error from mistaken belief that classical operations measure nonclassical systems. Note the incorrect *Kochen-Specker theorem*: actually a proof that hidden variable theories must be *contextual* but which was advertised as proof of their non-existence (see 'spin').

As Einstein foresaw in 1926: 'Your theory will one day get you into hot water', because 'when it comes to observation, you behave as if everything can be left as it was, that is, as if you could use the old descriptive language.'

Spin

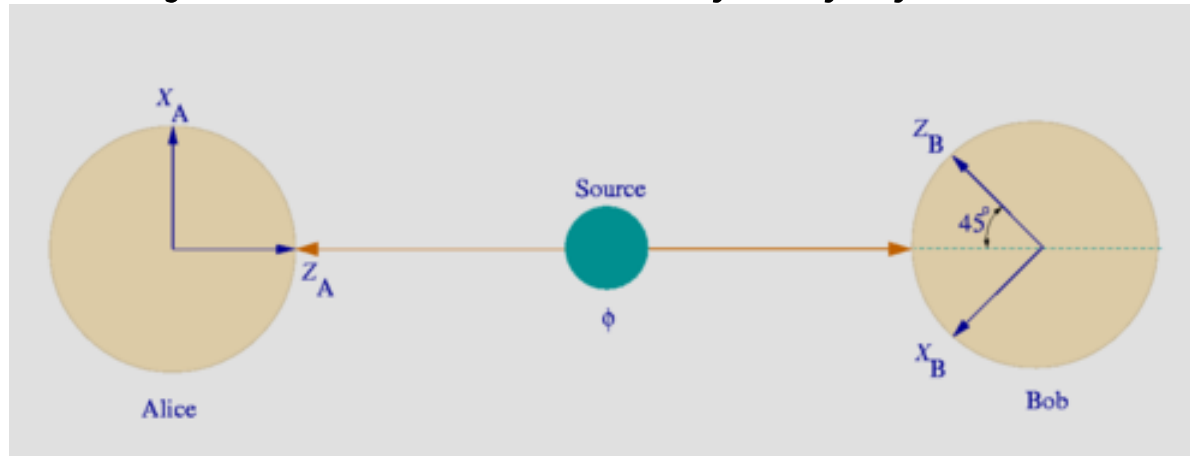


Pilot waves also make sense for particles with spin, i.e., for particles whose wave functions are *spinor-valued*. When directed toward Stern-Gerlach magnets, they emerge moving in a discrete set of directions e.g. 2 directions for a $\text{spin-}\frac{1}{2}$ particle, having 2 spin components. **Spin property of wave function not of particles.**

- Magnets designed/oriented so that incident wave packet will, due to Schrödinger evolution, separate into distinct packets - corresponding to spin components of wave function - moving in the discrete set of directions. Particle, depending on initial position, ends up in *one* of these packets. Probability distribution conveniently expressed in terms of quantum-mechanical spin operators (for a $\text{spin-}\frac{1}{2}$ particle given by the Pauli spin matrices).
- Clear from contextuality particles don't carry spin. Imagine $\text{spin-}\frac{1}{2}$ particle coming towards SG device that 'measures spin in z -direction'. Assume initial position such that it emerges from 'spin-up' port. Now mentally rotate SG apparatus 180° so **B**-field points in opposite direction and repeat. According to standard QM still a device that 'measures z -component of spin', but pilot-wave approach says particle now comes out of 'spin down' port (i.e. same direction as before).

Nonlocality

Definition: *a direct influence of one object on another, distant object, contrary to our expectation that an object is influenced directly only by its immediate surroundings.*



What Einstein-Podolsky-Rosen (EPR) experiment implies:

- Result of measurement on one side immediately predicts result on other (for parallel analyzers).
- If do not believe one side can have causal influence on the other, then require that results on both sides are determined in advance. But this has implications for non-parallel settings which conflict with quantum mechanics (*Bell*).

Thus Bell's analysis showed that *any account* of quantum phenomena needs to be nonlocal, not just any 'hidden variables' account i.e. nonlocality is implied by the predictions of standard quantum theory itself. Thus, if nature is governed by these predictions (which it is, according to real experiments) then nature is nonlocal.

Nonlocality and pilot waves

- Nonlocality of pilot-wave theory solely from nonlocality built into structure of standard QM, as provided by wave function on configuration space, an abstraction which combines or binds distant particles into a single irreducible reality.
- Velocity from guiding equation of any particle of a many-particle system will typically depend upon the positions of the other, possibly distant, particles whenever wave function of the system is *entangled*, i.e., not a product of single-particle wave functions.
- All usual results (such as EPR) follow from pilot-wave mathematical analysis using the concept of the 'conditional wave function' (Lecture 5).
- Only problem therefore acceptance of nonlocality without necessarily knowing 'mechanism'. Concept perfectly rational since leads to neither internal logical contradiction nor disagreement with any facts. Can still isolate systems sufficiently to study them, and nonlocal effects not significant at the large-scale level.

Doesn't violate special relativity: no *controllable* observable consequences (if in quantum equilibrium! - Valentini). In pilot-wave field theory on flat spacetime, can say subquantum nonlocality acts instantaneously across true 3-space, defining an absolute simultaneity and a true time t . In equilibrium, nonlocality and preferred rest frame invisible. Absolute uniform motion of physical reference frame undetectable as moving experimenter may say at rest by using Lorentz-transformed variables. 'Lorentz invariance' property of *equilibrium dynamics* not of space and time. Hmmm!

Implicate order and metaphysics

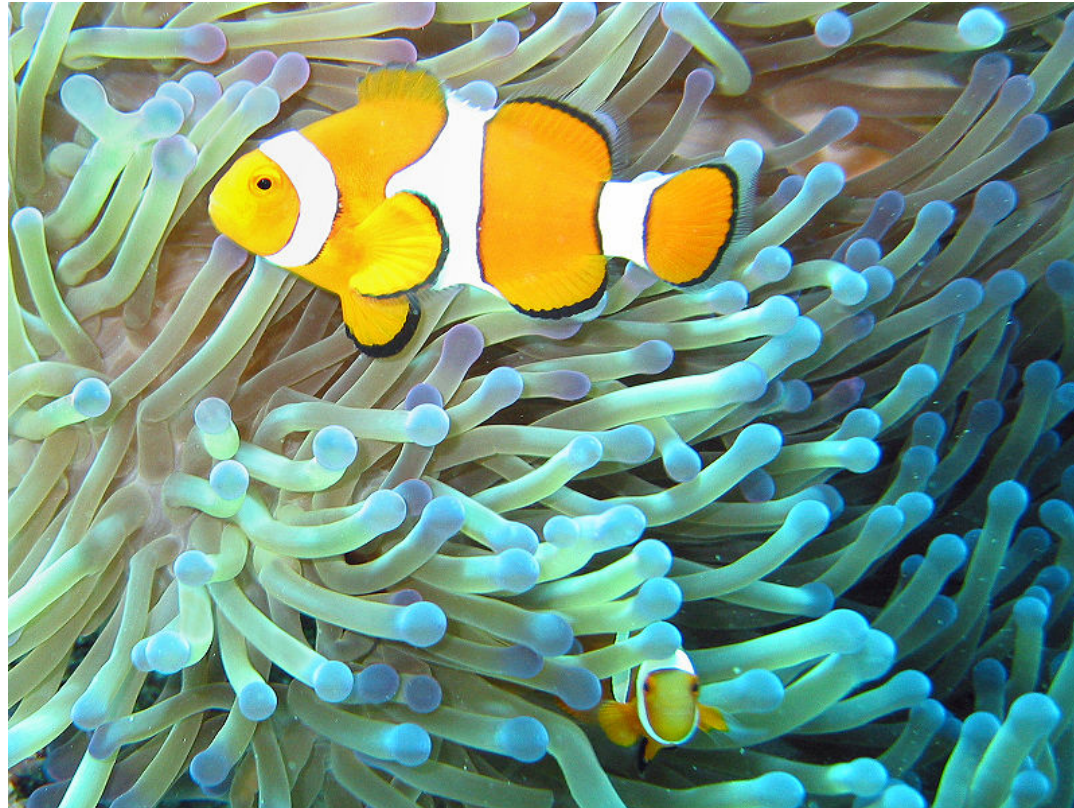
See Bohm's book '*Wholeness and the Implicate Order*' (1983) and Lecture 8.

- Local notion of order by means of a Cartesian grid (extended where necessary to curvilinear) is inadequate in the quantum domain, so Bohm suggests new notion of order, somewhat analagous to using a hologram to make an image, to replace the old, which is analagous to using a lens.
- Each region of the hologram makes possible an image of the whole object. The hologram does not look like the object, but gives rise to an image only when suitably illuminated. The whole object is 'enfolded' in each part of the hologram rather than being in point-to-point correspondence. Order in the hologram = *implicate*. Order in the object and image = *explicate*.
- Order of the whole universe is enfolded into each region. Suggestion that implicate order will have the kind of general necessity that is suitable for expressing the basic laws of physics, while the explicate order (what you see) is important only as a particular case.
- Implicit in Green's function! : $\psi(x, t) = \int G(x - x', t - t') \psi(x', t') dx'$.

'Undivided Wholeness in Flowing Movement'

Bohm popular with 'nutters'

Bohm's metaphysical inclinations have led to existence of huge 'Bohm industry' involving Eastern philosophy, consciousness, life, wholeness etc. See e.g. Friedman's *Bridging Science and Spirit* book - explores parallels between Bohm's thinking and that of well-known extra-dimensional entity Seth. This stuff is actually rather fun!

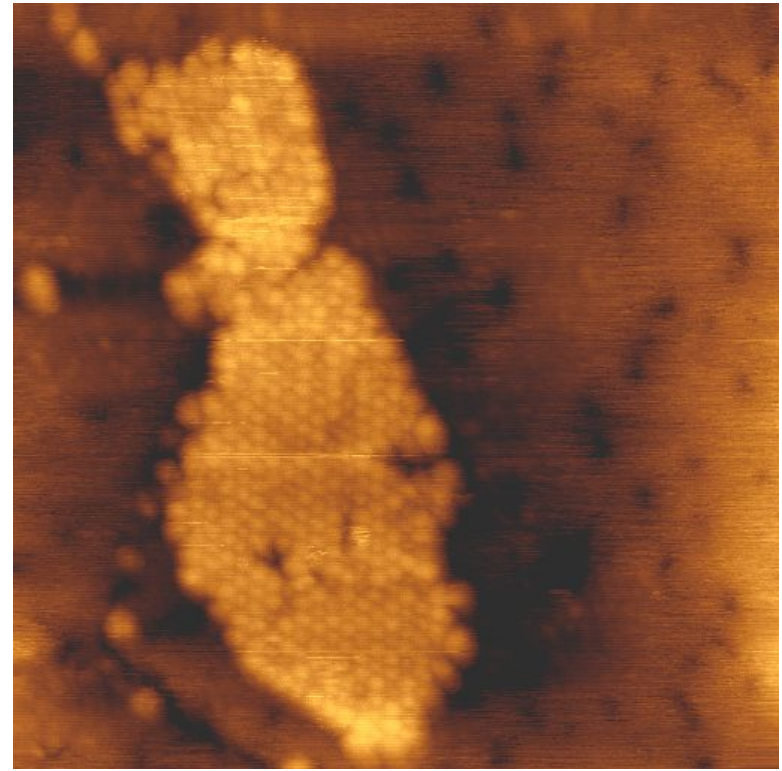
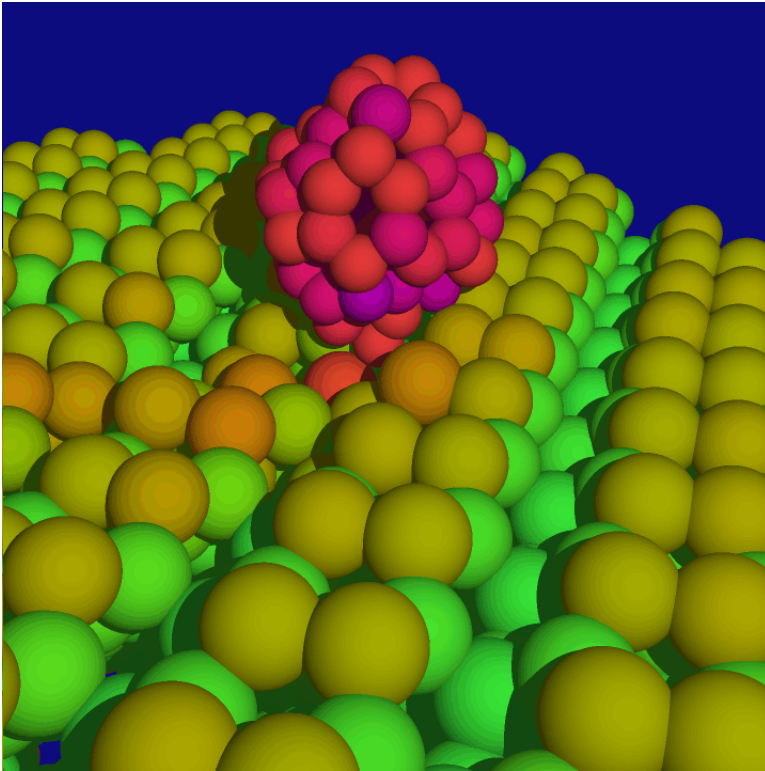


For Bohm, life is continuous flowing process of enfoldment and unfoldment involving relatively autonomous entities. DNA 'directs' environment to form living things. Life can be said to be implicate in ensembles of atoms that ultimately form life.

The best things about pilot waves

- Minimum benefit of pilot-wave theory, independent of debate about whether preferable formulation, is disproof of claim that QM implies particles can't exist before being measured. Since based on familiar concepts of realism, pilot waves give simple, intuitive, non-mystical, and natural answers to what in standard theory are perplexing philosophical questions.
- Pilot-wave approach gives better formulation of QM since defined more precisely than Copenhagen, which is based on theorems not expressed in precise mathematical terms but in words. Pilot waves subsume quantum concepts of measurement, complementarity, decoherence, and entanglement into mathematically precise guidance conditions on position variables.
- Like standard QM it implies existence of EPR type of nonlocality, but it avoids need for positing other types of alleged nonlocality, such as wave collapse; and also avoids need for splitting universes as in many-worlds (*utter madness!*).
- Can derive time-dependent wave function from trajectories (Lecture 6). Implies we are not dealing just with another interpretation or 'superfluous ideological superstructure', but rather with an alternative mathematical representation or picture of QM, equivalent in status to, say, Feynman's path-integral method.
- Can treat e.g. dwell/tunneling times, escape times/escape positions, scattering theory, quantum chaos. Difficult or impossible to analyze in pure wave model.

Molecular dynamics



- Experimentalists have been able to ‘diffract’ 60 C-atom fullerene molecules passing through small holes, so such objects definitely fall on the quantum side of the classical-quantum boundary.
- The TCM group in Cambridge is a leading centre for doing molecular dynamics (e.g. with the CASTEP code). So if trajectories don’t exist, and particles don’t have objective reality, what exactly do we imagine our molecular dynamics movies are depicting?

The Last Word



"This isn't right. It isn't even wrong." (Pauli)

Guess who he was referring to?



"..We consider it juvenile deviationism .. we don't waste our time ... [by] actually read[ing] the paper."

"..[Bohm] is a public nuisance.."

"..[Bohm] is a Trotskyite and a traitor."

"..to hope for hidden variables is as ridiculous as hoping that $2 \times 2 = 5$."

"Attempts have been made by Broglie, David Bohm, and others to construct theories based on hidden variables, but the theories are very complicated [!!!] and contrived. For example, the electron would definitely have to go through only one slit in the two-slit experiment. To explain that interference occurs only when the other slit is open, it is necessary to postulate a special force on the electron which exists only when that slit is open. Such artificial additions make hidden variable theories unattractive, and there is little support for them among physicists". [Encyclopedia Britannica (2007)]

Rest of the course

Lecture 2: 28th January 2009

Pilot-waves and the classical limit. Derivation and justification of the theory

Lecture 3: 4th February 2009

Elementary wave mechanics and pilot waves, with nice examples

Lecture 4: 11th February 2009

The theory of measurement and the origin of randomness

Lecture 5: 18th February 2009

Nonlocality, relativistic spacetime and quantum equilibrium

Lecture 6: 25th February 2009

Calculating things with quantum trajectories

Lecture 7: 4th March 2009

Not even wrong. Why does nobody like pilot-wave theory?

Lecture 8: 11th March 2009

Bohmian metaphysics : the implicate order and other arcana

Followed by a GENERAL DISCUSSION.

Slides/references on web site: www.tcm.phy.cam.ac.uk/~mdt26/pilot_waves.html

If I have provoked any curiosity today, please feel free to attend!